Markov Abstractions for PAC Reinforcement Learning in Non-Markov Decision Processes

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Abstract

Our work aims at developing reinforcement learning algorithms that do not rely on the Markov assumption. We consider the class of Non-Markov Decision Processes where histories can be abstracted into a finite set of states while preserving the dynamics. We call it a *Markov abstraction* since it induces a Markov Decision Process over a set of states that encode the non-Markov dynamics. This phenomenon underlies the recently introduced Regular Decision Processes (as well as POMDPs where only a finite number of belief states is reachable). In all such kinds of decision process, an agent that uses a Markov abstraction can rely on the Markov property to achieve optimal behaviour. We show that Markov abstractions can be learned during reinforcement learning. Our approach combines automata learning and classic reinforcement learning. For these two tasks, standard algorithms can be employed. We show that our approach has PAC guarantees when the employed algorithms have PAC guarantees, and we also provide an experimental evaluation.

1 Introduction

In the classic setting of Reinforcement Learning (RL), the agent is provided with the current *state* of the environment [Sutton and Barto, 2018]. States are a useful abstraction for agents, since predictions and decisions can be made according to the current state. This is RL under the Markov assumption, or *Markov RL*. Here we focus on the more realistic *non-Markov RL* setting [Hutter, 2009; Brafman and De Giacomo, 2019; Icarte *et al.*, 2019; Ronca and De Giacomo, 2021] where the agent is not given the current state, but can observe what happens in response to its actions. The agent can still try to regain the useful abstraction of states. However, now the abstraction has to be learned, as a map from every *history* observed by the agent to some *state* invented by the agent [Hutter, 2009].

We propose RL agents that learn a specific kind of abstraction called *Markov abstraction* as part of the overall learning process. Our approach combines automata learning and Markov RL in a modular manner, with Markov abstractions acting as an interface between the two modules. A key aspect of our contribution is to show how the sequence of intermediate automata built during learning induce partial Markov abstractions that can be readily used to guide exploration or exploitation. Our approach is Probably Approximately Correct (PAC), cf. [Kearns and Vazirani, 1994], whenever the same holds for the employed automata and Markov RL algorithms.

The idea of solving non-Markov tasks by introducing a Markovian state space can already be found in [Bacchus *et al.*, 1996] and more recently in [Brafman *et al.*, 2018; Icarte *et al.*, 2018]. RL in this setting has been considered [Icarte *et al.*, 2018; De Giacomo *et al.*, 2019; Gaon and Brafman, 2020; Xu *et al.*, 2020]. This setting is simpler than ours since transitions are still assumed to be Markov. The setting where both transitions and rewards are non-Markov has been considered in [Icarte *et al.*, 2019; Brafman and De Giacomo, 2019], with RL studied in [Icarte *et al.*, 2019; Abadi and Brafman, 2020; Ronca and De Giacomo, 2021]. Such RL techniques are based on automata learning. The approach in [Ronca and De Giacomo, 2021] comes with PAC guarantees, as opposed to the others which do not. Our approach extends the techniques in [Ronca and De Giacomo, 2021] in order to use the learned automata not only to construct the final policy, but also to guide exploration.

Abstractions from histories to states have been studied in [Hutter, 2009; Maillard *et al.*, 2011; Veness *et al.*, 2011; Nguyen *et al.*, 2013; Lattimore *et al.*, 2013; Hutter, 2016; Majeed and Hutter, 2018]. [Hutter, 2009] introduces the idea of abstractions from histories to states. Its algorithmic solution, as well as the one in [Veness *et al.*, 2011], is less general than ours since it assumes a bound on the length of the histories to consider. This corresponds to a subclass of automata. [Maillard *et al.*, 2011] provides a technique to select an abstraction from a given finite set of candidate abstractions; instead, we consider an infinite set of abstractions. [Nguyen *et al.*, 2013] considers a set of abstractions without committing to a specific way of representing them. As a consequence, they are not able to take advantage of specific properties of the chosen representation formalism, in the algorithm nor in the analysis. On the contrary, we choose automata, which allows us to take advantage of existing automata-learning techniques, and in particular of their properties such as the incremental construction. [Lattimore *et al.*, 2013] studies non-Markov RL in the case where the non-Markov decision process belongs to a compact class. Their results do not apply to our case because the class of decision processes admitting a Markov abstraction is not compact. [Majeed and Hutter, 2018] studies Q-learning with abstractions, but it assumes that abstractions are given. [Hutter, 2016] provides a result that we use in Section 3; however, none of their abstractions is required to preserve the dynamics, as we require for our Markov abstractions. Even in the case called 'exact state aggregation', their abstractions are only required to preserve rewards, and not to preserve observations. In this setting, it is unclear whether automata techniques apply.

Proofs and experimental details are given in the extended version [Ronca *et al.*, 2022].

2 Preliminaries

For x and z strings, xz denotes their concatenation. For Σ and Γ alphabets, $\Sigma\Gamma$ denotes the set of all strings $\sigma\gamma$ with $\sigma \in \Sigma$ and $\gamma \in \Gamma$. For f and g functions, fg denotes their composition. We write $f : X \rightsquigarrow Y$ to denote a function $f : X \times Y \to [0, 1]$ that defines a probability distribution $f(\cdot|x)$ over Y for every $x \in X$.

