Robotics 1

Kinematic control

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Robot motion control

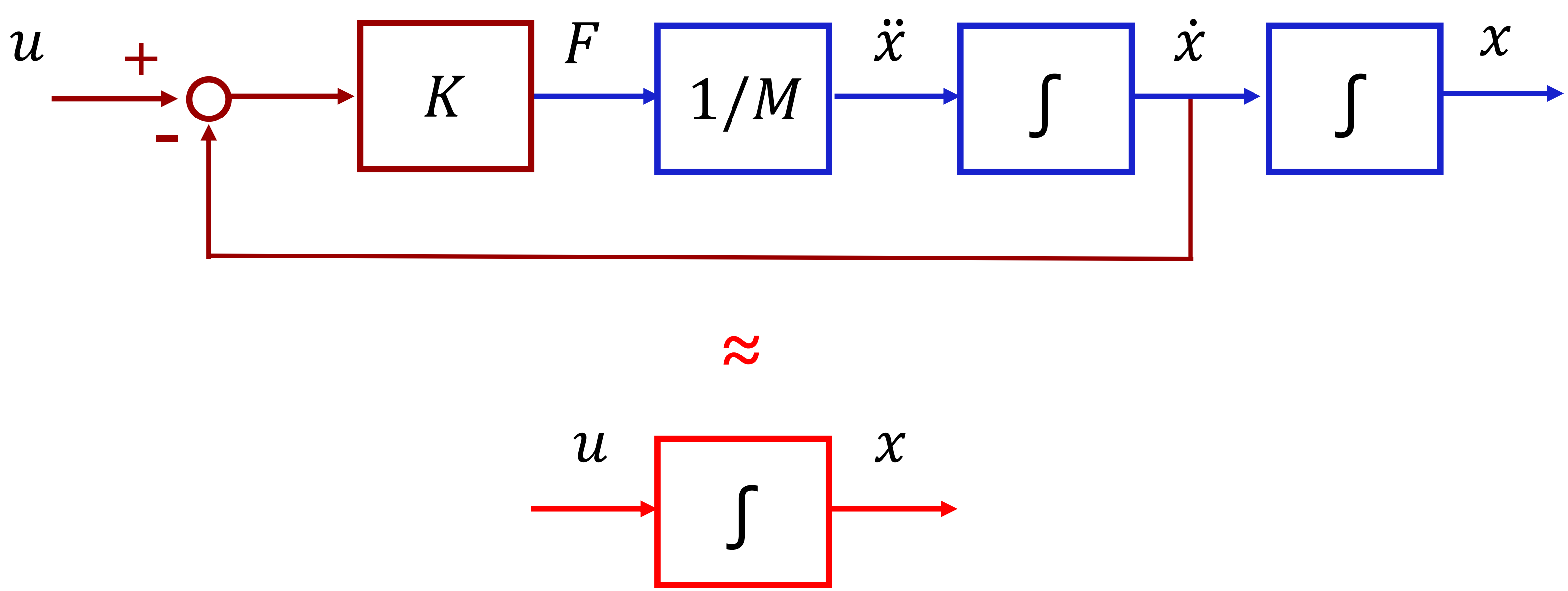
- need to "actually" realize a desired robot motion task ...
  - regulation of pose/configuration (constant reference)
  - trajectory following/tracking (time-varying reference)
- ... despite the presence of
  - external disturbances and/or unmodeled dynamic effects
  - initial errors (or arising later due to disturbances) w.r.t. desired task
  - discrete-time implementation, uncertain robot parameters, ...
- we use a general control scheme based on
  - feedback (from robot state measures, to impose asymptotic stability)
  - feedforward (nominal commands generated in the planning phase)
- the error driving the feedback part of the control law can be defined either in Cartesian or in joint space
  - control action always occurs at the joint level (where actuators drive the robot), but performance has to be evaluated at the task level

Kinematic control of robots

- a robot is an electro-mechanical system driven by actuating torques produced by the motors
- it is possible, however, to consider a kinematic command (most often, a velocity) as control input to the system...
- ...thanks to the presence of low-level feedback control at the robot joints that allows imposing commanded reference velocities (at least, in the "ideal case")
- these feedback loops are present in industrial robots within a "closed" control architecture, where users can only specify reference commands of the kinematic type
- in this way, performance can be very satisfactory, provided the desired motion is not too fast and/or does not require large accelerations

An introductory example

- a mass M in linear motion: $M\ddot{x} = F$
- low-level feedback: $F = K(u - \dot{x})$, with u = reference velocity
- equivalent scheme for $K \to \infty$: $\dot{x} \approx u$
- in practice, valid in a limited frequency "bandwidth" $\omega \leq K/M$



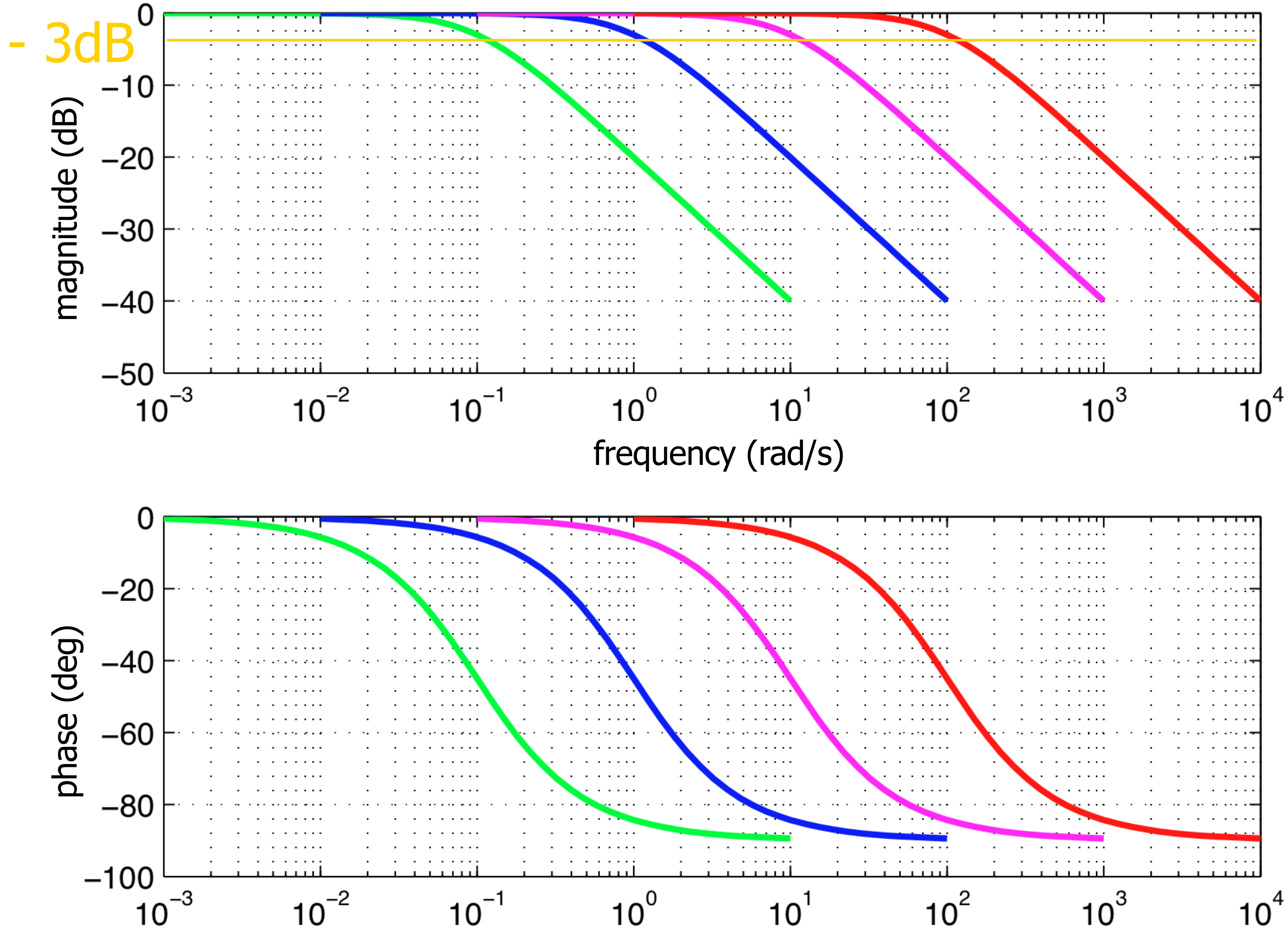
Frequency response
of the closed-loop system

- Bode diagrams of $P(s) = \dfrac{v(s)}{u(s)} = \dfrac{sx(s)}{u(s)}$ for $K/M = 0.1, 1, 10, 100$

actual velocity

$$\frac{\dot{x}}{u}$$

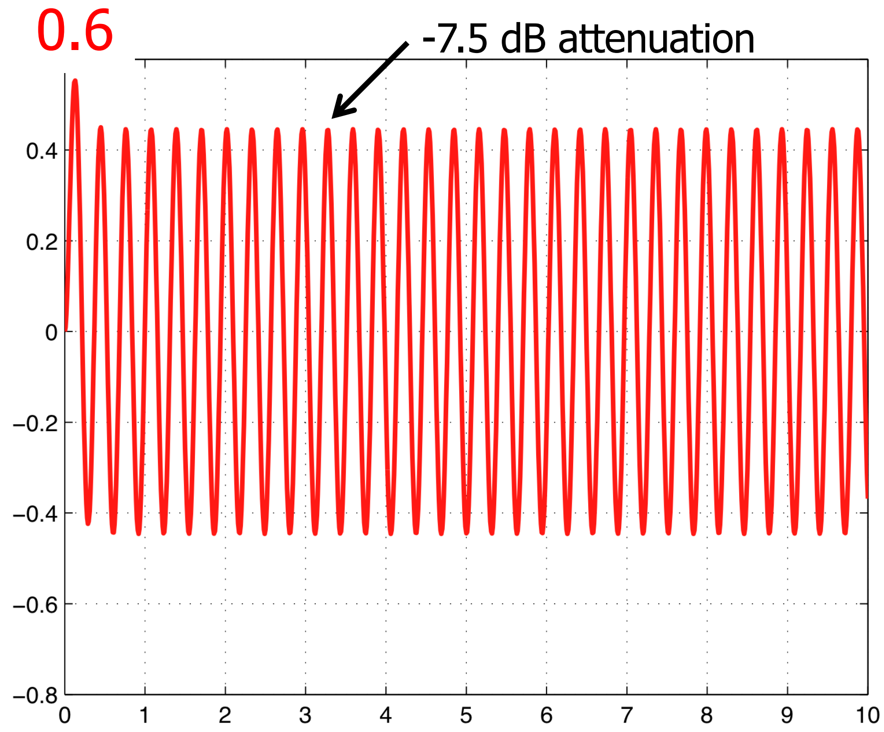
commanded reference velocity



Time response

- setting $K/M = 10$ (bandwidth), we show two possible time responses to unit sinusoidal velocity reference commands at different ω

