Robotics 1

Dynamic control of a single axis

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Dynamic control (single axis)

- when dynamic issues associated to the desired motion become relevant, one should consider robot mass/inertia and dissipative effects (friction) in the control design
- for a multi-dof articulated robot, the dynamics of each link is subject also to forces/torques due to
  - motion couplings with other links (inertial, centrifugal)
  - its own motion simultaneous with that of other links (Coriolis)
  - static loads (gravity, contact forces)
- the effects of these nonlinear couplings and loads can be partly "masked" in the dynamic behavior of a joint axis/motor load
  - if transmissions with high reduction ratios ($N \geq 100$) are used
- we will consider next the dynamic control design for a single joint axis of a robot (decentralized approach)

Dynamic model of a single robot axis

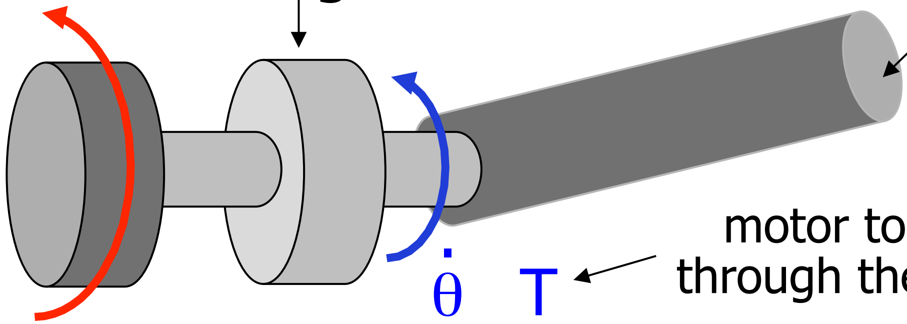
actuator
a DC motor, driven
by armature voltage

reduction gear N:1

link
(load)

