CONTROL SYSTEMS - 27/10/2021

[time 2 hours and 30 minutes; no textbooks; no programmable calculators]

1) Given the plant

$$\mathbf{P}(s) = \frac{1}{s^2(s-1)}$$

design a minimal dimensional controller $\mathbf{G}(s)$ such that

(i) the feedback system $\mathbf{W}(s) = \frac{\mathbf{PG}(s)}{1+\mathbf{PG}(s)}$ is asymptotically stable (check with Nyquist criterion)

(iii) the open-loop system $\mathbf{PG}$ has phase margin $m_\phi \geqslant 40°$.

For the feedback system $\mathbf{W}(s)$ calculate the steady-state error to an input $\mathbf{v}(t) = t$.

2) Given

$$\mathbf{P}(s) = \frac{s+2}{s-1}$$

design a minimal dimensional controller such that the feedback system $\mathbf{W}(s) = \frac{\mathbf{PG}(s)}{1+\mathbf{PG}(s)}$ has

- zero steady-state ouput response to additive output disturbances $\mathbf{d}(t) = t$

- poles with real part $\leqslant -4$.

Draw the root locus of $\mathbf{PG}(s)$.

3) For the system

$$\begin{aligned} \dot{\mathbf{x}}_1 &= -\mathbf{x}_1 + \mathbf{x}_2 + \mathbf{u} \\ \dot{\mathbf{x}}_2 &= -\mathbf{x}_2 \\ \mathbf{y} &= \mathbf{x}_1 \end{aligned} \tag{1}$$

i) compute the forced output response $\mathbf{y}(t)$ and, if it exists, the steady-state output response $\mathbf{y}^{(ss)}(t)$ to the input $\mathbf{u}(t) = \sin t$

ii) compute an input $\mathbf{u}(t)$ such that $\mathbf{y}(T) = 0$ with initial condition $\mathbf{x}(0) = (1,0)^\top$ and $T = 1$ sec.