Non-Markov Decision Processes. A *Non-Markov Decision Process* (NMDP), cf. [Brafman and De Giacomo, 2019], is a tuple $\mathcal{P} = \langle A, O, R, \zeta, \mathbf{T}, \mathbf{R}\rangle$ with components defined as follows. A is a finite set of *actions*, O is a finite set of *observations*, $R \subseteq \mathbb{R}_{\geq 0}$ is a finite set of non-negative *rewards*, ζ is a special symbol that denotes *episode termination*. Let the elements of $\mathcal{H} = (AOR)^*$ be called *histories*, and let the elements of $\mathcal{E} = (AOR)^* A\zeta$ be called *episodes*. Then, $\mathbf{T} : \mathcal{H} \times A \rightsquigarrow (O \cup \{\zeta\})$ is the *transition function*, and $\mathbf{R} : \mathcal{H} \times A \times O \rightsquigarrow R$ is the *reward function*. The transition and reward functions can be combined into the *dynamics function* $\mathbf{D} : \mathcal{H} \times A \rightsquigarrow (OR \cup \{\zeta\})$, which describes the probability to observe next a certain pair of observation and reward, or termination, given a certain history and action. Namely, $\mathbf{D}(or|h, a) = \mathbf{T}(o|h, a) \cdot \mathbf{R}(r|h, a, o)$ and $\mathbf{D}(\zeta|h, a) = \mathbf{T}(\zeta|h, a)$. We often write an NDMP directly as $\langle A, O, R, \zeta, \mathbf{D}\rangle$. A *policy* is a function $\pi : \mathcal{H} \rightsquigarrow A$. The *uniform policy* π_{u} is the policy defined as $\pi_{\mathrm{u}}(a|h) = 1/|A|$ for every a and every h. The *dynamics of* $\mathcal{P}$ *under a policy* π describe the probability of an episode remainder, given the history so far, when actions are chosen according to a policy π; it can be recursively computed as $\mathbf{D}_\pi(aore|h) = \pi(a|h) \cdot \mathbf{D}(or|h, a) \cdot \mathbf{D}_\pi(e|haor)$, with base case $\mathbf{D}_\pi(a\zeta|h) = \pi(a|h)\cdot\mathbf{D}(\zeta|h, a)$. Since we study episodic reinforcement learning, we require episodes to terminate with probability one, i.e., $\sum_{e\in\mathcal{E}} \mathbf{D}_\pi(e|\varepsilon) = 1$ for every policy π.[1] This requirement ensures that the following value functions take a finite value. The *value of a policy* π *given a history* h, written $\mathbf{v}_\pi(h)$, is the expected sum of future rewards when actions are chosen according to π given that the history so far is h; it can be recursively computed as $\mathbf{v}_\pi(h) = \sum_{aor} \pi(a|h)\cdot\mathbf{D}(o, r|h, a)\cdot(r+\mathbf{v}_\pi(haor))$. The *optimal value given a history* h is $\mathbf{v}_*(h) = \max_\pi \mathbf{v}_\pi(h)$, which can be expressed without reference to any policy as $\mathbf{v}_*(h) = \max_a \left(\sum_{or} \mathbf{D}(o, r|h, a) \cdot (r + \mathbf{v}_*(haor))\right)$. The *value of an action* a *under a policy* π *given a history* h, written $\mathbf{q}_\pi(h, a)$, is the expected sum of future rewards when the next action is a and the following actions are chosen according to π, given that the history so far is h; it is $\mathbf{q}_\pi(h, a) = \sum_{or} \mathbf{D}(o, r|h, a) \cdot (r+\mathbf{v}_\pi(haor))$. The *optimal value of an action* a *given a history* h is $\mathbf{q}_*(h, a) = \max_\pi \mathbf{q}_\pi(h, a)$, and it can be expressed as $\mathbf{q}_*(h, a) = \sum_{or} \mathbf{D}(o, r|h, a) \cdot (r + \mathbf{v}_*(haor))$. A policy π is *optimal* if $\mathbf{v}_\pi(\varepsilon) = \mathbf{v}_*(\varepsilon)$. For $\epsilon > 0$, a policy π is ϵ*-optimal* if $\mathbf{v}_\pi(\varepsilon) \geq \mathbf{v}_*(\varepsilon) - \epsilon$.

Markov Decision Processes. A *Markov Decision Process (MDP)* [Bellman, 1957; Puterman, 1994] is a decision process where the transition and reward functions (and hence the dynamics function) depend only on the last observation in the history, taken to be an arbitrary observation when the history is empty. Thus, an observation is a complete description of the state of affairs, and it is customarily called a *state* to emphasise this aspect. Hence, we talk about a set S of states in place of a set O of observations. All history-dependent functions—e.g., transition and reward functions, dynamics, value functions, policies—can be seen as taking a single state in place of a history.