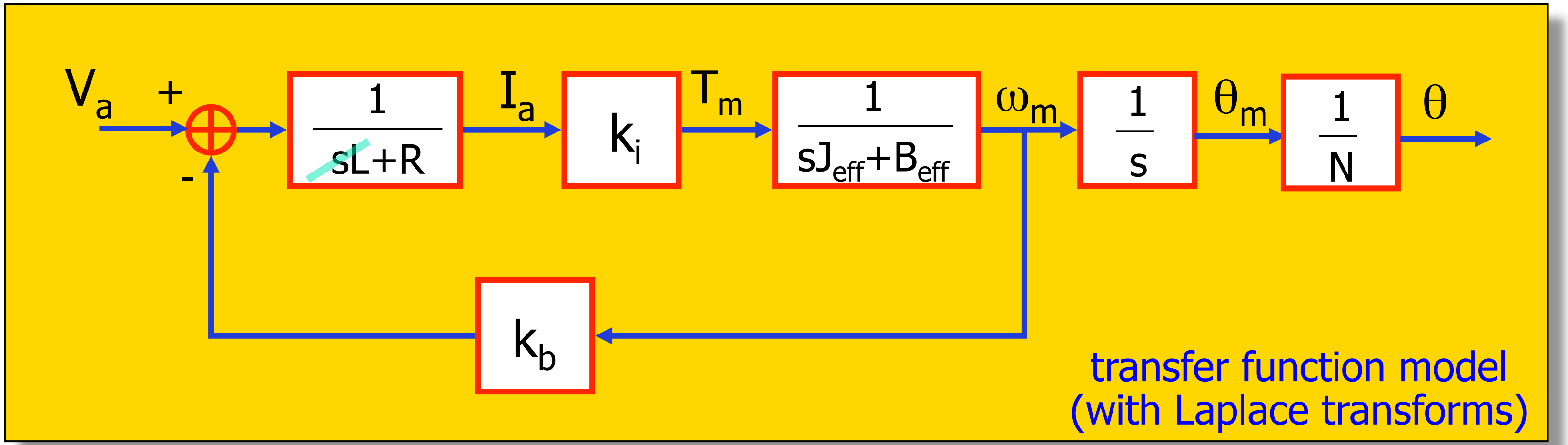
motor torque

$\dot{\theta}_m$ T_m



$\dot{\theta}$ T

motor torque reflected
through the reduction gear

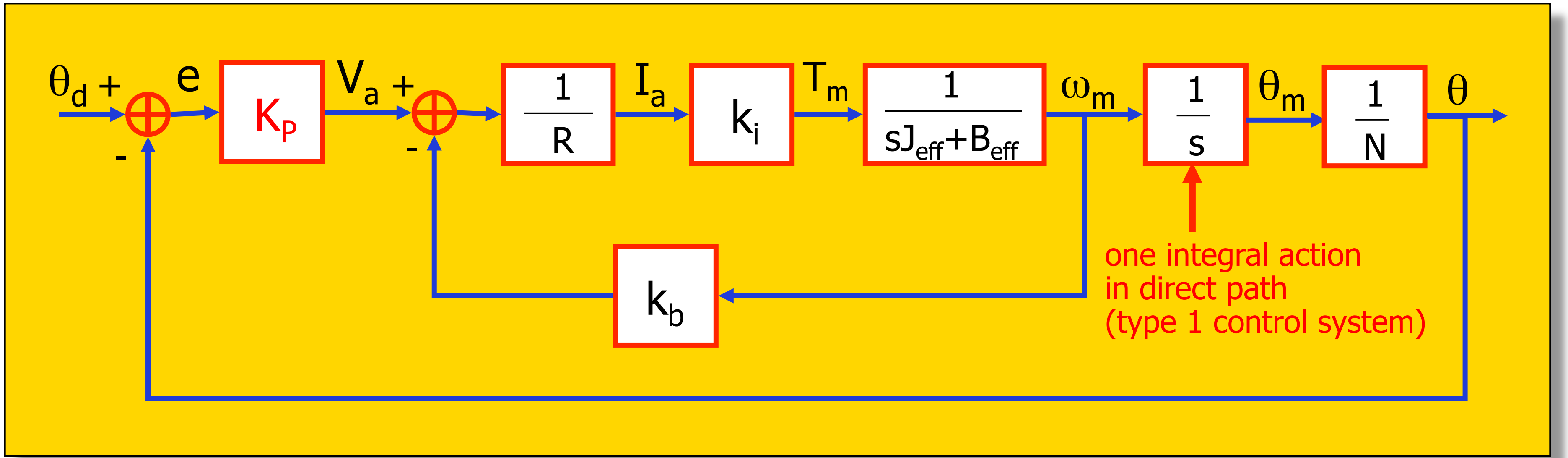


effective inertia $J_{eff} = J_m + 1/N^2 \, J_l$

$B_{eff} = B_m + 1/N^2 \, B_l$ effective viscous friction

$\approx 10^{-4}$

P control
(Proportional to the error)



closed-loop transfer function

$$\frac{\theta(s)}{\theta_d(s)} = \frac{\theta/e}{1+\theta/e} = \frac{K_P\, k_i}{N R\, J_{eff}} \; \frac{1}{s^2 + \dfrac{R\, B_{eff} + k_i\, k_b}{R\, J_{eff}}\, s + \dfrac{K_P\, k_i}{N R\, J_{eff}}}$$

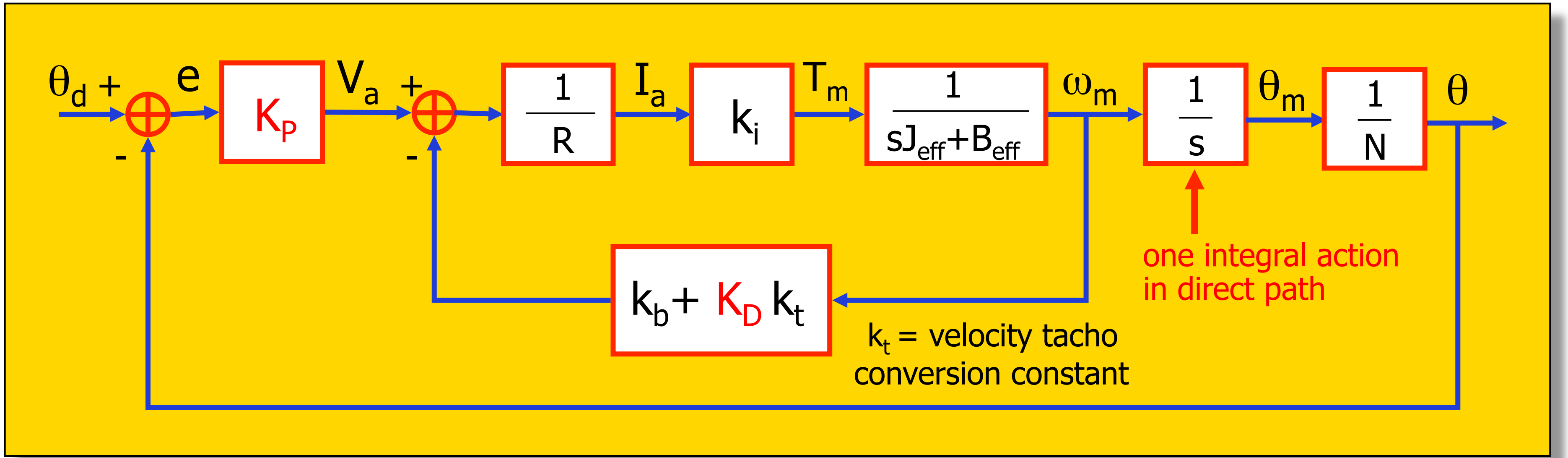
always ASYMPTOTICALLY STABLE for $K_P > 0$

Comments on P controller

- for θ_d = constant, the steady-state error is always zero
  - type 1 control system
- just one control design parameter (the gain K_P)
  - the (two) closed-loop poles cannot be independently assigned
  - in particular, the natural frequency ω_n and damping ratio ζ of this (complex) pole pair are coupled
- transient response and/or disturbance rejection features may not be satisfactory

note: variable measured for feedback is most often the motor position θ_m (where the encoder is usually mounted) $\Rightarrow \theta = \theta_m/N$

