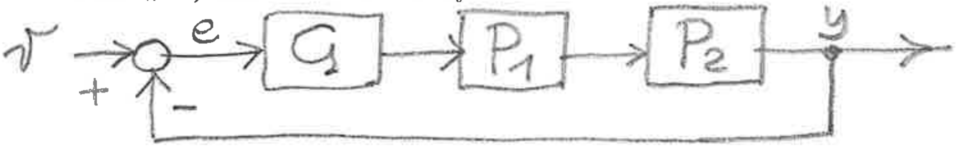
CONTROL SYSTEMS - 5/6/2023

[time 3 hours; no textbooks; no programmable calculators; all the mathematical passages must be motivated and clearly explained]

Ex. # 1) Given the feedback system



with $\mathbf{P}_1(s) = \frac{1}{s-1}$ and $\mathbf{P}_2(s) = \frac{1}{s+1}$, design a one-dimensional proper controller $\mathbf{G}(s)$ such that

(i) the feedback system $\mathbf{W}(s) = \frac{\mathbf{P}_1\mathbf{P}_2\mathbf{G}(s)}{1+\mathbf{P}_1\mathbf{P}_2\mathbf{G}(s)}$ is asymptotically stable (use Nyquist criterion for stability) with steady-state error $|\mathbf{e}_{ss}(t)| \leqslant 1$

(ii) $|G(j\omega)|_{dB} \leqslant 26dB$ for all ω

(iii) the open-loop system $\mathbf{P}_1\mathbf{P}_2\mathbf{G}$ has maximal phase margin (give a detailed and well-motivated explanation, providing the final value of the phase margin).

For the feedback system $\mathbf{W}(s)$ calculate the steady-state error $\mathbf{e}_{ss}(t)$ to a unit step input $\mathbf{v}(t)$.

Ex. # 2) Given the feedback system

with $\mathbf{P}(s) = \frac{s^2+b_1s+b_2}{s^3+1}$, $b_1, b_2 \in \mathbb{R}$,

(i) determine for which values b_1, b_2 there does not exist a controller $\mathbf{G}(s)$ such that the feedback system $\mathbf{W}(s) = \frac{\mathbf{PG}(s)}{1+\mathbf{PG}(s)}$ is (internally) asymptotically stable with steady state error $\mathbf{e}_{ss}(t) \equiv 0$ to constant inputs $\mathbf{v}(t)$

(ii) set $b_1 = 3$ and $b_2 = 2$ and draw accurately the root locus of $\mathbf{P}(s)$

(iii) with b_1, b_2 as in point **(ii)** design a controller $\mathbf{G}(s)$ with minimal dimension such that $\mathbf{W}(s) = \frac{\mathbf{PG}(s)}{1+\mathbf{PG}(s)}$ is asymptotically stable with steady state error $\mathbf{e}_{ss}(t) \equiv 0$ to constant inputs $\mathbf{v}(t)$.

Ex. # 3) Given the system $\mathbf{P} : \dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u},\ \mathbf{y} = C\mathbf{x}$ with

$$A = \begin{pmatrix} -1 & 1 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & -2 \end{pmatrix},\ B = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}, C = \begin{pmatrix} 0 & 3 & 0 \end{pmatrix},$$

(i) check instability of $\mathbf{P}$

(ii) does there exist a controller $\mathbf{u} = F\mathbf{x} + \mathbf{v}$ such that the closed-loop system is asymptotically stable? if yes, determine F

(iii) does there exist a controller $\mathbf{u} = K\mathbf{y} + \mathbf{v}$ such that the closed-loop system is asymptotically stable? if yes, determine K. Is it possible to determine this K with the help of the root locus of $\mathbf{P}(s)$? Motivate the answer

(iii) does there exist a controller $\mathbf{u} = F\hat{\mathbf{x}} + \mathbf{v}$, $\dot{\hat{\mathbf{x}}} = \mathrm{G}\hat{\mathbf{x}} + \mathrm{Q}\mathbf{y}$ such that the closed-loop system is asymptotically stable? if yes, determine F, G, Q.