Episodic RL. Given a decision process $\mathcal{P}$ and a required accuracy $\epsilon > 0$, Episodic Reinforcement Learning (RL) for $\mathcal{P}$ and ϵ is the problem of an agent that has to learn an ϵ-optimal policy for $\mathcal{P}$ from the data it collects by interacting with the environment. The interaction consists in the agent iteratively performing an action and receiving an observation and a reward in response, until episode termination. Specifically, at step i, the agent performs an action a_i, receiving a pair of an observation o_i and a reward r_i, or the termination symbol ζ, according to the dynamics of the decision process $\mathcal{P}$. This process generates an episode of the form $a_1o_1r_1a_2o_2r_2 \dots a_n\zeta$. The collection of such episodes is the data available to the agent for learning.

Complexity Functions. Describing the behaviour of a learning algorithm requires a way to measure the complexity of the instance $\mathcal{I}$ to be learned, in terms of a set of numeric parameters that reasonably describe the complexity of the instance—e.g., number of states of an MDP. Thus, for every instance $\mathcal{I}$, we have a list $\vec{c}_\mathcal{I}$ of values for the parameters. We use the complexity description to state properties for the algorithm by means of complexity functions. A *complexity function* f is an integer-valued non-negative function of the complexity parameters and possibly of other parameters. Two distinguished parameters are accuracy $\epsilon > 0$, and probability of failure $\delta \in (0, 1)$. The dependency of complexity functions on these parameters is of the form $1/\epsilon$ and $\ln(1/\delta)$. When writing complexity functions, we will hide the specific choice of complexity parameters by writing $\mathcal{I}$ instead of $\vec{c}_\mathcal{I}$, and we will write ϵ and δ for $1/\epsilon$ and $\ln(1/\delta)$. For instance, we will write $f(\mathcal{I}, \epsilon, \delta)$ in place of $f(\vec{c}_\mathcal{I}, 1/\epsilon, \ln(1/\delta))$.

Automata. A *Probabilistic Deterministic Finite Automaton (PDFA)*, cf. [Balle *et al.*, 2014], is a tuple $\mathcal{A} = \langle Q, \Sigma, \Gamma, \tau, \lambda, q_0\rangle$ where: Q is a finite set of states; Σ is a finite alphabet; Γ is a finite set of termination symbols not in Σ; $\tau : Q \times \Sigma \to Q$ is the (deterministic) transition function; $\lambda : Q \times (\Sigma \cup \Gamma) \to [0, 1]$ is the probability function, which defines

[1] A constant probability p of terminating at every step amounts to a discount factor of $1 - p$, see [Puterman, 1994].

a probability distribution $\lambda(\cdot|q)$ over $\Sigma \cup \Gamma$ for every state $q \in Q$; $q_0 \in Q$ is the initial state. The iterated transition function $\tau^* : Q \times \Sigma^* \to Q$ is recursively defined as $\tau^*(q, \sigma_1 \dots \sigma_n) = \tau^*(\tau(q, \sigma_1), \sigma_2 \dots \sigma_n)$ with base case $\tau^*(q, \varepsilon) = q$; furthermore, $\tau^*(w)$ denotes $\tau^*(q_0, w)$. It is required that, for every state $q \in Q$, there exists a sequence $\sigma_1, \dots, \sigma_n$ of symbols of Σ such that $\lambda(\tau^*(q, \sigma_1 \dots \sigma_{i-1}), \sigma_i) > 0$ for every $i \in [1, n]$, and $\lambda(\tau(q, \sigma_1 \dots \sigma_n), \gamma) > 0$ for $\gamma \in \Gamma$—to ensure that every string terminates with probability one. Given a string $x \in \Sigma^*$, the automaton represents the probability distribution $\mathcal{A}(\cdot|x)$ over $\Sigma^*\Gamma$ defined recursively as $\mathcal{A}(\sigma y|x) = \lambda(\tau(x), \sigma) \cdot \mathcal{A}(y|x\sigma)$ with base case $\mathcal{A}(\gamma|x) = \lambda(\tau(x), \gamma)$ for $\gamma \in \Gamma$.

3 Markov Abstractions

Operating directly on histories is not desirable. There are exponentially-many histories in the episode length, and typically each history is observed few times, which does not allow for computing accurate statistics. A solution to this problem is to abstract histories to a reasonably-sized set of states while preserving the dynamics. We fix an NMDP $\mathcal{P} = \langle A, O, R, \zeta, \mathbf{D} \rangle$.