PD control
(Proportional-Derivative)



closed-loop transfer function

$$\frac{\theta(s)}{\theta_d(s)} = \frac{\theta/e}{1+\theta/e} = \frac{K_P\, k_i}{N R\, J_{eff}} \frac{1}{s^2 + \dfrac{R B_{eff} + k_i(k_b + K_D k_t)}{R\, J_{eff}}\, s + \dfrac{K_P\, k_i}{N R\, J_{eff}}}$$

always ASYMPTOTICALLY STABLE for K_P, $K_D > 0$

Comments on PD controller

- for θ_d = constant, $\dot{e} = -\dot{\theta}$, this scheme implements a PD action on the position error
- for $\theta_d \neq$ constant, in order to obtain a "true" PD action on the position error e (on the load side), the input reference to the control loop should be modified as

$$\theta_d + \dot{\theta}_d (N k_t K_D)/K_P$$

often neglected for large K_P

- K_P and K_D are chosen so as to yield smooth/fast transients
  - damping ratio $\zeta \geq 0.7$ (at $\zeta = 1$, two coincident negative real poles)
  - natural frequency $\omega_n < 0.5\,\omega_r$, where ω_r is the (lowest) resonance frequency of the joint assembly structure (with "braked" motor)
    - such a resonance (caused by the un-modeled elasticity of the transmission gears) should non be excited by the control law
    - current industrial robots have typically $f_r = \omega_r / 2\pi = 4 \div 20$ Hz