$\omega = 2$ rad/s

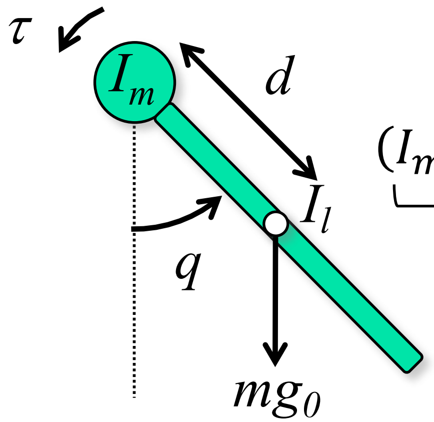


$\omega = 20$ rad/s

actually realized velocities

A more detailed example
including nonlinear dynamics

- single link (a thin rod) of mass m, center of mass at d from joint axis, inertia M (motor + link) at the joint, rotating in a vertical plane (the gravity torque at the joint is configuration dependent)



dynamic model

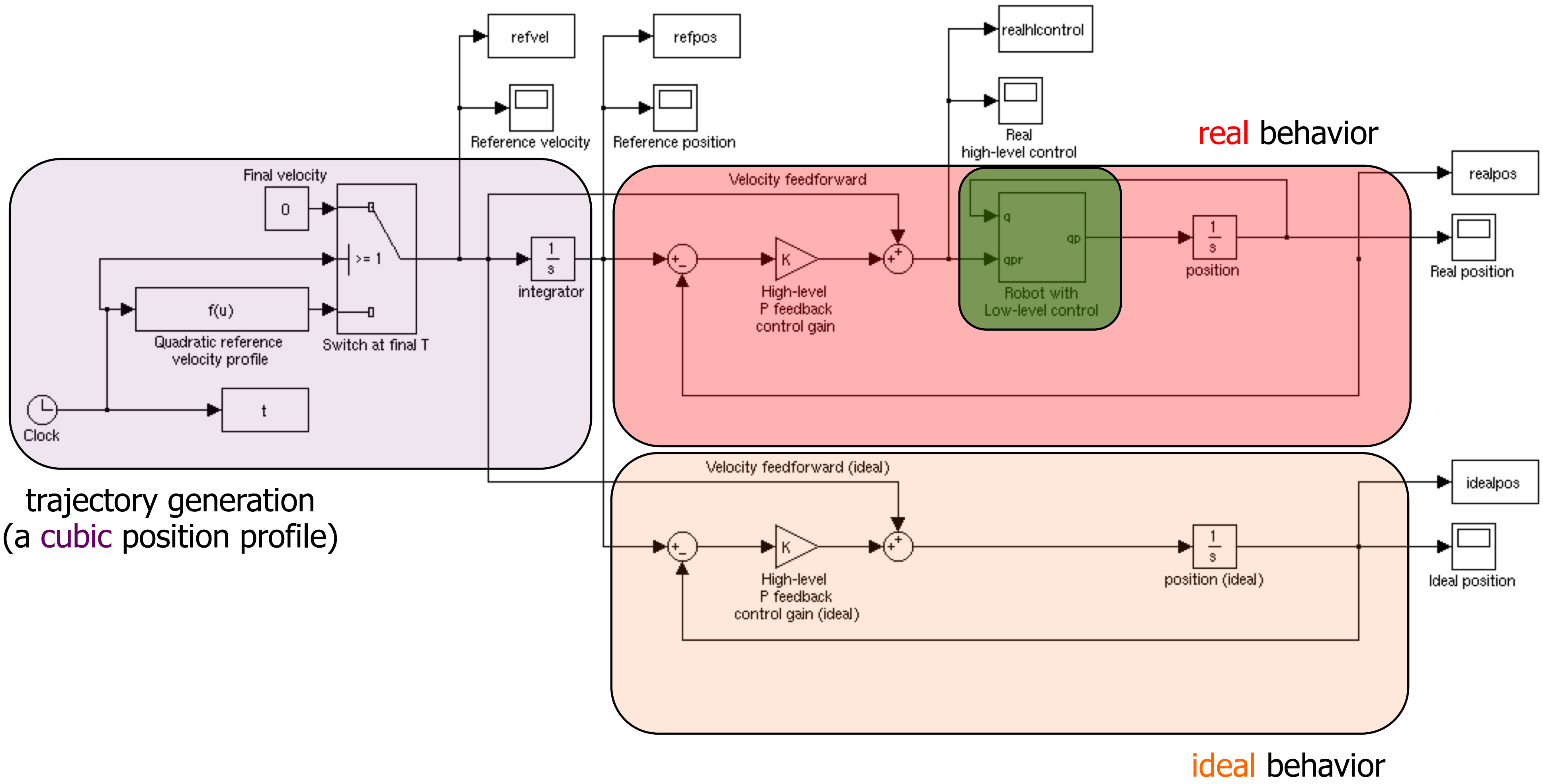
$$\underbrace{(I_m + I_l + md^2)}_{M}\ddot{q} + mg_o d \sin q = \tau$$

$g_0 = 9.81\ [m/s^2]$
$m = 10\ [kg]$
$d = l/2 = 0.2\ [m]$
$I_l = ml^2/12 = 0.1333[kgm^2]$
$I_m = 0.5333[kgm^2]$
$(= I_l + md^2)$
$\Longrightarrow M = 1.0667[kgm^2]$

- fast low-level feedback control loop based on a PI action on the velocity error + an approximate acceleration feedforward
- kinematic control loop based on a P feedback action on the position error + feedforward of the velocity reference at the joint level
- evaluation of tracking performance for rest-to-rest motion tasks with "increasing dynamics" = higher accelerations

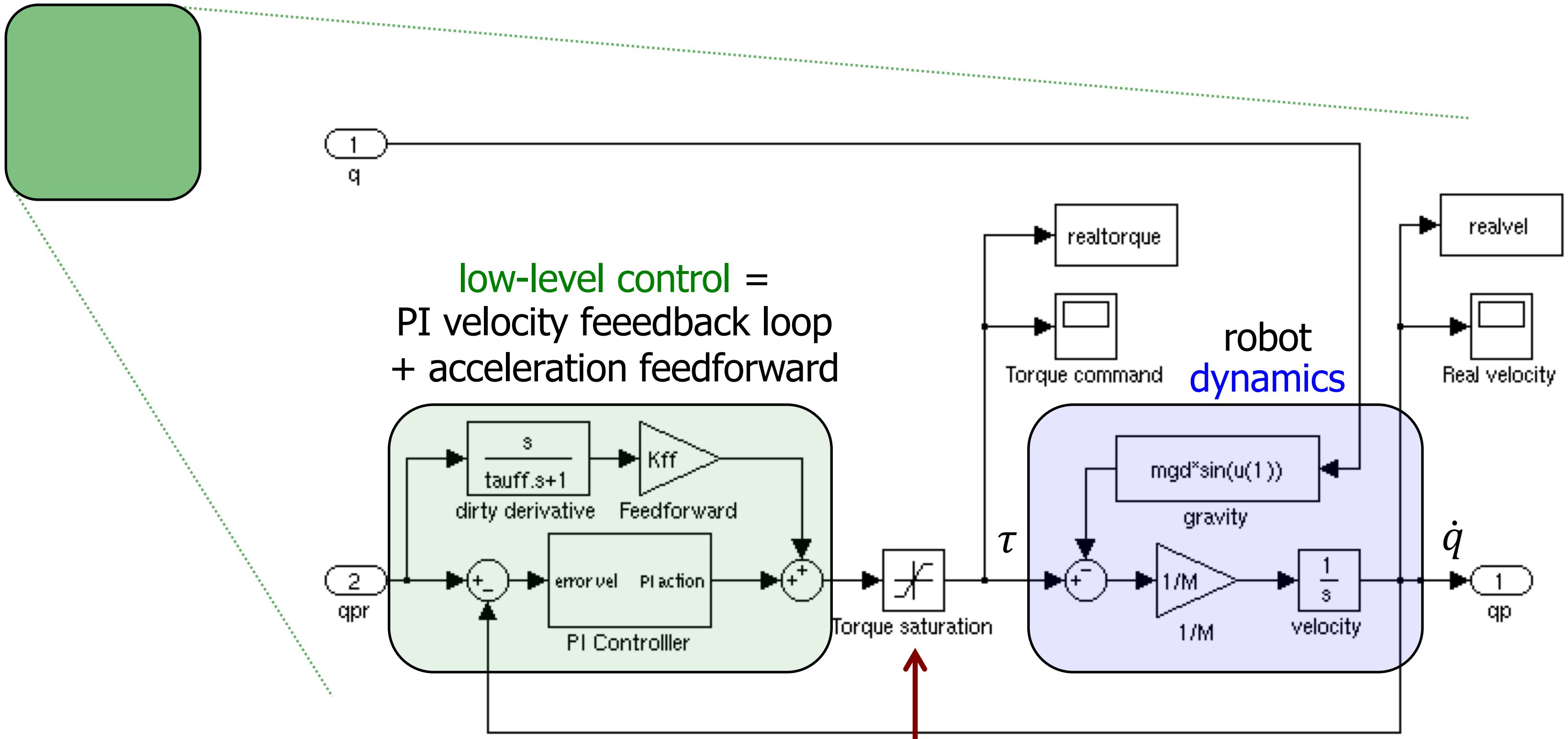
A more detailed example
differences between the ideal and real case

- Simulink scheme



A more detailed example
robot with low-level control

- Simulink scheme



low-level control =
PI velocity feeedback loop
+ acceleration feedforward

robot
dynamics

actuator
saturation at 100 [Nm]