Definition 1. *A Markov abstraction over a finite set of states S is a function $\alpha : \mathcal{H} \to S$ such that, for every two histories h_1 and h_2, $\alpha(h_1) = \alpha(h_2)$ implies $\mathbf{D}(\cdot|h_1, a) = \mathbf{D}(\cdot|h_2, a)$ for every action a.*

Given an abstraction α, its repeated application α^* transforms a given history by replacing observations by the corresponding states as follows:

$$\alpha^*(a_1 o_1 r_1 \dots a_n o_n r_n) = s_0 a_1 s_1 r_1 \dots a_n s_n r_n,$$
$$\text{where } s_i = \alpha(h_i) \text{ and } h_i = a_1 o_1 r_1 \dots a_i o_i r_i.$$

Now consider the probability $P_\alpha(s_i, r_i | h_{i-1}, a_i)$ obtained by marginalisation as:

$$P_\alpha(s_i, r_i | h_{i-1}, a_i) = \sum_{o : \alpha(h_{i-1} a_i o) = s_i} \mathbf{D}(o, r_i | h_{i-1}, a_i).$$

Since the dynamics $\mathbf{D}(o, r_i | h_{i-1}, a_i)$ are the same for every history mapped to the same state, there is an MDP $\mathcal{M}_\mathcal{P}^\alpha$ with dynamics $\mathbf{D}^\alpha$ such that $P_\alpha(s_i r_i | h_{i-1}, a_i) = \mathbf{D}^\alpha(s_i r_i | \alpha(h_{i-1}), a_i)$. The *induced MDP* $\mathcal{M}_\mathcal{P}^\alpha$ can be solved in place of $\mathcal{P}$. Indeed, the value of an action in a state is the value of the action in any of the histories mapped to that state [Hutter, 2016, Theorem 1]. In particular, if π is an ϵ-optimal policy for $\mathcal{M}_\mathcal{P}^\alpha$, then $\pi\alpha$ is an ϵ-optimal policy for $\mathcal{P}$.

3.1 Related Classes of Decision Processes

We discuss how Markov abstractions relate to existing classes of decision processes.

MDPs. MDPs can be characterised as the class of NMDPs where histories can be abstracted into their last observation. Namely, they admit $\alpha(haor) = o$ as a Markov abstraction.

RDPs. A *Regular Decision Process (RDP)* can be defined in terms of the temporal logic on finite traces LDL_f [Brafman and De Giacomo, 2019] or in terms of finite transducers [Ronca and De Giacomo, 2021]. The former case reduces to

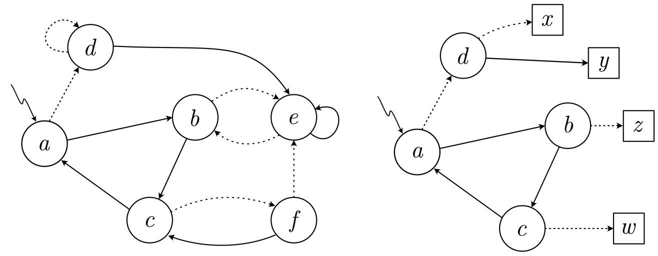


Figure 1: Target automaton (left), and hypothesis automaton (right). Solid edges denote transitions for input 1, and dashed edges for 0.

the latter by the well-known correspondence between LDL_f and finite automata. In terms of finite transducers, an RDP is an NMDP $\mathcal{P} = \langle A, O, R, \zeta, \mathbf{D} \rangle$ whose dynamics function can be represented by a finite transducer T that, on every history h, outputs the function $\mathbf{D}_h : A \rightsquigarrow (OR \cup \{\zeta\})$ induced by $\mathbf{D}$ when its first argument is h. Here we observe that the iterated version of the transition function of T is a Markov abstraction.

POMDPs. A *Partially-Observable Markov Decision Process* (POMDP), cf. [Kaelbling *et al.*, 1998], is a tuple $\mathcal{P} = \langle A, O, R, \zeta, X, \mathbf{T}, \mathbf{R}, \mathbf{O}, x_0 \rangle$ where: A, O, R, ζ are as in an NMDP; X is a finite set of *hidden states*; $\mathbf{T} : X \times A \rightsquigarrow X$ is the *transition function*; $\mathbf{R} : X \times A \times O \rightsquigarrow R$ is the *reward function*; $\mathbf{O} : X \times A \times \rightsquigarrow (O \cup \{\zeta\})$ is the *observation function*; $x_0 \in X$ is the initial hidden state. To define the dynamics function—i.e., the function that describes the probability to observe next a certain pair of observation and reward, or termination, given a certain history of observations and action—it requires to introduce the *belief function* $\mathbf{B} : \mathcal{H} \rightsquigarrow X$, which describes the probability of being in a certain hidden state given the current history. Then, the dynamics function can be expressed in terms of the belief function as $\mathbf{D}(or|h, a) = \sum_x \mathbf{B}(x|h) \cdot \mathbf{O}(o|x, a) \cdot \mathbf{R}(r|x, a, o)$ and $\mathbf{D}(\zeta|h, a) = \sum_x \mathbf{B}(x|h) \cdot \mathbf{O}(\zeta|x, a)$. Policies and value functions, and hence the notion of optimality, are as for NMDPs. For each history h, the probability distribution $\mathbf{B}(\cdot|h)$ over the hidden states is called a *belief state*. We note the following property of POMDPs.