Simulation data
Matlab/Simulink

```
% Simulation parameters for the first (base) joint of the Stanford robot arm
% motor (U9M4T)
Ki = 0.043;    % torque/current constant [Nm/A]
Bm = 0.00008092;        % viscous friction coefficient [Nm s/rad]
Kb = 0.04297;           % back emf constant [V s/rad]
L = 0.000100;           % inductance of the equivalent armature circuit [H], negli
R = 1.025;     % resistance of the equivalent armature circuit [Ohm]
Ja = 0.000056;          % inertia of motor+tachometer assembly [Nm s^2/rad]
% velocity tachometer (Photocircuits 030/105)
Kt = 0.02149;           % tachometer conversion constant [V s/rad]
% reduction
n = 0.01;      % inverse of reduction ratio (= 1/N)
% load
Jl = 5;        % inertia on the link side [Nm s^2/rad] (varies from 1.4 to 6.17)
Bl = 0;        % viscous friction coefficient on the link side (N/A)
omr = 25.13;% resonant frequency (at nominal Jl) [rad/s] (4 Hz)
% computed parameters
Beff = Bm + Bl*n^2;     % effective viscous friction coefficient
Jeff = Ja + Jl*n^2;     % effective inertia
% reference input
qdes = 1;      % desired joint angle value (for step input case) [rad]
Kram = 2;      % angular coefficient (for position ramp input) [rad/s]
% possible "hard" nonlinearities
Fm = 0.042;  % dry friction torque [Nm]
D = 0.0087;  % reduction gear backlash [rad] (0.5 deg)
Tmax = 4;    % motor torque saturation level [Nm]
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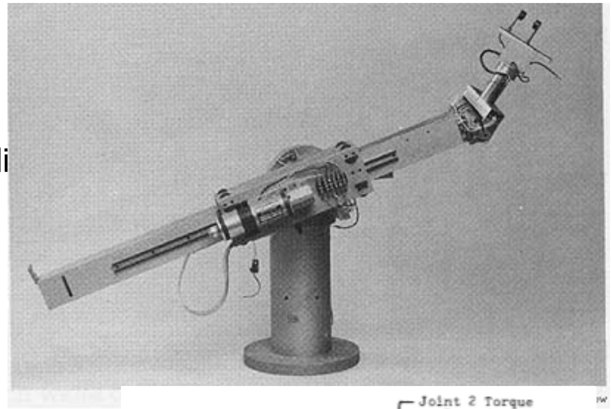
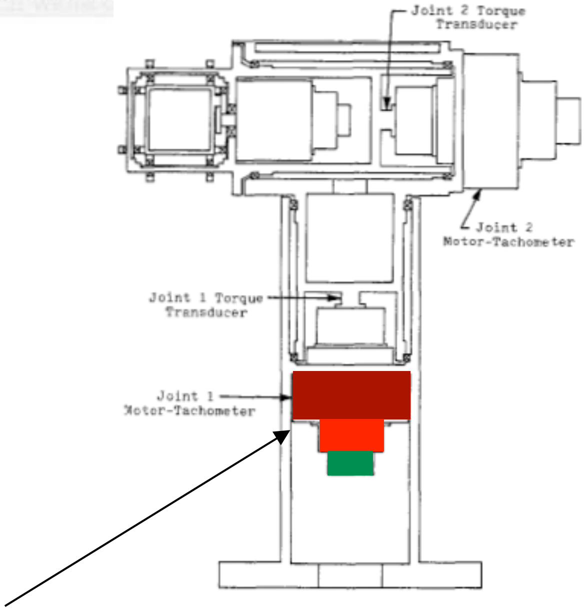
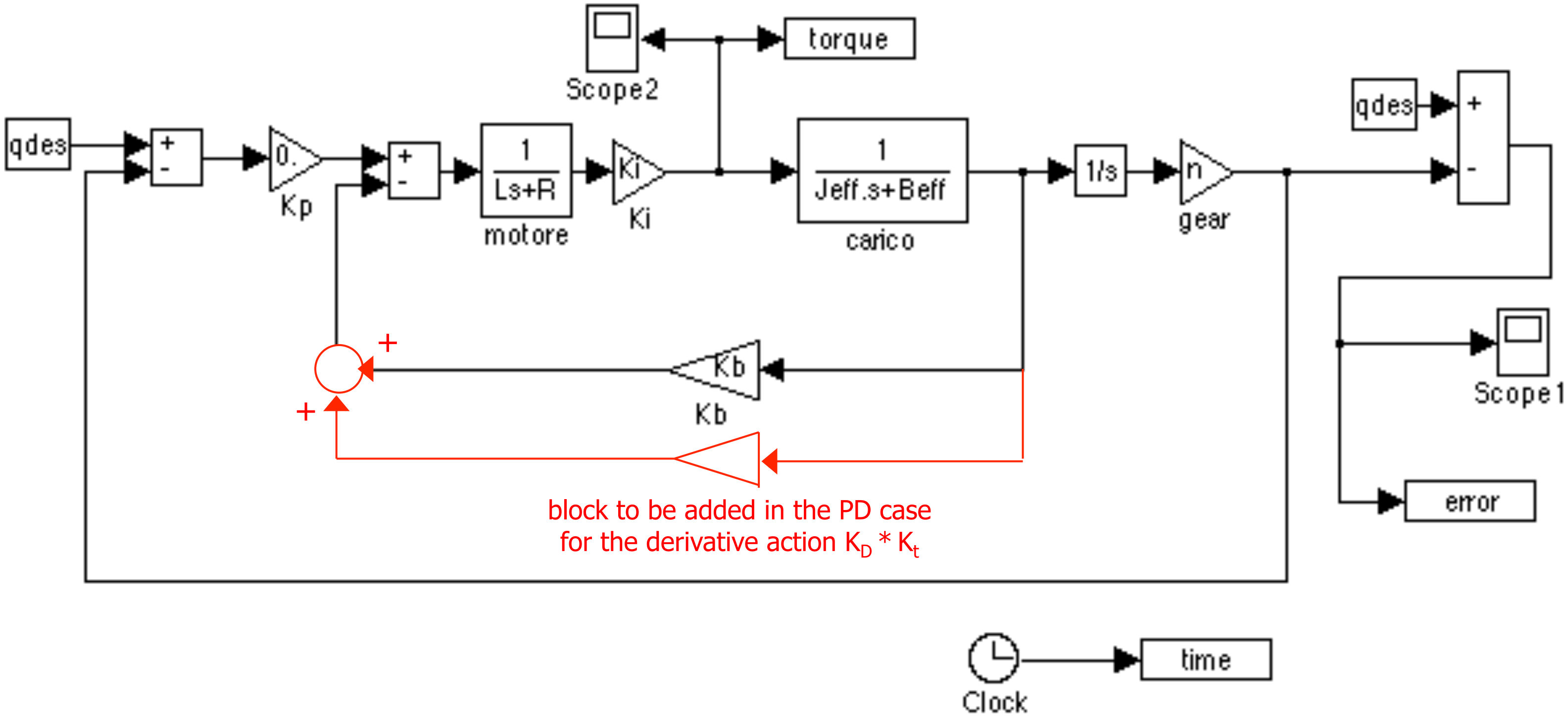




motor, velocity tachometer, optical encoder

Simulink block diagram

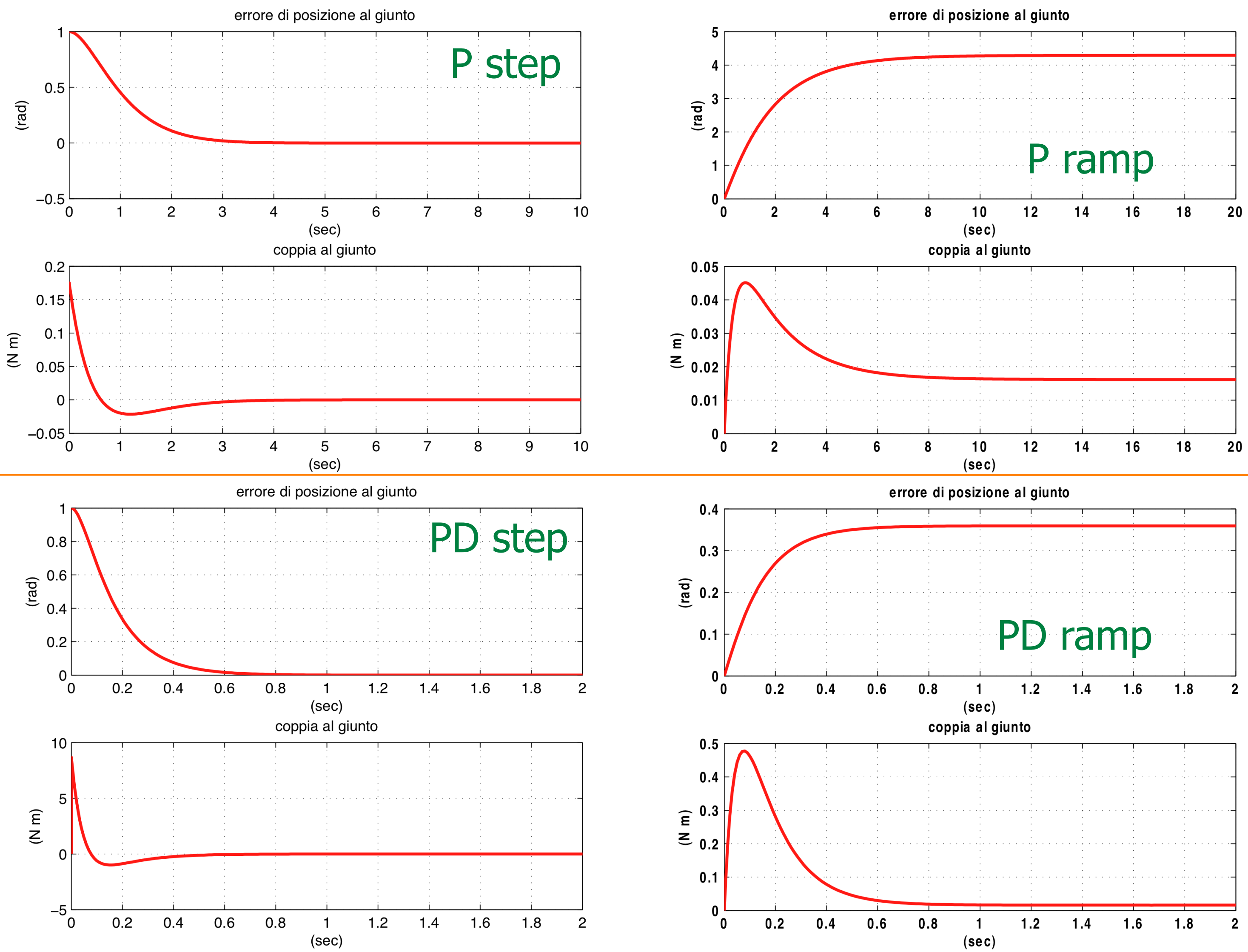
dynamic model and P/PD control



- P control law: K_P = 4.2 (the maximum value that guarantees motion transients without oscillations)