$$M\ddot{q} + mg_o d \sin q = \tau$$

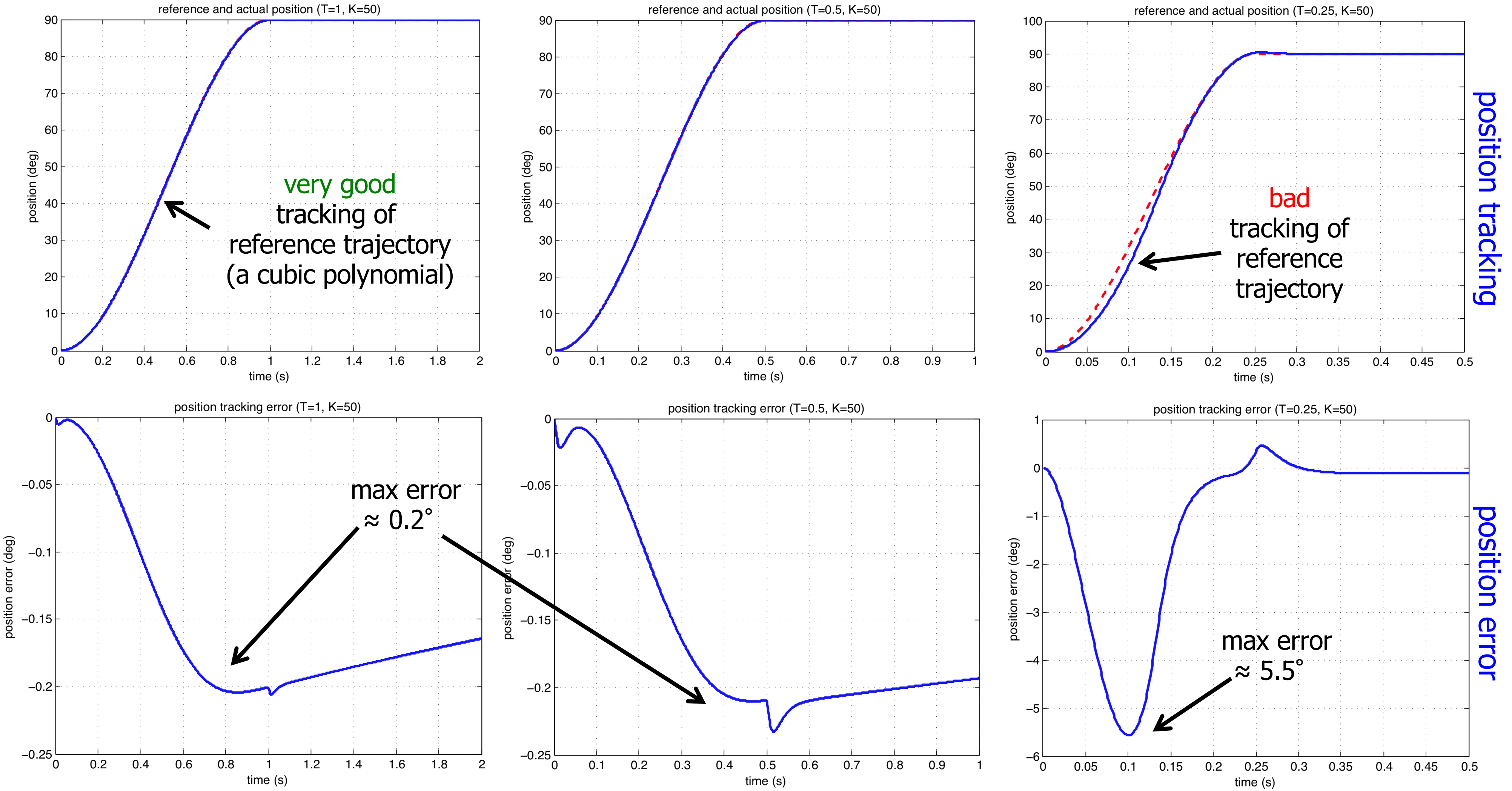
$$\Downarrow$$

$$\ddot{q} = \frac{1}{M}(\tau - mg_o d \sin q)$$

Simulation results

rest-to-rest motion from downward to horizontal position

- in $T = 1$ s
- in $T = 0.5$ s
- in $T = 0{,}25$ s



Simulation results

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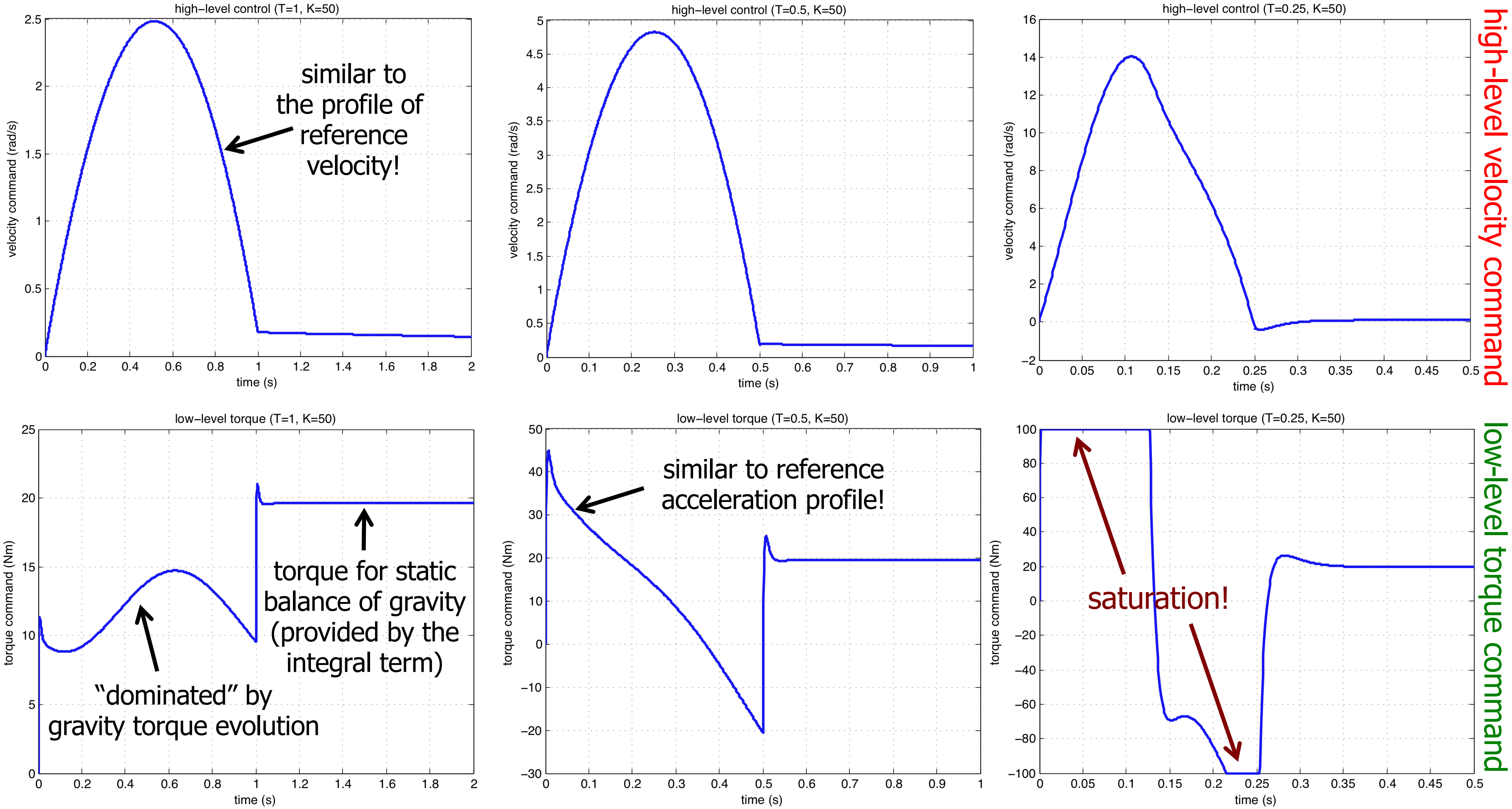

Simulation results

rest-to-rest motion from downward to horizontal position

- in $T = 1$ s
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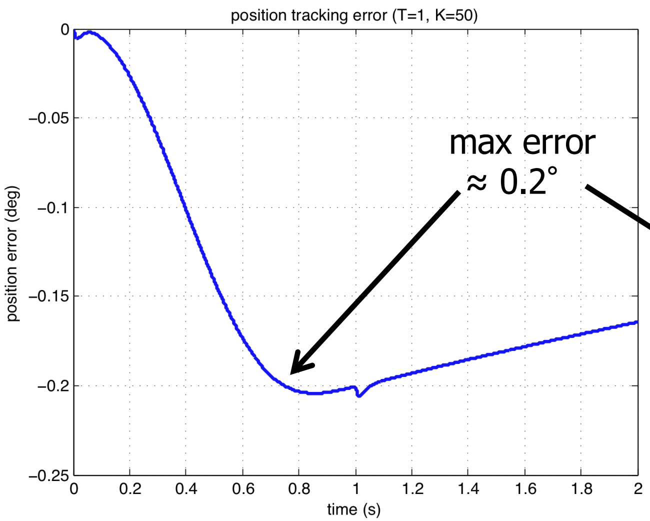
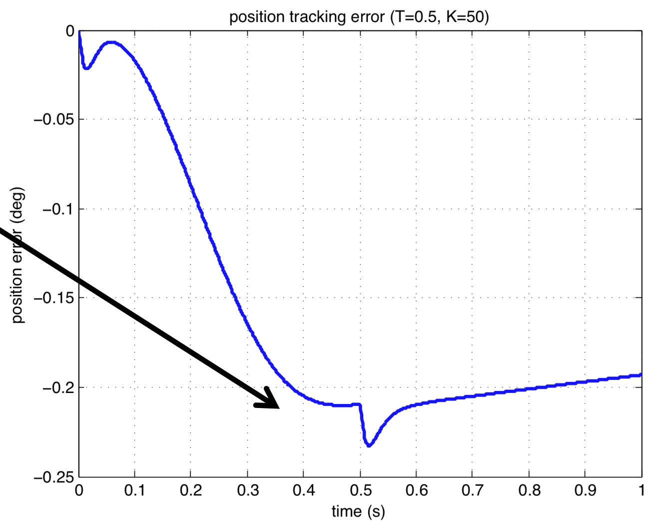
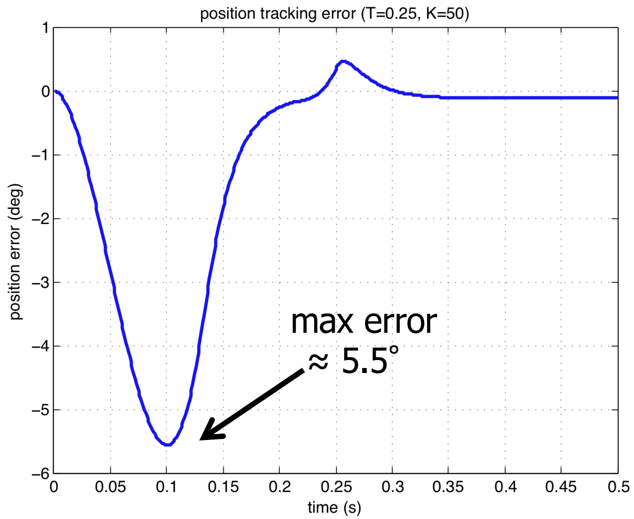


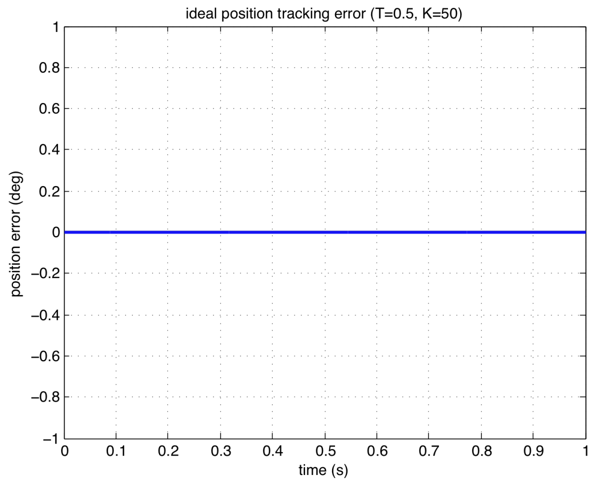




real position errors increase when reducing too much motion time (⇒ too high accelerations)

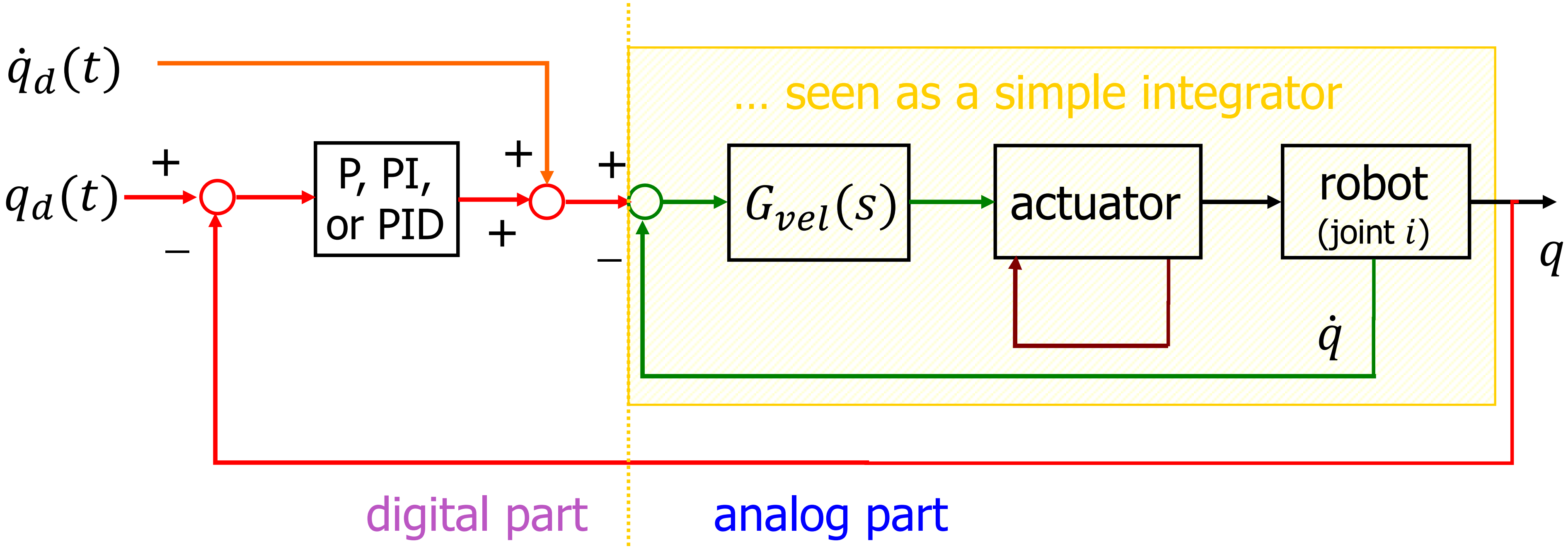
while ideal position errors
(based only on kinematics)
remain always the same!!

