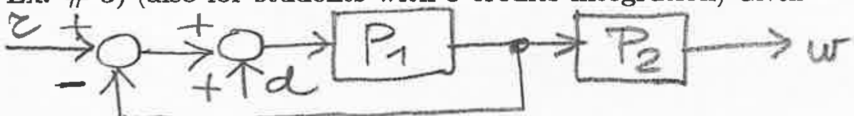
CONTROL SYSTEMS (b) - 8/1/2024

[time 3 hours; no textbooks or notes; no programmable calculators; all the mathematical passages must be motivated and clearly explained]

Ex. # 1) Draw accurately the root locus of $\mathbf{P}(s) = \frac{(s+\frac{2}{3})(s+\frac{4}{3})}{(s-3)^2(s+9)}$, using the Routh table for determining the crossing points of the imaginary axis (there is a singular point at $s = -1$). Moreover,
(i) design a controller $\mathbf{G}(s) = K$ such that the feedback system $\frac{\mathbf{G}(s)\mathbf{P}(s)}{1+\mathbf{G}(s)\mathbf{P}(s)}$ is asymptotically stable with all negative real poles
(ii) design a one-dimensional controller $\mathbf{G}(s)$ such that the feedback system $\frac{\mathbf{G}(s)\mathbf{P}(s)}{1+\mathbf{G}(s)\mathbf{P}(s)}$ is asymptotically stable with all real poles $< -\frac{4}{3}$
(iii) design a two-dimensional controller $\mathbf{G}(s)$ such that the feedback system $\frac{\mathbf{G}(s)\mathbf{P}(s)}{1+\mathbf{G}(s)\mathbf{P}(s)}$ is asymptotically stable with all real poles < -10
(iv) design a two-dimensional controller $\mathbf{G}(s)$ such that the feedback system $\frac{\mathbf{G}(s)\mathbf{P}(s)}{1+\mathbf{G}(s)\mathbf{P}(s)}$ is asymptotically stable with all real negative poles and steady-state error $\mathbf{e}_{ss}(t) = 0$ to inputs $\mathbf{v}(t) = \cos(t)$.

Ex. # 2) With $\mathbf{P}(s) = -\frac{1}{s(10s+1)}$ design a controller $\mathbf{G}(s)$ such that
(i) the feedback system $\frac{\mathbf{G}(s)\mathbf{P}(s)}{1+\mathbf{G}(s)\mathbf{P}(s)}$ is asymptotically stable (use Nyquist criterion for proving stability) with steady-state error $|\mathbf{e}_{ss}(t)| \leqslant 0.1$ to inputs $\mathbf{v}(t) = t$
(ii) the open-loop system $\mathbf{G}(s)\mathbf{P}(s)$ has crossover frequency $\omega_t^* = 1$ rad/sec and phase margin $m_\phi^* \geqslant 40°$
(iii) $\mathbf{G}(s)$ has minimal dimension and $|\mathbf{G}(j\omega)|_{dB} \leqslant 36$ dB for all $\omega \geqslant 0$.

Ex. # 3) **(also for students with 3-credits integration)** Given



with $\mathbf{P}_1(s) = \frac{s+2}{s+1}$ and $\mathbf{P}_2(s) = \frac{1}{s+2}$
(i) determine a state space realization S with two states $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2)^\top$, input $\mathbf{r}$, disturbance $\mathbf{d}$ and output $\mathbf{w}$.
For S with $\mathbf{d} = 0$:
(ii) compute the set of reachable states from the origin with control $\mathbf{r}$; determine if $x_f = (1, 1 + \frac{1}{e})^\top$ is reachable from $x_0 = (0, 1)^\top$ and, if yes, compute the corresponding control input function $\mathbf{r}(t)$
(iii) **(only for students with 3-credits integration)** find the controllable and observable subsystems.