- PD control law: K_P = 209, K_D = 15.4 (such as to obtain a ≈ critically damped transient behavior)

P/PD control results

step (1 rad) and ramp (2 rad/s) responses



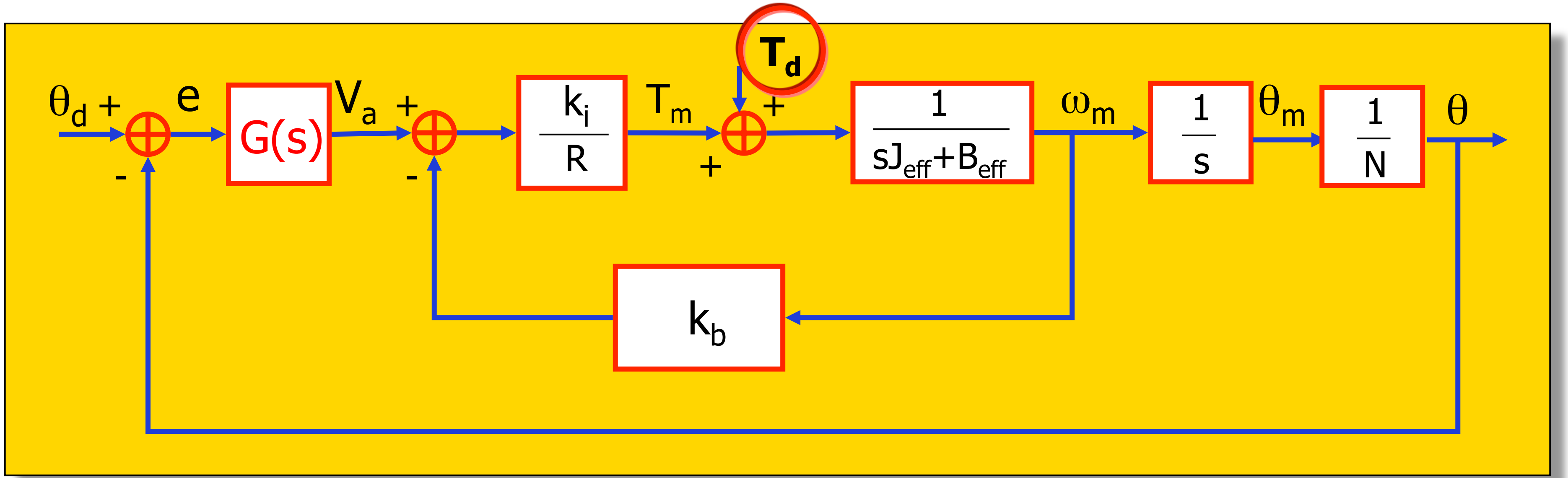
position error

control torque

position error

control torque

General case (n joints)



$\mathbf{T_d}$ = disturbance torque due to inertial couplings with other links/axes, centrifugal/Coriolis terms, and gravity (only position-dependent)

in order to obtain zero error at steady state at least for a constant disturbance (robot at rest, under gravity), an integral action should be added in the direct path before the disturbance entry point (astatic control behavior)

⇒ G(s) = PID controller

PID control
(Proportional-Integral-Derivative)

- $G(s) = K_P + K_I/s + K_D\, s$
  - as usual, the derivative (anticipative) action must be low-pass filtered in order to be physically realizable
- closed-loop transfer function

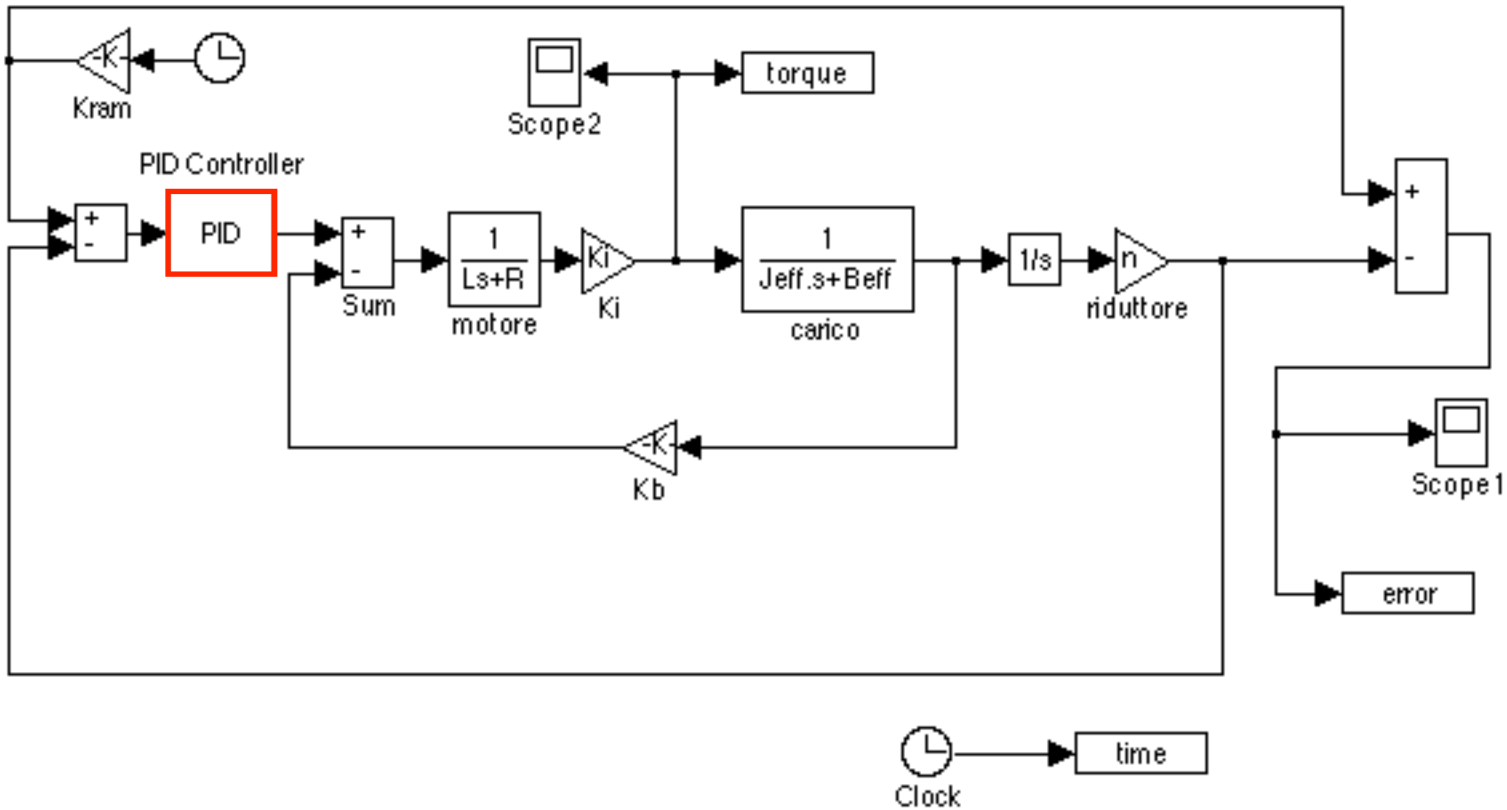