here ≡ 0, thanks to the initial matching between robot and reference trajectory

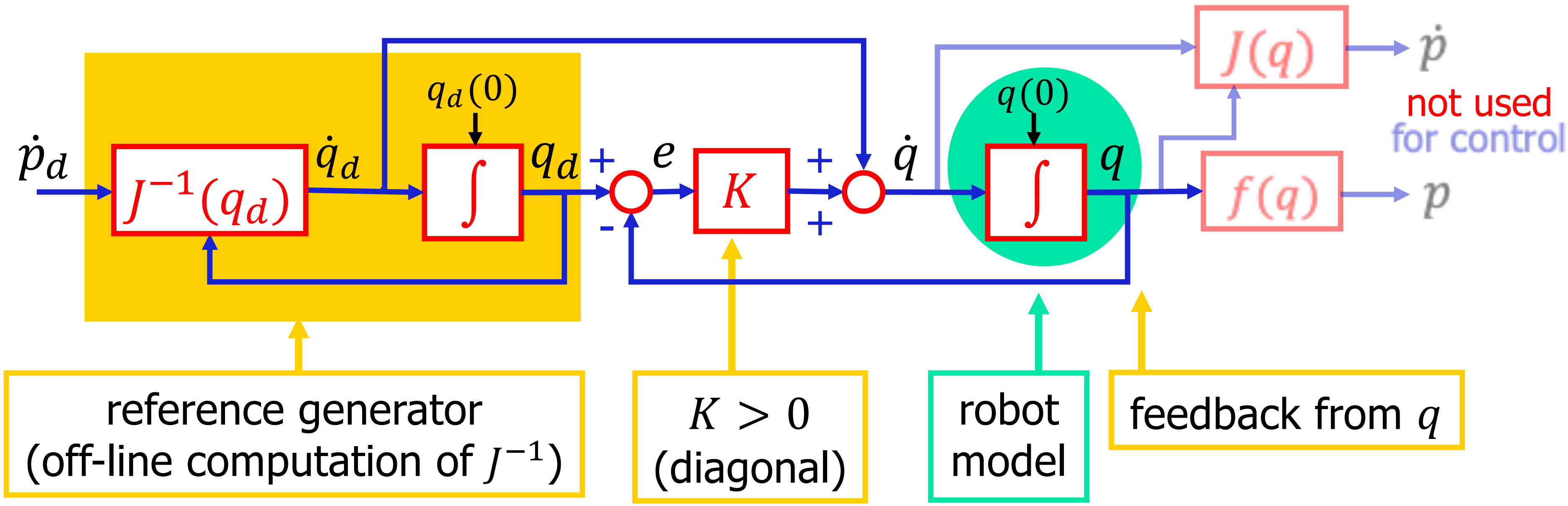


Control loops in industrial robots

- analog loop of large bandwidth on motor current ($\propto$ torque)
- analog loop on velocity ($G_{vel}(s)$, typically a PI)
- digital feedback loop on position, with velocity feedforward
- this scheme is local to each joint (decentralized control)



Kinematic control of joint motion



$$e = q_d - q \;\Rightarrow\; \dot{e} = \dot{q}_d - \dot{q} = \dot{q}_d - (\dot{q}_d + K(q_d - q)) = -Ke$$

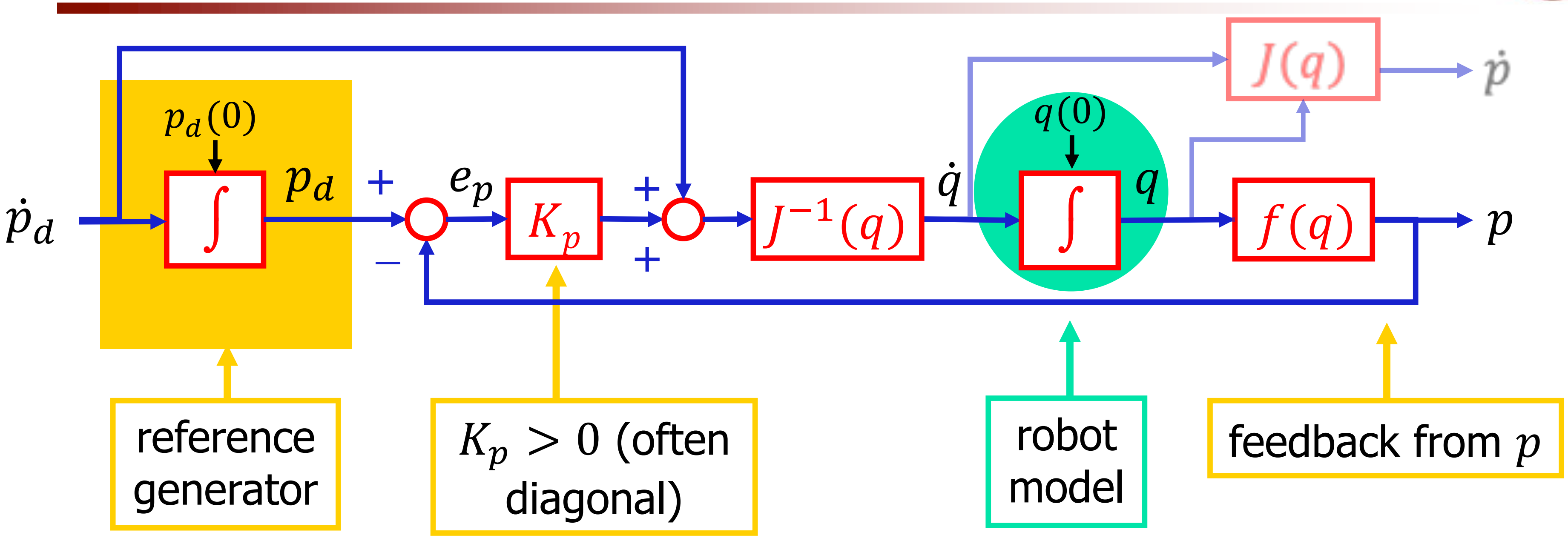
decoupled $e_i \to 0$ $(i = 1, \cdots, n)$ exponentially, $\forall e(0)$

$$e_p = p_d - p \;\Rightarrow\; \dot{e}_p = \dot{p}_d - \dot{p} = J(q_d)\dot{q}_d - J(q)(\dot{q}_d + K(q_d - q))$$

$$\begin{array}{c} q \approx q_d \\ e_p \to J(q)e \end{array} \;\Rightarrow\; \dot{e}_p \approx -J(q)K\,J^{-1}(q)e_p$$

coupled Cartesian error dynamics

Kinematic control of Cartesian motion



$$e_p = p_d - p \quad \Rightarrow \quad \dot{e}_p = \dot{p}_d - \dot{p} = \dot{p}_d - \cancel{J(q)J^{-1}(q)}\left(\dot{p}_d + K_p(p_d - p)\right) = -K_p e_p$$

- decoupled $e_{p,i} \to 0$ $(i = 1, \cdots, m)$ exponentially, $\forall e_p(0)$
- needs on-line computation of the inverse(*) $J^{-1}(q)$
- real-time + singularities issues

(*) or pseudoinverse if $m < n$

Simulation

features of kinematic control laws

desired reference trajectory:

two types of tasks

1. straight line
2. circular path

both with constant speed

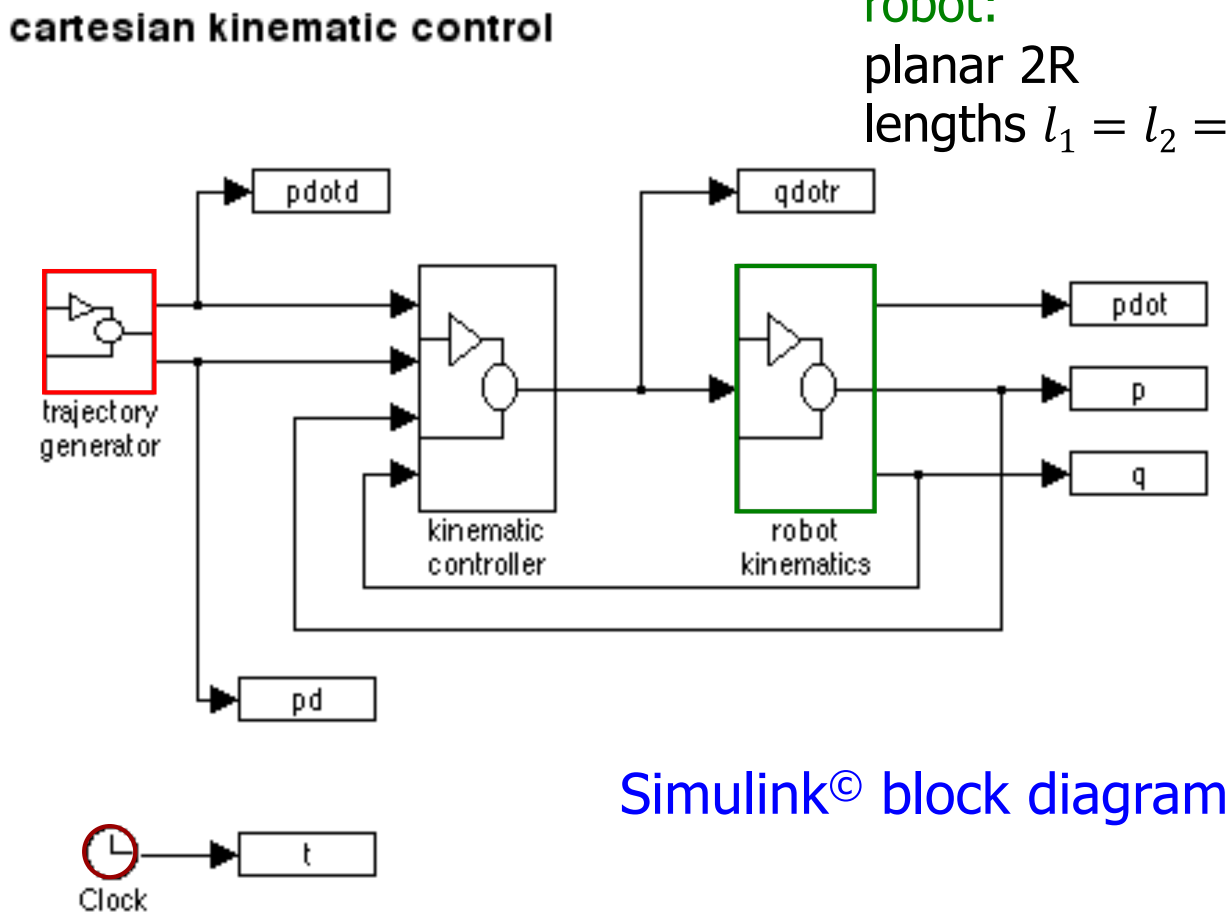
numerical integration method:

fixed step Runge-Kutta at 1 msec

robot:

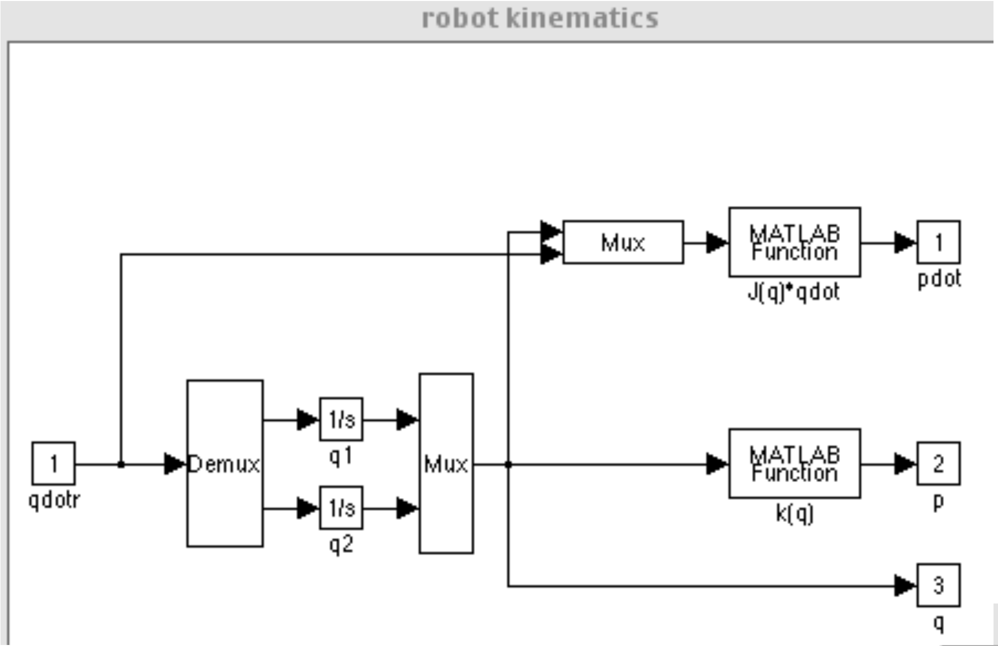
planar 2R

lengths $l_1 = l_2 = 1$



Simulink© block diagram

Simulink blocks



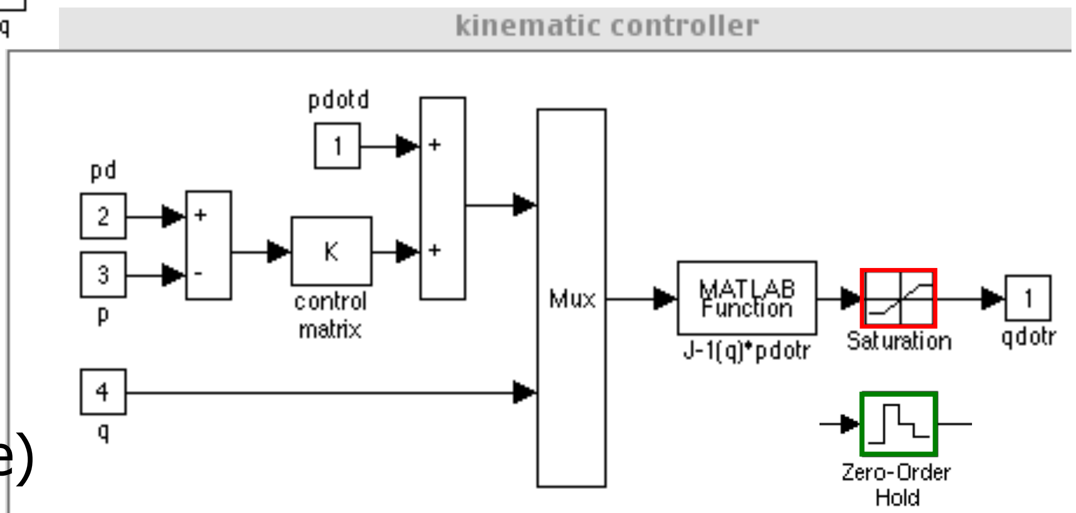
calls to Matlab functions

k(q)=dirkin (user)

J(q)=jac (user)

J^{-1}(q)=inv(jac) (library)

- a saturation (for task 1.) or a sample and hold (for task 2.) added on joint velocity commands
- system initialization of kinematics data, desired trajectory, initial state, and control parameters (in init.m file)



never put "numbers" inside the blocks !

Matlab functions

dirkin.m

```
function [p] = dirkin(q)

global l1 l2

px=l1*cos(q(1))+l2*cos(q(1)+q(2));
py=l1*sin(q(1))+l2*sin(q(1)+q(2));
```

jac.m

```
function [J] = jac(q)

global l1 l2

J(1,1)=-l1*sin(q(1))-l2*sin(q(1)+q(2))
J(1,2)=-l2*sin(q(1)+q(2));
J(2,1)=l1*cos(q(1))+l2*cos(q(1)+q(2));
J(2,2)=l2*cos(q(1)+q(2));
```

init.m

```
% controllo cartesiano di un robot 2R
% initialization

clear all; close all
global l1 l2

% lunghezze bracci robot 2R

l1=1; l2=1;

% velocità cartesiana desiderata (costante)

vxd=0; vyd=0.5;

% tempo totale

T=2;

% configurazione desiderata iniziale

q1d0=-45*pi/180; q2d0=135*pi/180;

pd0=dirkin([q1d0 q2d0]');
pxd0=pd0(1); pyd0=pd0(2);

% configurazione attuale del robot

q10=-45*pi/180; q20=90*pi/180;

p0=dirkin([q10 q20]');

% matrice dei guadagni cartesiani

K=[20 20]; K=diag(K);

%saturazioni di velocità ai giunti (input in deg/sec, convertito in rad/sec)

vmax1=120*pi/180; vmax2=90*pi/180;
```

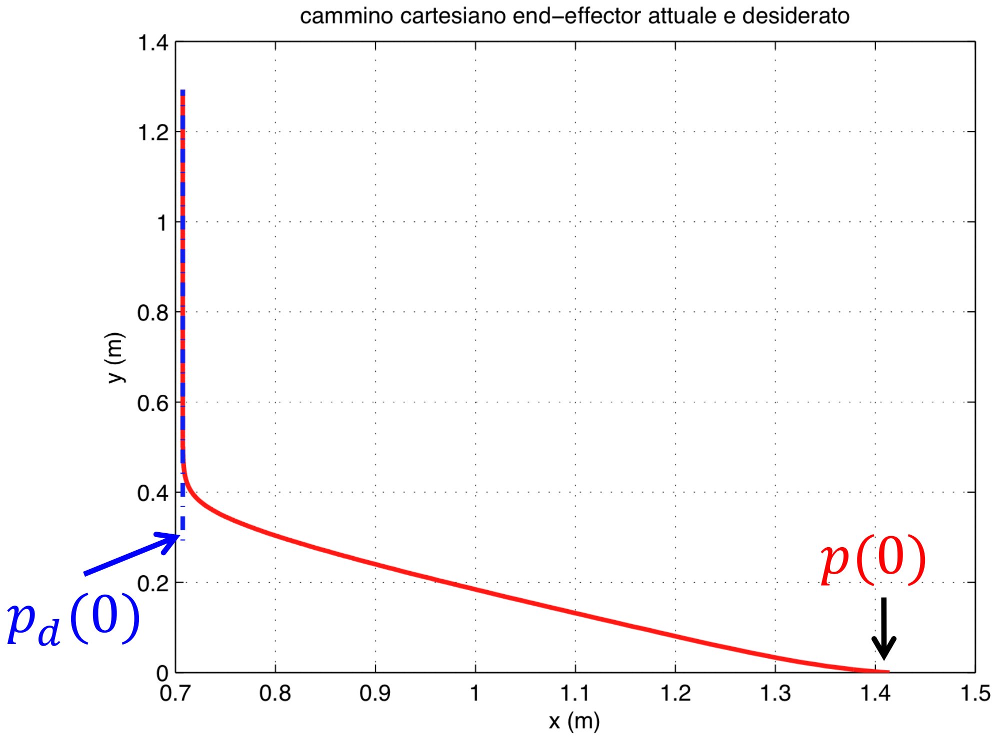
init.m
script
(for task 1.)

Simulation data for task 1

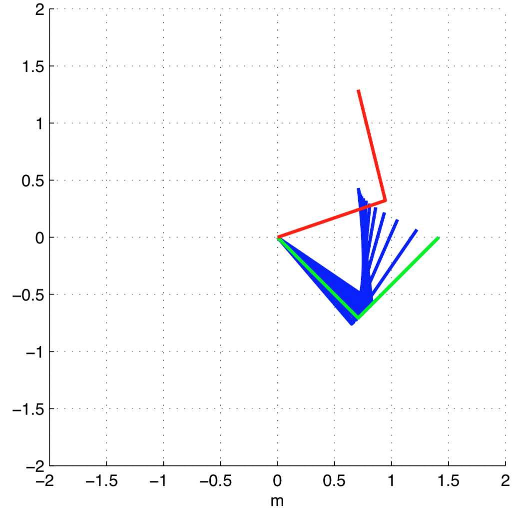
- straight line path with constant velocity
  - $x_d(0) = 0.7$ m, $y_d(0) = 0.3$ m; $v_{d,y} = 0.5$ m/s, for $T = 2$ s
- large initial error on end-effector position
  - $q(0) = (-45°, 90°) \Rightarrow e_p(0) = (-0.7, 0.3)$ m
- Cartesian control gains
  - $K_p = \text{diag}\{20, 20\}$
- (a) without joint velocity command saturation
- (b) with saturation ...
  - $v_{max,1} = 120°$/s, $v_{max,2} = 90°$/s

Results for task 1a

straight line: initial error, **no** saturation

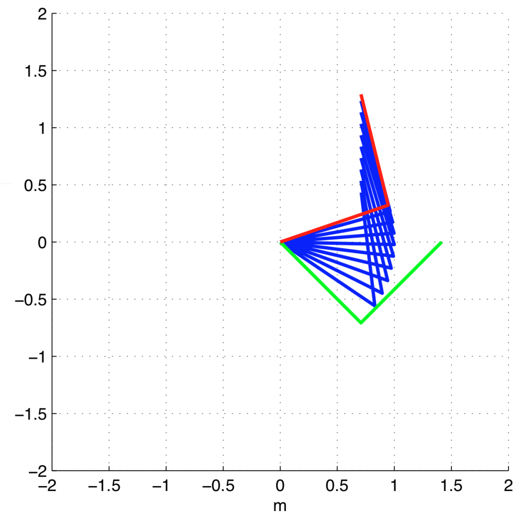


path executed by the robot end-effector (actual and desired)



initial transient phase (about 0.2 s)