Theorem 1. *If a POMDP has a finite set of reachable belief states, then the function that maps every history to its belief state is a Markov abstraction.*

Proof sketch. The function $\alpha(h) = \mathbf{B}(\cdot|h)$ is a Markov abstraction, as it can be verified by inspecting the expression of the dynamics function of a POMDP given above. Specifically, $\alpha(h_1) = \alpha(h_2)$ implies $\mathbf{B}(\cdot|h_1) = \mathbf{B}(\cdot|h_2)$, and hence $\mathbf{D}(\cdot|h_1, a) = \mathbf{D}(\cdot|h_2, a)$ for every action a. □

4 Our Approach to Non-Markov RL

Our approach combines automata learning with Markov RL. We first describe the two modules separately, and then present the RL algorithm that combines them. For the section we fix an NMDP $\mathcal{P} = \langle A, O, R, \zeta, \mathbf{D} \rangle$, and assume that it admits a Markov abstraction α on states S.

4.1 First Module: Automata Learning

Markov abstractions can be learned via automata learning due to the following theorem.

Theorem 2. *There exist a transition function $\tau : S \times AOR \to S$ and an initial state s_0 such that, for every Markov policy π on S, the dynamics $\mathbf{D}_{\pi\alpha}$ of $\mathcal{P}$ are represented by an automaton $\langle S, AOR, A\zeta, \tau, \lambda, s_0 \rangle$ for some probability function λ. Furthermore, $\tau^* = \alpha$.*

Proof sketch. The start state is $s_0 = \alpha(\varepsilon)$. The transition function is defined as $\tau(s, aor) = \alpha(h_s aor)$ where h_s is an arbitrarily chosen history such that $\alpha(h_s) = s$. Clearly $\tau^* = \alpha$. Then, the probability function is defined as $\lambda(s, aor) = \pi(a|s) \cdot \mathbf{D}(or|h_s a)$ and $\lambda(s, a\zeta) = \pi(a|s) \cdot \mathbf{D}(\zeta|h_s a)$. It can be shown by induction that the resulting automaton represents $\mathbf{D}_{\pi\alpha}$ regardless of the choice of the representative histories h_s, since all histories mapped to the same state determine the same dynamics function. □

We present an informal description of a generic PDFA-learning algorithm, capturing the essential features of the algorithms in [Ron *et al.*, 1998; Clark and Thollard, 2004; Palmer and Goldberg, 2007; Balle *et al.*, 2013; Balle *et al.*, 2014]. We will highlight the characteristics that have an impact on the rest of the RL algorithm. To help the presentation, consider Figure 1. The figure shows the transition graph of a target automaton (left), and the *hypothesis graph* built so far by the algorithm (right). In the hypothesis graph we distinguish *safe* and *candidate* nodes. Safe nodes are circles in the figure. They are in a one-to-one correspondence with nodes in the target, they have all transitions defined, and they are not going to change. Candidate nodes are squares in the figure. Their transitions are not defined, and hence they form the frontier of the learned part of the graph. The graph is extended by *promoting* or *merging* a candidate. If the algorithm tests that a candidate node is distinct from every other safe state in the hypothesis graph, then it is promoted to safe. Upon promotion, a candidate is added for each possible transition from the just-promoted safe node. This effectively extends the automaton by pushing the frontier. If the algorithm tests that a candidate is equivalent to a safe node already in the hypothesis, then the candidate is merged into the safe node. The merge amounts to deleting the candidate node and redirecting all its incoming edges to the safe node. Thus, the statistical core of the algorithm consists in the tests. The test between two nodes is based on the input strings having prefixes that map to the two nodes respectively. A sufficient number of strings yields a good accuracy of the test. Assuming that all tests yield the correct result, it is easy to see that every new hypothesis is closer to the target. We focus on the approach in [Balle *et al.*, 2014], which has the following guarantees.

Guarantees 1. *There are complexity functions K, N, T_s such that, for every δ and every target automaton $\mathcal{A}$, the automata learning algorithm builds a sequence of hypotheses $\mathcal{A}_1, \dots, \mathcal{A}_n$ satisfying the following conditions:* (soundness) *with probability at least $1-\delta$, for every $i \in [1, n]$, there is a transition-preserving bijection between the safe states of $\mathcal{A}_i$ and a subset of states of the target $\mathcal{A}$;* (incrementality) *for every $i \in [2, n]$, the hypothesis $\mathcal{A}_i$ contains all safe states and all transitions between safe states of $\mathcal{A}_{i-1}$;* (liveness) *every candidate s is either promoted or merged within reading an expected number $K(\mathcal{A}, \delta)$ of strings having a prefix that maps to s;* (boundedness) *the number n of hypotheses is at most $N(\mathcal{A})$;* (computational cost) *every string x is processed in time $T_\mathrm{s}(\mathcal{A}, \delta, |x|)$.*

We will ensure that statistics for a state q are no longer updated once it is promoted to safe. This preserves the guarantees, and makes the algorithm insensitive to changes in the distribution $\lambda(q', \cdot)$ of the target state q' corresponding to q that take place after q is promoted.