$$\frac{\theta(s)}{\theta_d(s)} = \frac{(K_D\, s^2 + K_P\, s + K_I)\, k_i}{NRJ_{eff}\, s^3 + (NRB_{eff} + Nk_b k_i + K_D k_i)s^2 + k_i K_P s + k_i K_I}$$

- asymptotic stability if and only if (Routh criterion)

$$0 < K_I < \underbrace{K_P/RJ_{eff}}_{>0}\, \underbrace{(RB_{eff} + K_D k_i/N + k_b k_i)}_{>0}$$

- control system of type 2 and astatic w.r.t. disturbance

Simulink block diagram
dynamic model and PID control



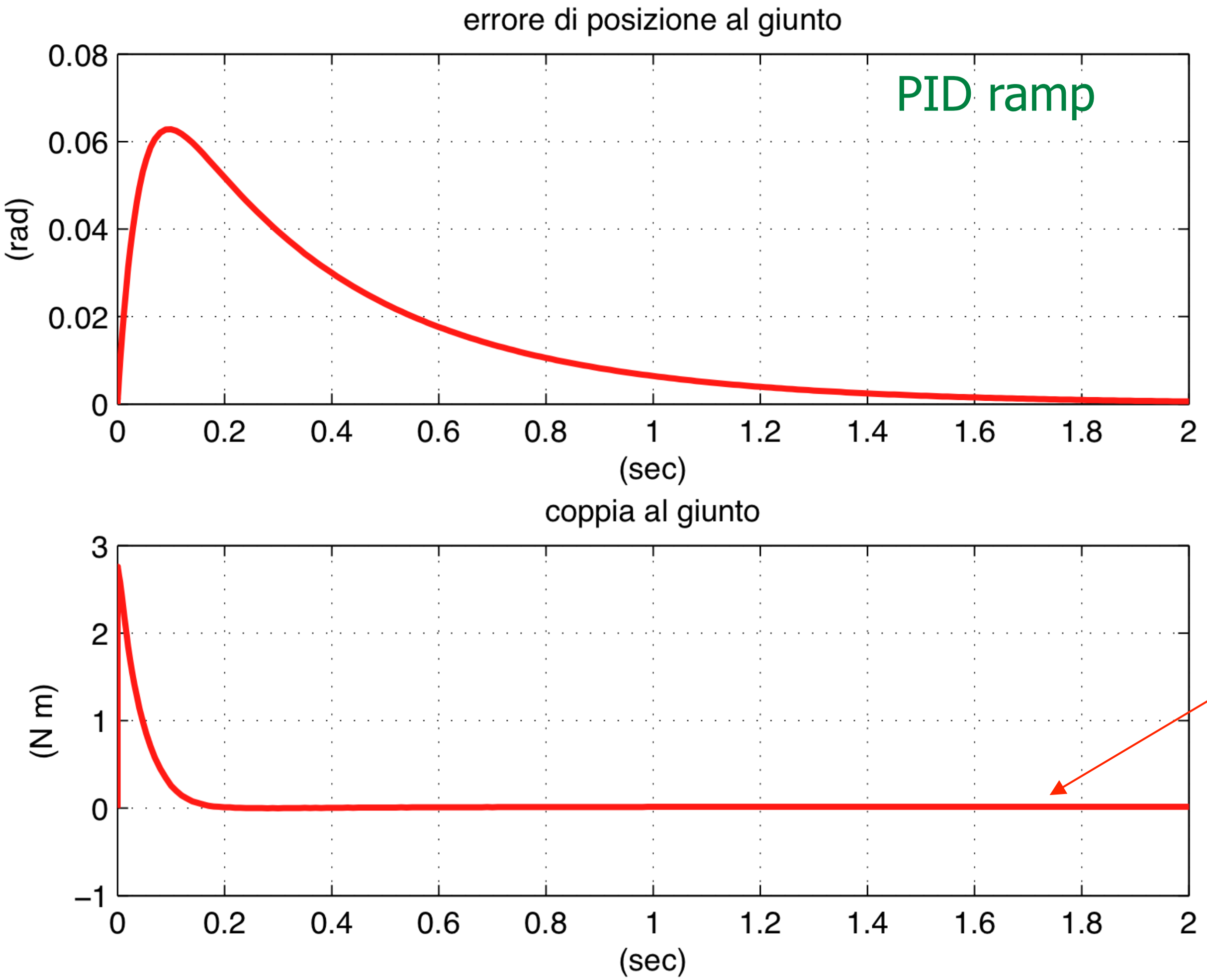
- gain after some tuning: K_P = 209 (as for PD law), K_D = 33, K_I = 296
- type 2 control system ⇒ zero steady-state error on position ramp inputs

PID control results

ramp (2 rad/s) response

position error

control torque



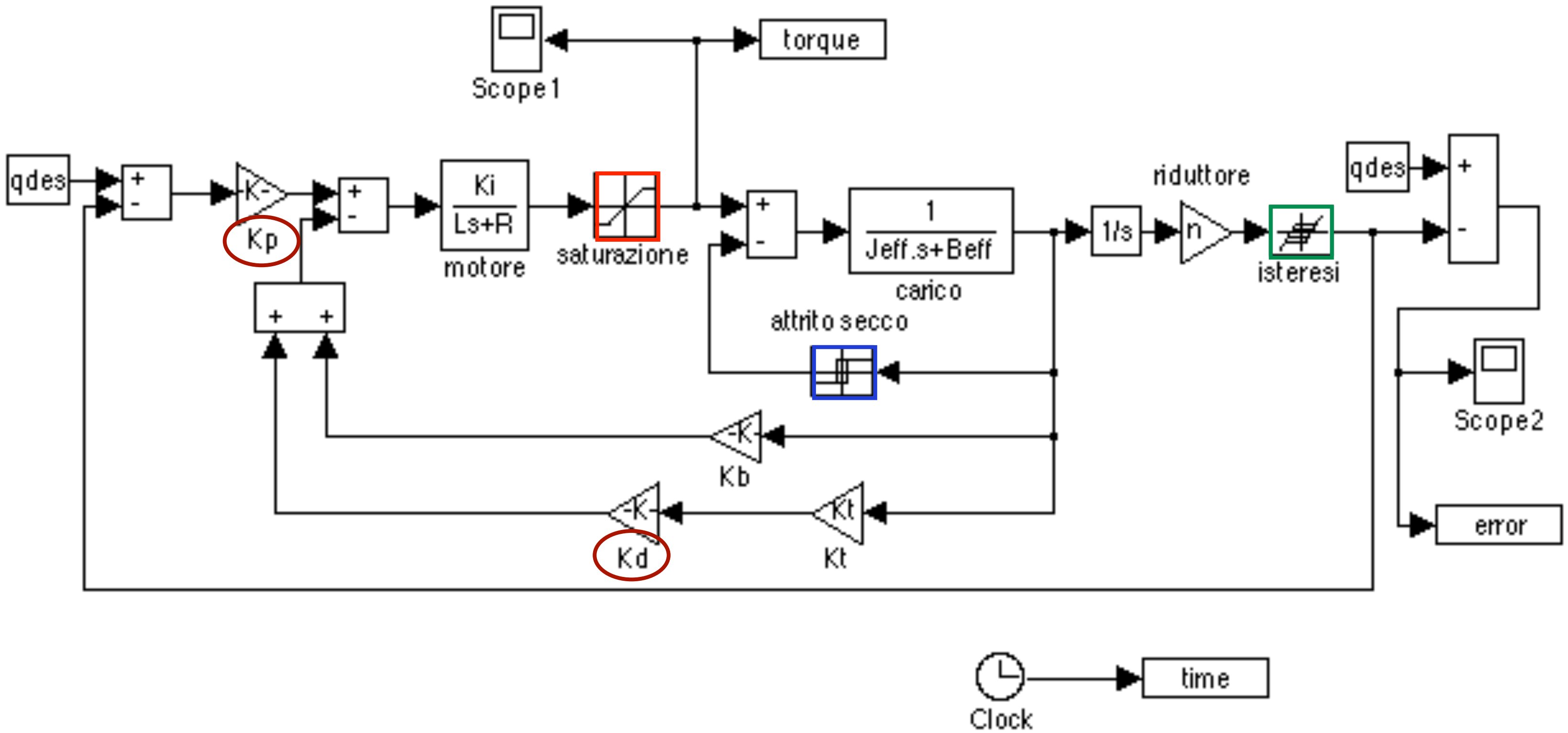
note: the torque at steady-state is NOT zero (≅ 0.02 Nm)

Final remarks

- there are many non-linear physical phenomena that cannot be directly considered in control design and analysis based on linear models