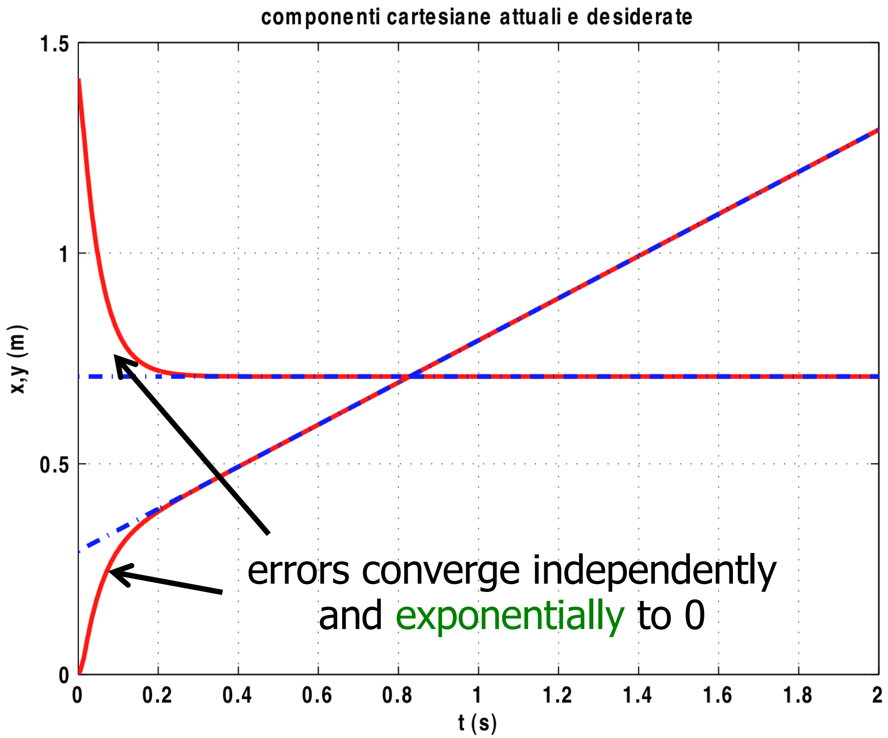
stroboscopic view of motion (start and end configurations)

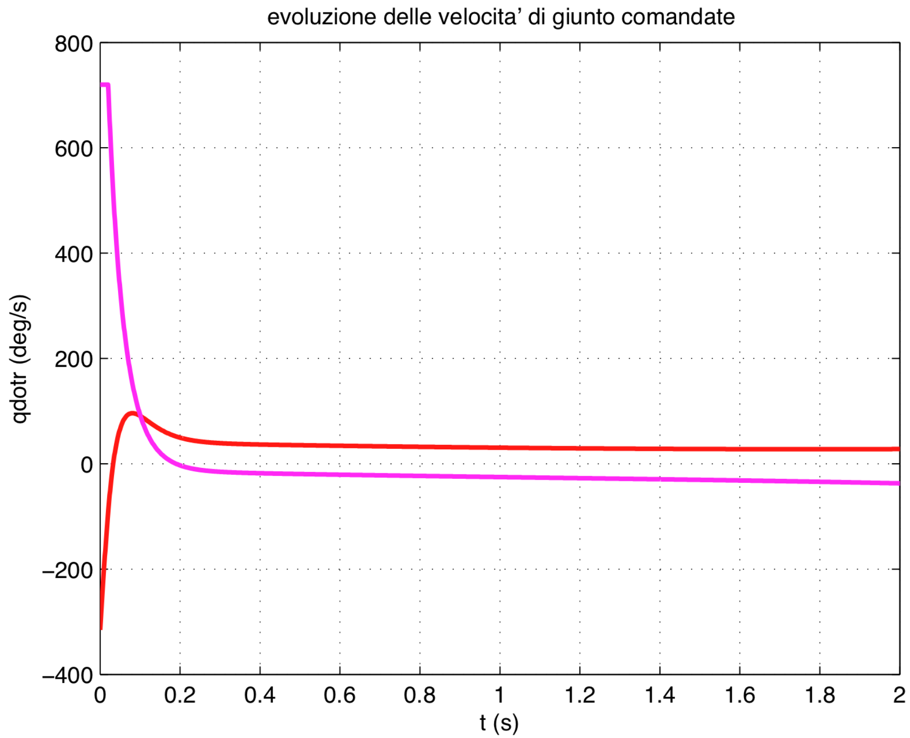


trajectory following phase (about 1.8 s)

Results for task 1a (cont)

straight line: initial error, **no** saturation



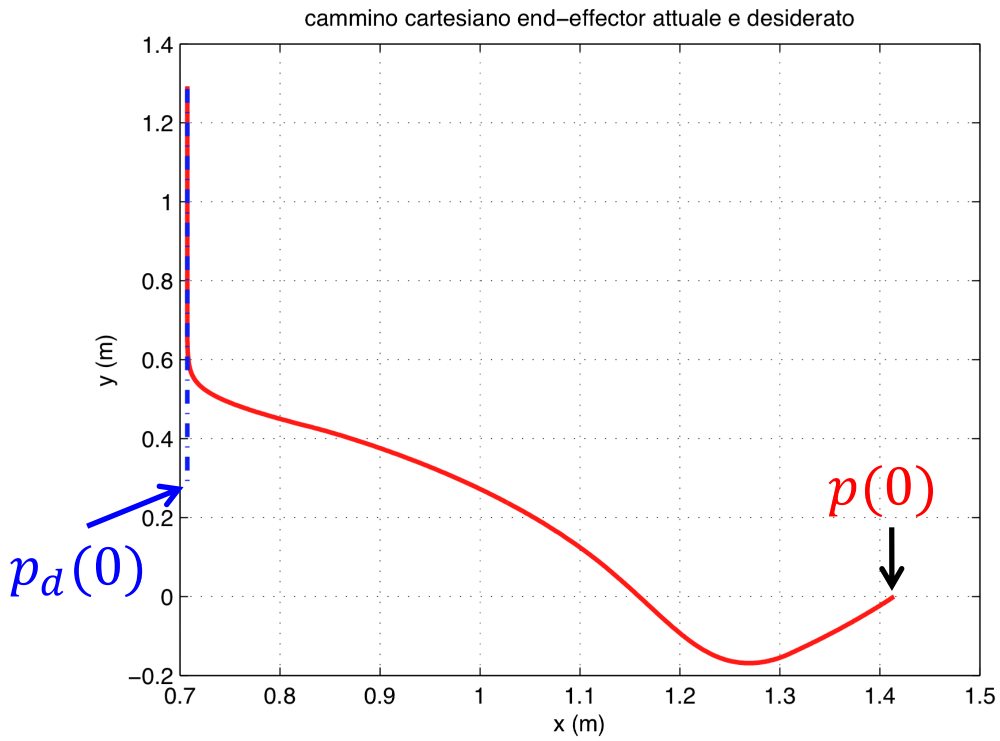


p_x, p_y actual and desired

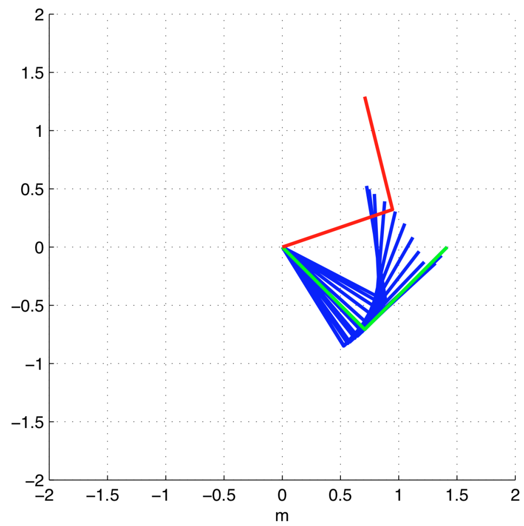
control inputs $\dot{q}_{r1}$, $\dot{q}_{r2}$

Results for task 1b

straight line: initial error, **with** saturation

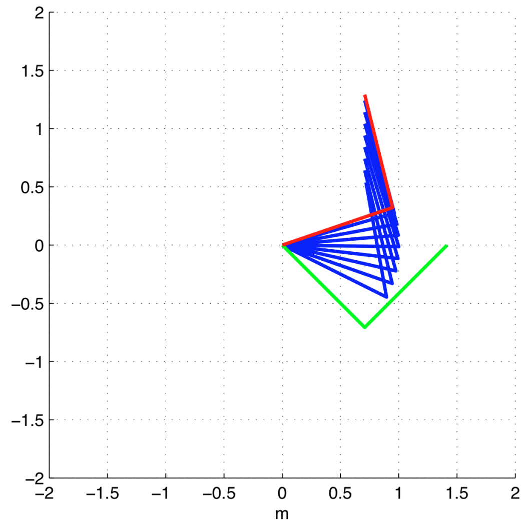


path executed by the robot end-effector (actual and desired)



initial transient phase (about 0.5 s)

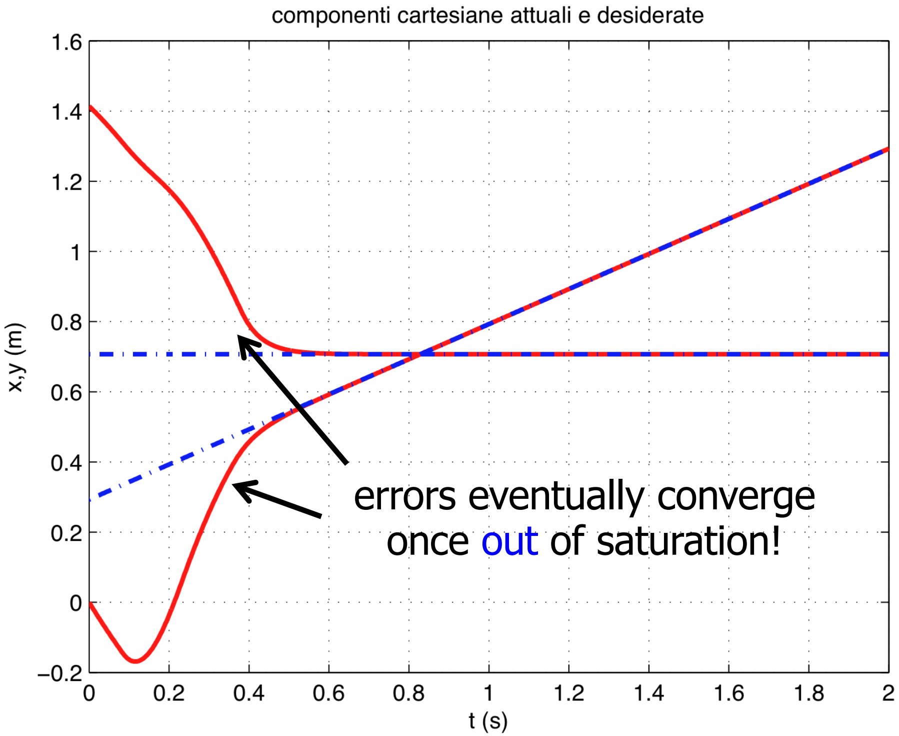
stroboscopic view of motion (start and end configurations)

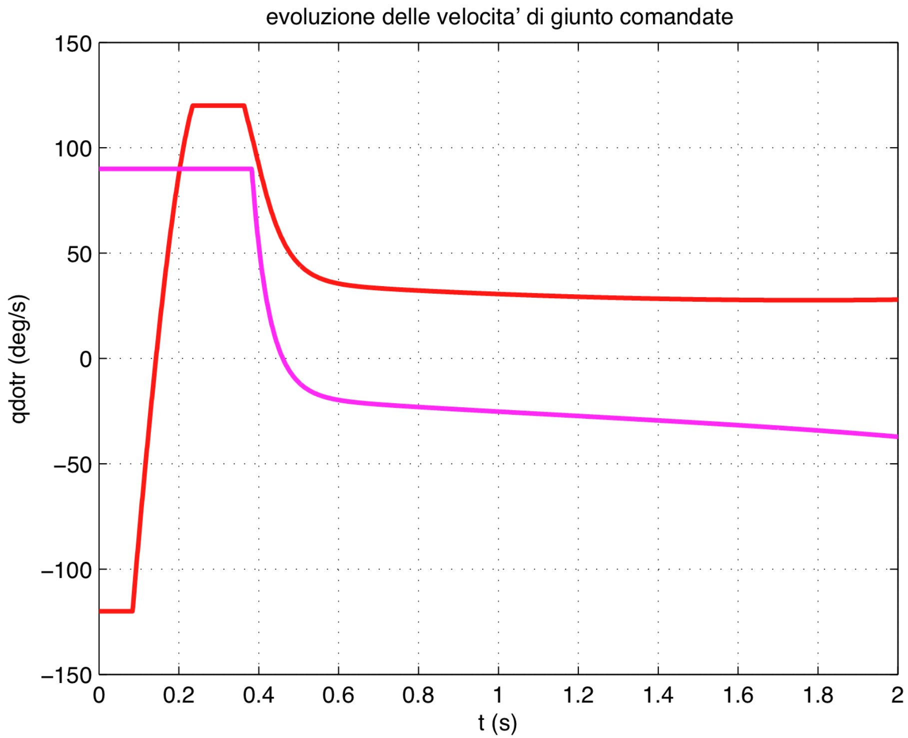


trajectory following phase (about 1.5 s)