Since a hypothesis automaton $\mathcal{A}_i$ contains candidate states, that do not have outgoing transitions, the Markov abstraction α_i obtained as the iteration τ_i^* of its transition function is not a complete Markov abstraction, but a partial one. As a consequence, the MDP $\mathcal{M}_i$ induced by α_i is also partial. Specifically, it contains states from which one cannot proceed, the ones deriving from candidate nodes.

4.2 Second Module: Markov RL

When a Markov RL agent is confined to operate in a subset S' of the states S of an MDP, it can always achieve one of two mutually-exclusive results. The first result is that it can find a policy that is near-optimal for the entire MDP, ignoring the states not in S'. Otherwise, there is a policy that leads to a state not in S' sufficiently often. This is the essence of the Explore or Exploit lemma from [Kearns and Singh, 2002]. The property is used explicitly in the E^3 algorithm [Kearns and Singh, 2002], and more implicitly in algorithms such as RMax [Brafman and Tennenholtz, 2002]. Indeed, these algorithms can be employed to satisfy the following guarantees.

Guarantees 2. *There are complexity functions E, T_e such that, for every ϵ, every δ, and every MDP $\mathcal{M}$ where the agent is restricted to operate in a subset S' the states, the two following conditions hold:* (explore or exploit) *within $E(\mathcal{M}, \epsilon, \delta)$ episodes, with probability at least $1-\delta$, either the agent finds an ϵ-optimal policy or there is a state not in S' that is visited at least once;* (computational cost) *every episode e is processed in time $T_\mathrm{e}(\mathcal{M}, \epsilon, \delta, |e|)$.*

4.3 Overall Approach

We will have a Markov agent operating in the MDP induced by the current partial abstraction, in order to either find a near-optimal policy (when the abstraction is sufficiently complete) or to visit candidate states (to collect strings to extend the abstraction). Concretely, we propose Algorithm 1, that employs the modules `AutomataLearning` and `MarkovRL` to solve non-Markov RL. It starts with an empty abstraction α (Line 1), and then it loops, processing one episode per iteration. Line 3 corresponds to the beginning of an episode, and hence the algorithm sets the current history to the empty history. Lines 4–8 process the prefix of the episode that maps to the safe states; on this prefix, the Markov RL agent is employed. Specifically, the agent is first queried for the action to take (Line 5), the action is then executed (Line 6), the resulting transition is shown to the agent (Line 7), and the current history is extended (Line 8). Lines 9–12 process the remainder of the episode, for which we do not have an abstraction

Algorithm 1: `NonMarkovRL`

1 $\alpha \leftarrow \emptyset$;
2 **loop**
3 $\quad h \leftarrow \varepsilon$;
4 $\quad$ **while** $\alpha(h)$ *is safe* **and** *episode is not over* **do**
5 $\quad\quad a \leftarrow$ `MarkovRL.choose()`;
6 $\quad\quad o, r \leftarrow$ perform action a;
7 $\quad\quad$ `MarkovRL.observe`$(\alpha(h), a, r, \alpha(haor))$;
8 $\quad\quad h \leftarrow haor$;
9 $\quad$ **while** *episode is not over* **do**
10 $\quad\quad a \leftarrow$ pick an action according to π_{u};
11 $\quad\quad o, r \leftarrow$ perform action a;
12 $\quad\quad h \leftarrow haor$;
13 $\quad \alpha \leftarrow$ `AutomataLearning.consume`(h);
14 $\quad$ `MarkovRL.update`(α);

yet. First, an action is chosen according to the uniform policy π_{u} (Line 10), the action is then executed (Line 11), and the current history is extended (Line 12). Lines 13–14 process the episode that has just been generated, before moving to the next episode. In Line 13 the automata learning algorithm is given the episode, and it returns the possibly-updated abstraction α. Finally, in Line 14, the Markov agent is given the latest abstraction, and it has to update its model and/or statistics to reflect any change in the abstraction. We assume a naive implementation of the update function, that amounts to resetting the Markov agent when the given abstraction is different from the previous one.

The algorithm has PAC guarantees assuming PAC guarantees for the employed modules. In particular, let K, N, T_{s} be as in Guarantees 1, and let E, T_{e} be as in Guarantees 2. In the context of Algorithm 1, the target automaton is the automaton $\mathcal{A}^{\mathrm{u}}_{\mathcal{P}}$ that represents the dynamics of $\mathcal{P}$ under the uniform policy π_{u}, and the MDP the Markov agent interacts with is the MDP $\mathcal{M}^{\alpha}_{\mathcal{P}}$ induced by α. Furthermore, let $L_{\mathcal{P}}$ be the expected episode length for $\mathcal{P}$.