  - actuator saturations
  - transmission/gear backlash (delay, hysteresis)
  - dry friction and static friction
  - sensor quantization (encoder)
  - ...
- approximate mathematical models can be obtained and then simulated in combination with the already designed control law, for a more realistic validation of system behavior and control performance
- similarly, uncertainties on nominal parameters of robot kinematics/dynamics can be included in the simulation

Simulink block diagram

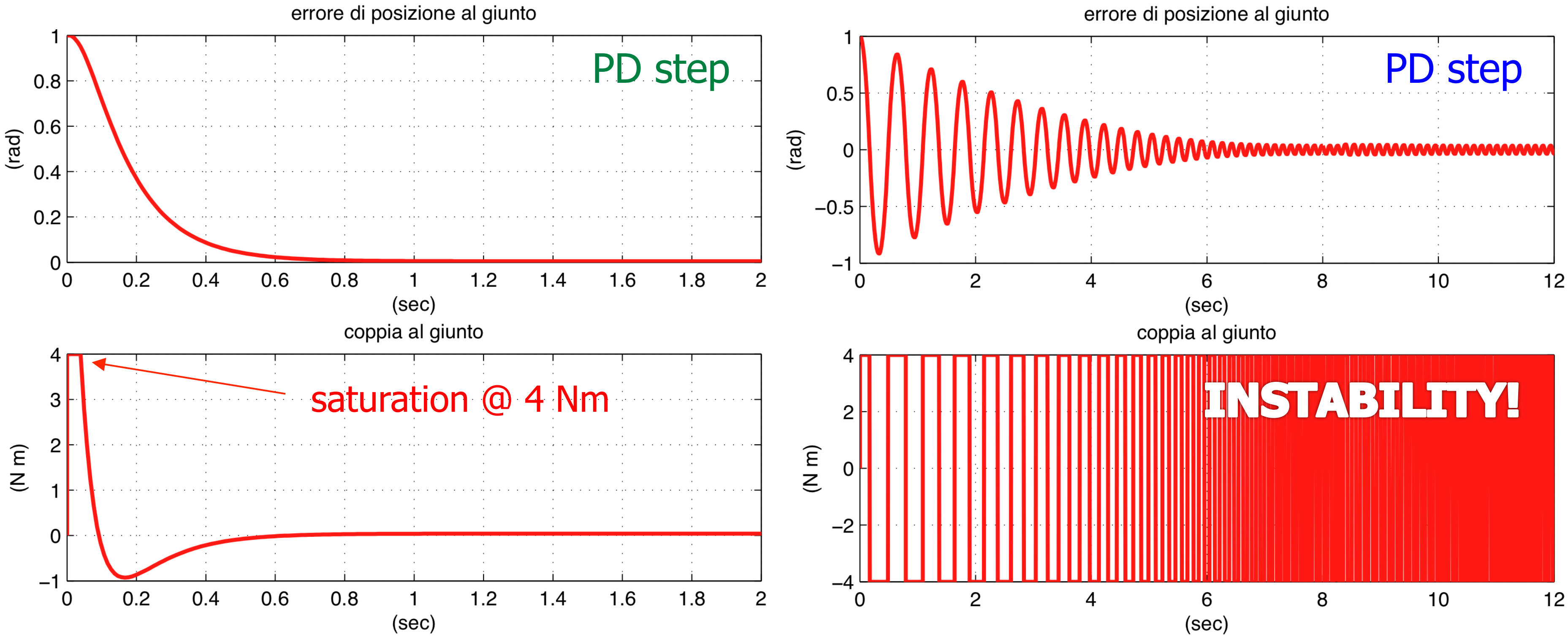
dynamic model with nonlinear phenomena and PD control



- actuator saturation, dry friction, backlash in reduction gears
- PD control

PD control results

step (1 rad) response with non-idealities



same PD gains as before

gears are always engaged
(already when motion starts)

with larger P gain....

gears initially engaged, but not
when velocity inversion occurs
→ "chattering" due to backlash