Results for task 1b (cont)

straight line: initial error, **with** saturation





p_x, p_y actual and desired

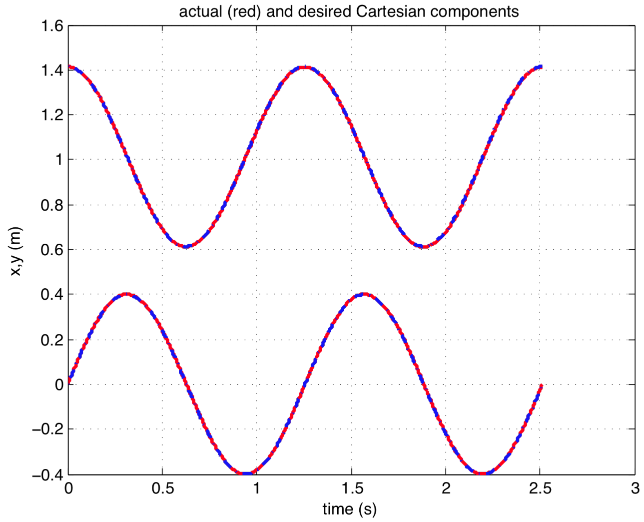
control inputs $\dot{q}_{r1}, \dot{q}_{r2}$
(saturated at $\pm v_{max,1}, \pm v_{max,2}$)

Simulation data for task 2

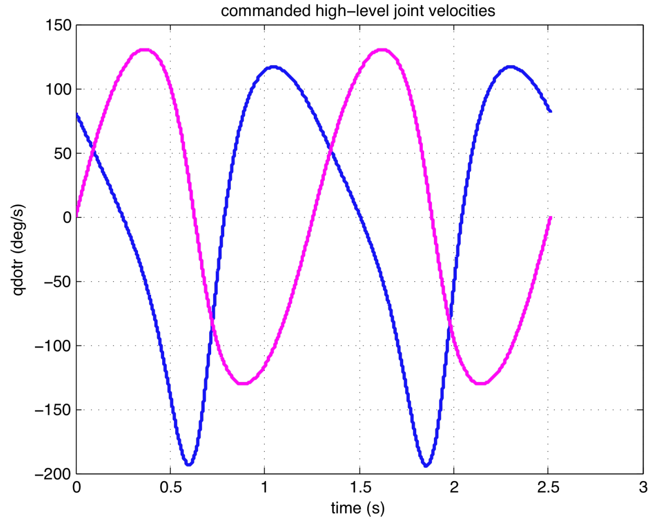
- circular path with constant velocity
  - centered at $(1.014, 0)$ with radius $R = 0.4$ m;
  - $v = 2$ m/s, performing two rounds ⇒ $T \approx 2.5$ s
- zero initial error on Cartesian position (“match”)
  - $q(0) = (-45°, 90°) \Rightarrow e_p(0) = 0$
- (a) ideal continuous case (1 kHz), even without feedback
- (b) with sample and hold (ZOH) of $T_{hold} = 0.02$ s (joint velocity command updated at 50 Hz), but without feedback
- (c) as before, but with Cartesian feedback using the gains
  - $K_p = \text{diag}\{25, 25\}$

Results for task 2a

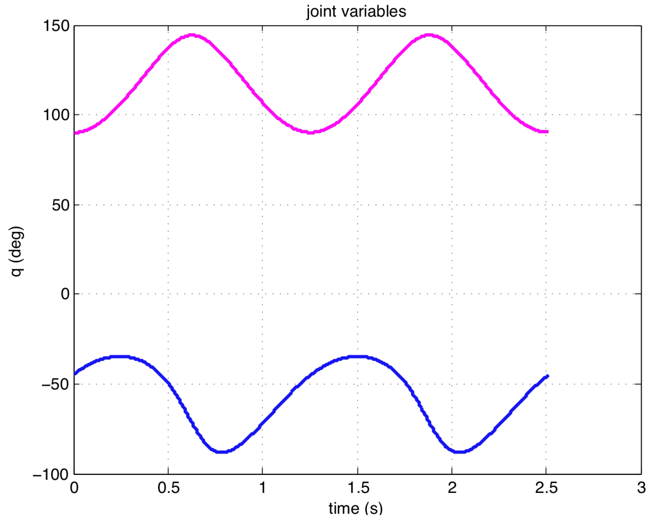
circular path: no initial error, **continuous** control (ideal case)



p_x, p_y actual and desired

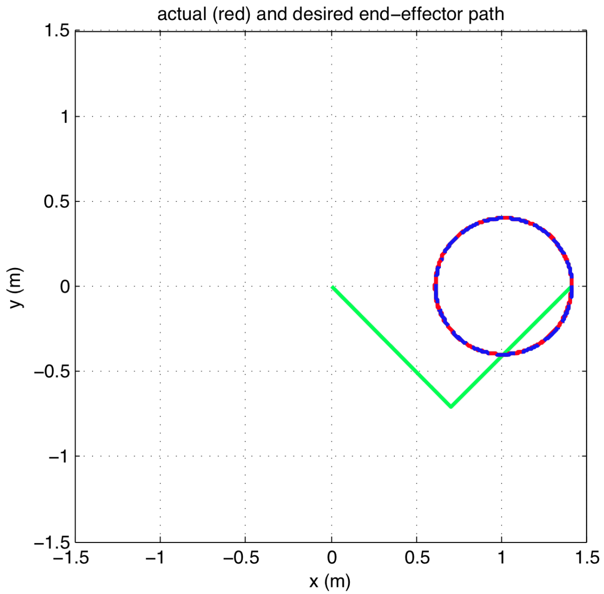


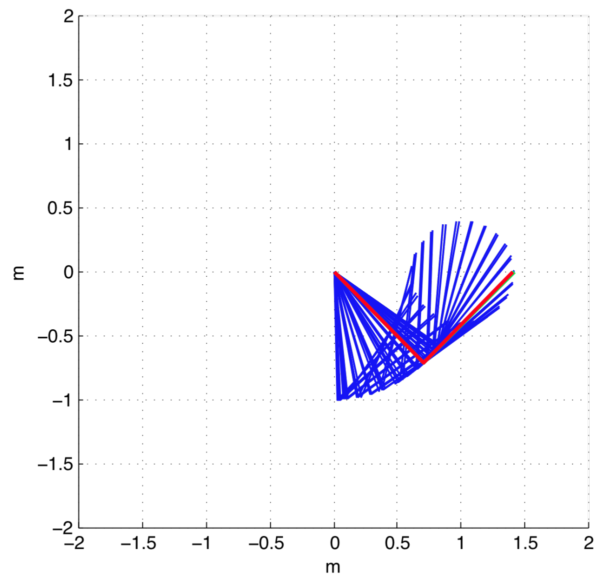
control inputs $\dot{q}_{r1}$, $\dot{q}_{r2}$



joint variables q_1, q_2

zero tracking error is kept at all times

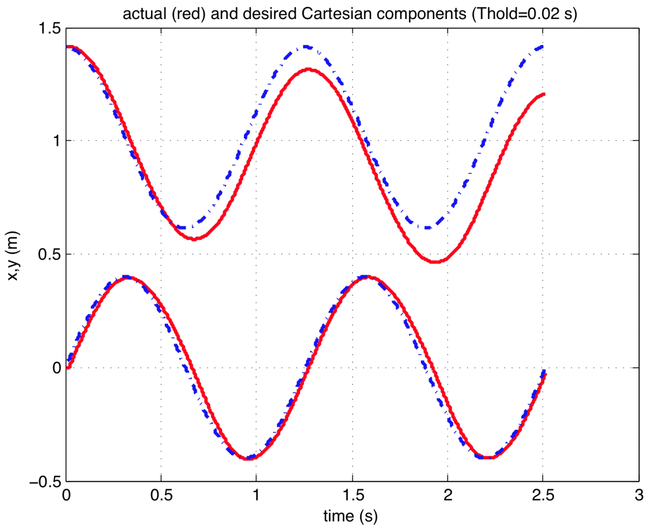


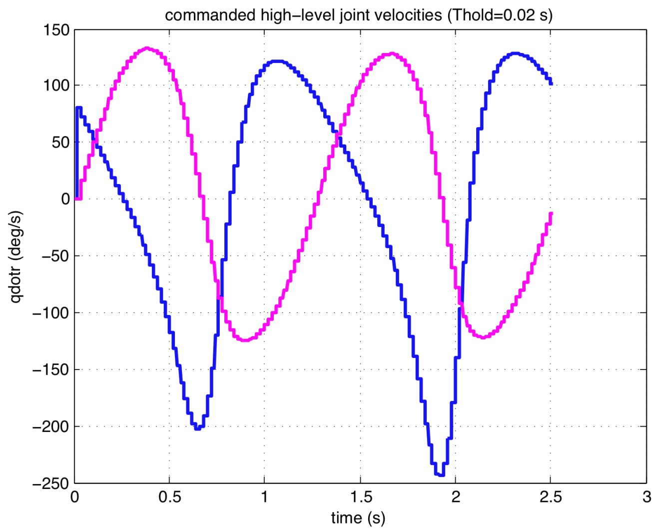


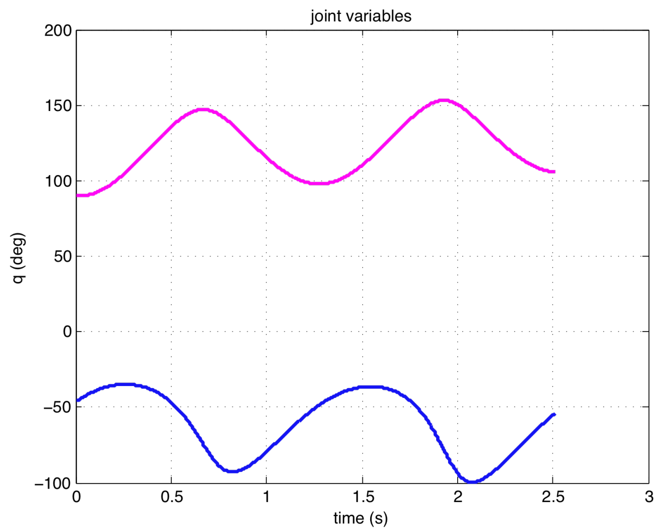
final configuration (after two rounds) coincides with initial configuration