Theorem 3. *For any given ϵ and δ, and for any NMDP $\mathcal{P}$ admitting a Markov abstraction α, Algorithm 1 has probability at least $1-\delta$ of solving the Episodic RL problem for $\mathcal{P}$ and ϵ, using a number of episodes*

$$O\big(N(\mathcal{A}^{\mathrm{u}}_{\mathcal{P}}) \cdot \lceil E(\mathcal{M}^{\alpha}_{\mathcal{P}}, \epsilon, \delta')/K(\mathcal{A}^{\mathrm{u}}_{\mathcal{P}}, \delta/2) \rceil\big)$$

with $\delta' = \delta/(2 \cdot N(\mathcal{A}^{\mathrm{u}}_{\mathcal{P}}))$, and a number of computation steps that is proportional to the number of episodes by a quantity

$$O\big(T_{\mathrm{s}}(\mathcal{A}^{\mathrm{u}}_{\mathcal{P}}, \delta/2, L_{\mathcal{P}}) + T_{\mathrm{e}}(\mathcal{M}^{\alpha}_{\mathcal{P}}, \epsilon, \delta', L_{\mathcal{P}})\big).$$

Proof sketch. An execution of the algorithm can be split into stages, with one stage for each hypothesis automaton. By the boundedness condition, there are at most $N(\mathcal{A}^{\mathrm{u}}_{\mathcal{P}})$ stages. Consider an arbitrary stage. Within $E(\mathcal{M}^{\alpha}_{\mathcal{P}}, \epsilon, \delta')$ episodes, the Markov agent either exploits or explores. If it exploits, i.e., it finds an ϵ-optimal policy π for the MDP induced by the current partial abstraction α_i, then $\pi\alpha_i$ is ϵ-optimal for $\mathcal{P}$. Otherwise, a candidate state is explored $K(\mathcal{A}^{\mathrm{u}}_{\mathcal{P}}, \delta/2)$ times, and hence it is promoted or merged. The probability δ of failing is partitioned among the one run of the automata learning algorithm, and the $N(\mathcal{A}^{\mathrm{u}}_{\mathcal{P}})$ independent runs of the Markov RL algorithm. Regarding the computation time, note that the algorithm performs one iteration per episode, it operates on one episode in each iteration, calling the two modules and performing some other operations having a smaller cost. $\square$

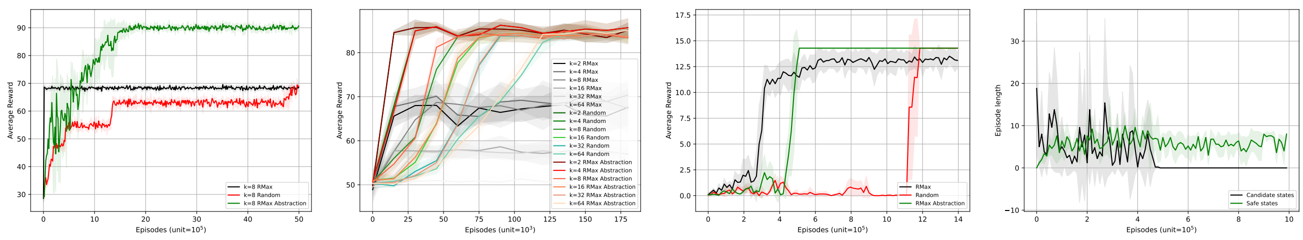
5 Empirical Evaluation

We show an experimental evaluation of our approach.[2] We employ the state-of-the-art stream PDFA learning algorithm [Balle *et al.*, 2014] and the classic RMax algorithm [Brafman and Tennenholtz, 2002]. They satisfy Guarantees 1 and 2. We consider the domains from [Abadi and Brafman, 2020]: Rotating MAB, Malfunction MAB, Cheat MAB, and Rotating Maze; a variant of Enemy Corridor [Ronca and De Giacomo, 2021]; and two novel domains: Reset-Rotating MAB, and Flickering Grid. Reset-Rotating MAB is a variant of Rotating MAB where failing to pull the correct arm brings the agent back to the initial state. Flickering Grid is a basic grid with an initial and a goal position, but where at some random steps the agent is unable to observe its position. All domains are parametrised by k, which makes the domains more complex as it is assigned larger values.

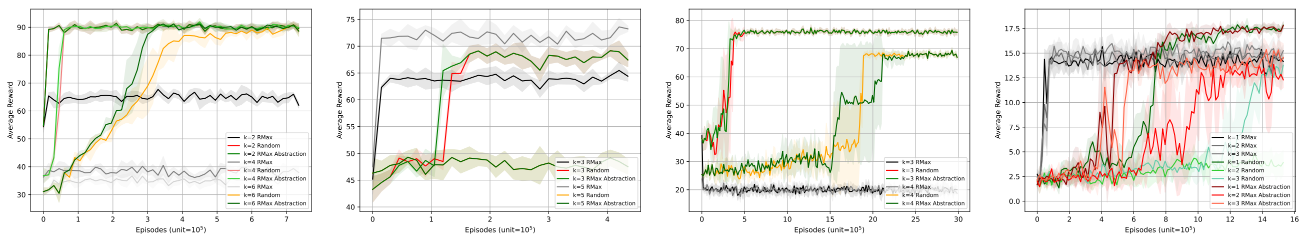
Figure 2 shows our performance evaluation. Plots include three different approaches. First, our approach referred to as *RMax Abstraction*, i.e., the RMax Markov agent provided with partial Markov abstractions as they are incrementally learned. Second, a *Random Sampling* approach, equivalent to what is proposed in [Ronca and De Giacomo, 2021], that always explores at random. Third, the RMax agent as a baseline for the performance of a purely Markov agent, that does not rely on Markov abstractions. Results for each approach are averaged over 5 trainings and show the standard deviation. At each training, the agent is evaluated at every 15k training episodes. Each evaluation is an average over 50 episodes, where for each episode we measure the accumulated reward divided by the number of steps taken. Notice that the RMax Abstraction agent takes actions uniformly at random on histories where the Markov abstraction is not yet defined, otherwise taking actions greedily according to the value function.

The results for Reset-Rotating MAB (Figure 2a) show the advantage of the exploration strategy of our approach, compared to Random Sampling. In fact, in this domain, random exploration does not allow for exploring states that are far from the initial state. The results for Enemy Corridor (Figure 2b) show that our approach scales with the domain size. Namely, in Enemy Corridor we are able to increase the corridor size k up to 64. The results for Flickering Grid (Figure 2c) show that our approach works in a domain with partial observability, that is natural to model as a POMDP. This is in line with our Theorem 1. The Markov abstraction for Flickering Grid maps histories to the current cell; intuitively, the

[2]The results shown in this paper are reproducible. Source code, instructions, and definitions of the experiments are available at: github.com/whitemech/markov-abstractions-code-ijcai22. Experiments were carried out on a server running Ubuntu 18.04.5 LTS, with 512GB RAM, and an 80 core Intel Xeon E5-2698 2.20GHz. Each training run takes one core. The necessary amount of compute and time depends mostly on the number of episodes set for training.



(a) Reset-Rotating MAB (b) Enemy Corridor (c) Flickering Grid (d) Safe vs. candidate states



(e) Rotating MAB (f) Malfunction MAB (g) Cheat MAB (h) Rotating Maze

Figure 2: Empirical evaluation of Markov Abstractions.

agent learns to compute its position even if sometimes it is not explicitly provided by the environment. For this domain, Figure 2d shows the number of steps the agent is in a safe state against when it is not. The figure exemplifies the incremental process of learning a Markov abstraction, as the time spent by the agent out of the safe region is high during the first episodes, and it decreases as more histories are collected and more safe states get to be learned by the stream PDFA learning algorithm; we see the agent achieves optimal behaviour around $5 \cdot 10^5$ episodes, when it no longer encounters candidate states, since all relevant states have been learned.

In the domains from [Abadi and Brafman, 2020] (Figures 2e-2h) our approach has poor performance overall. We are able to show convergence in all domains except Rotating Maze with $k = 2, 3$ and Malfunction MAB with $k = 5$. However, it requires a large number of episodes before achieving a close-to-optimal average reward. We observe that all these domains have a low distinguishability between the probability distribution of the states. The performance of PDFA-learning is greatly dependent on such parameter, cf. [Balle *et al.*, 2013]. Furthermore, in these domains, the value of distinguishability decreases with k. On the contrary, in Reset-Rotating MAB, Enemy Corridor, and Flickering Grid, distinguishability is high and independent of k. Therefore, we conclude that a low distinguishability is the source of poor performance. In particular, it causes the automata learning module to require a large number of samples.

We compare our approach against two related approaches. First, we compare against [Abadi and Brafman, 2020] that combines deterministic finite automata learning with history clustering to build a Mealy Machine that represents the underlying decision process, while employing MCTS to compute policies for the learned models. Comparing to the results published in [Abadi and Brafman, 2020] for Rotating MAB, Malfuction MAB, and Rotating Maze, our approach has better performance in Rotating MAB, and worse performance in the other domains. Second, we compare against [Icarte *et al.*, 2019], also based on automata learning. Our approach outperforms theirs in all the domains mentioned in this paper, and their approach outperforms ours in all their domains. Specifically, their algorithm does not converge in our domains, and our algorithm does not converge in theirs. Our poor performance is explained by the fact that all domains in [Icarte *et al.*, 2019] have a low distinguishability, unfavourable to us. On the other hand, their poor performance in our domains might be due to their reliance on local search methods.

6 Conclusion and Future Work

We presented an approach for learning and solving non-Markov decision processes, that combines standard RL with automata learning in a modular manner. Our theoretical results show that, for NMDPs that admit a Markov abstraction, ϵ-optimal policies can be computed with PAC guarantees, and that its dynamics can be accurately represented by an automaton. The experimental evaluation shows the feasibility of the approach. The comparison with the random-exploration baseline shows the advantage of a smarter exploration based on the automaton as it is still being learned, which is an important aspect introduced in this paper. Some of the domains illustrate the difficulty of learning when distinguishability is low—an issue of the underlying automata learning algorithm. At the same time, other domains show good scalability of the overall approach when distinguishability stays high.

The difficulties encountered by the PDFA-learning algorithm in domains with low distinguishability suggest that the existing PDFA-learning techniques should be further developed in order to be effective in RL. We see it as an interesting direction for future work. A second direction is to make our approach less reliant on the automata learning component. Specifically, an agent could operate directly on the histories for which the current Markov abstraction is yet undefined.

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