Results for task 2b

circular path: no initial error, **ZOH** at 50 Hz, **no** feedback





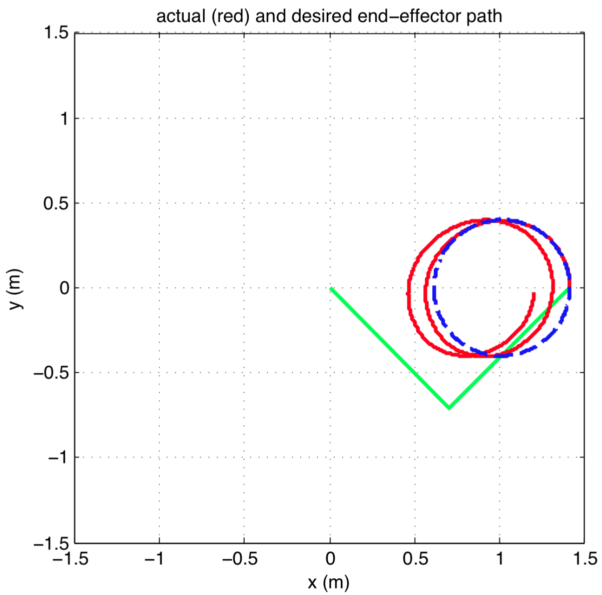


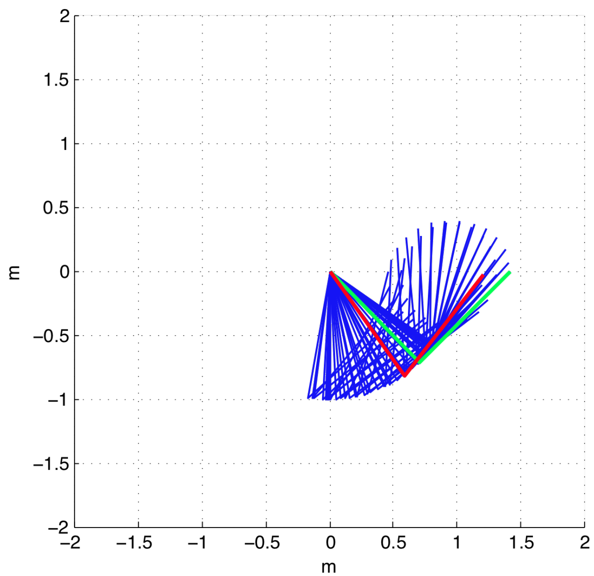
p_x, p_y actual and desired

control inputs $\dot{q}_{r1}$, $\dot{q}_{r2}$

joint variables q_1, q_2

a drift occurs along the path due to the "linearization error" along the path tangent

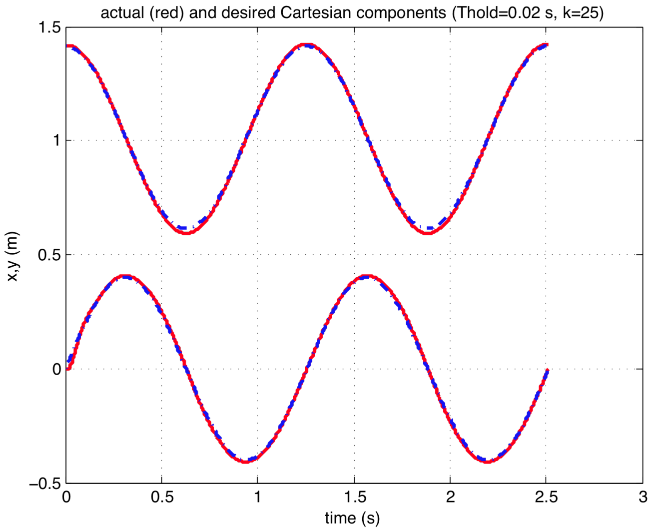




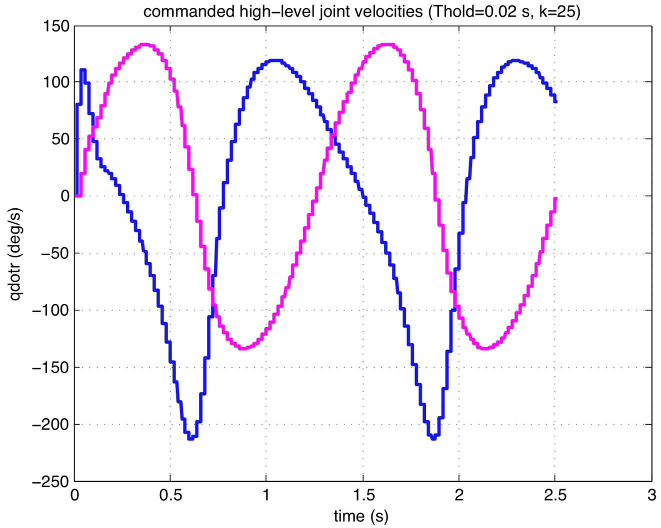
final configuration (after two rounds) differs from initial configuration

Results for task 2c

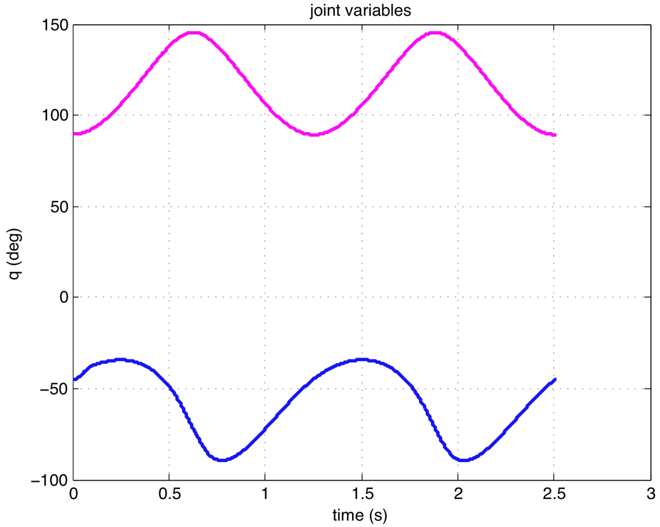
circular path: no initial error, **ZOH** at 50 Hz, **with** feedback



p_x, p_y actual and desired

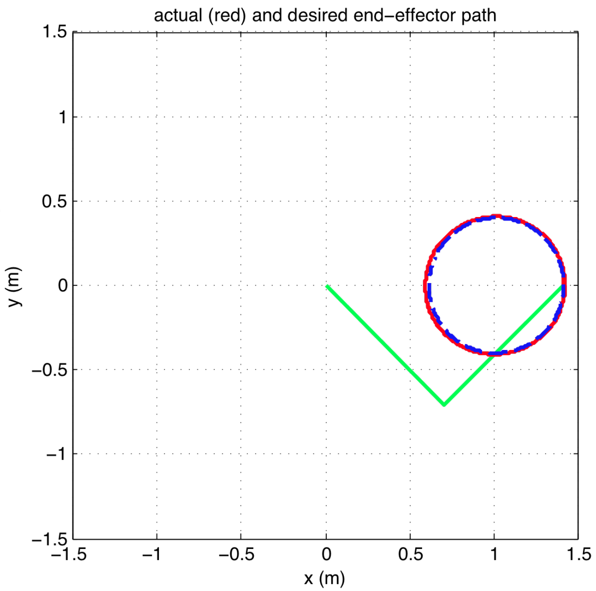


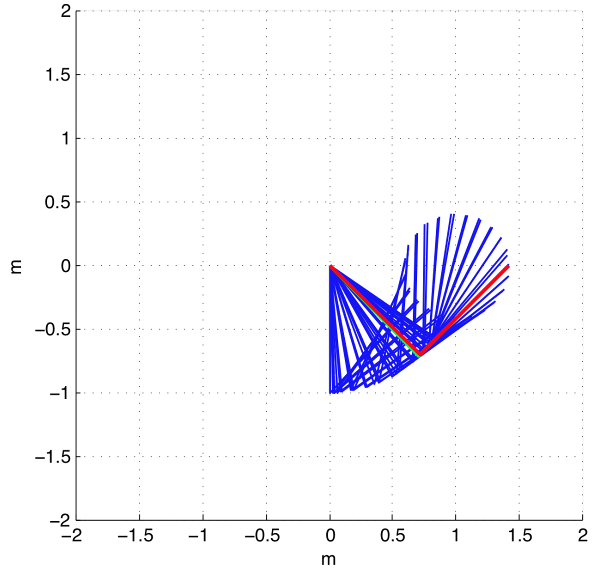
control inputs $\dot{q}_{r1}$, $\dot{q}_{r2}$



joint variables q_1, q_2

(almost) the same performance of the continuous case is recovered!!

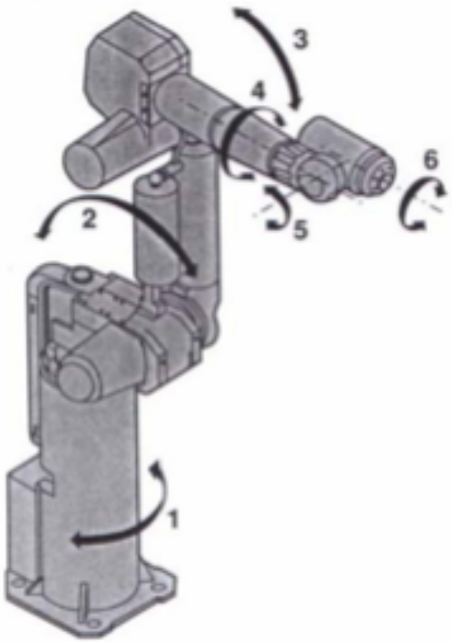


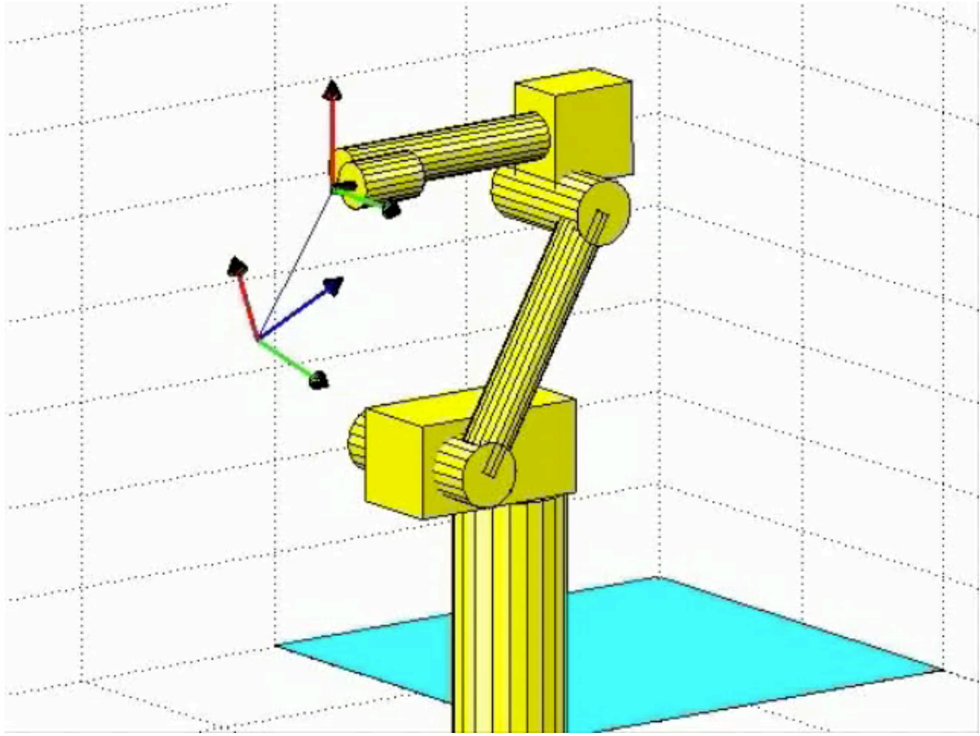


note however that larger P gains will eventually lead to unstable behavior (*see:* stability problems for discrete-time control systems)

3D simulation

video





kinematic control of Cartesian motion of Fanuc 6R (Arc Mate S-5) robot

simulation and visualization in Matlab

Kinematic control of KUKA LWR

video

kinematic control of Cartesian motion with redundancy exploitation
velocity vs. acceleration level