Robotics I

April 5, 2022

Exercise 1

Consider the spatial 4R robot shown in Fig. 1.

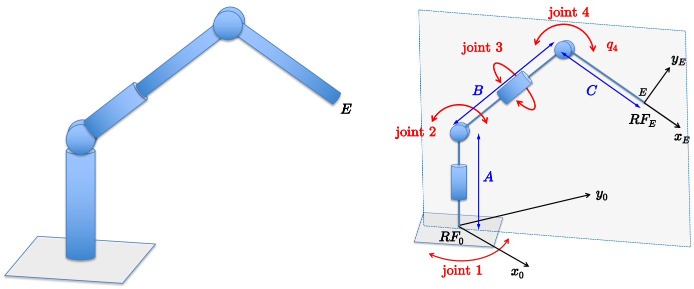

Figure 1: A 4R robot and its kinematic skeleton.

- Assign a set of frames to the robot according to the Denavit-Hartenberg convention and provide the associated table of parameters. Keep the reference frame RF_0 as shown in the figure.
- Determine the homogeneous transformation ${}^4\boldsymbol{T}_E$ from the assigned Denavit-Hartenberg frame RF_4 to the end-effector frame RF_E shown in the figure.
- Compute the symbolic expression of the position ${}^0\boldsymbol{p}_E(\boldsymbol{q})$ of the origin of the end-effector frame by using the minimum amount of operations. Show all intermediate passages. For $A = B = C = 1$, give the numerical value of the position ${}^0\boldsymbol{p}_E$ when $\boldsymbol{q} = (\pi/2, \pi/2, 0, 0)$.
- Compute the angular part of the geometric Jacobian, namely the 3×4 matrix $\boldsymbol{J}_A(\boldsymbol{q})$ such that

$$\boldsymbol{\omega}_E = \boldsymbol{J}_A(\boldsymbol{q})\dot{\boldsymbol{q}},$$

  and find all its singularities.
- Find the symbolic expression (as a function of the configuration $\boldsymbol{q}$) of a non-trivial joint velocity $\dot{\boldsymbol{q}}_0 \neq \boldsymbol{0}$ such that $\boldsymbol{\omega}_E = \boldsymbol{J}_A(\boldsymbol{q})\dot{\boldsymbol{q}}_0 = \boldsymbol{0}$ for all possible $\boldsymbol{q}$.

Exercise 2

Consider the motion profile in Fig. 2 for a generic robot joint, parametrized by the amplitude $J > 0$ and the duration $T > 0$. This time profile represents the motion jerk, namely the third time derivative of the joint position $q(t)$, for $t \in [0, T]$.

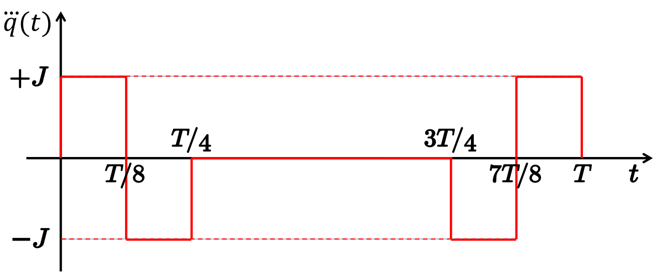

Figure 2: The jerk profile $\dddot{q}(t)$ of the joint motion.

- For a (rest-to-rest) motion with zero boundary conditions on velocity and acceleration, determine the value of the net displacement $\Delta = q(T) - q(0)$ as a function of J and T.
- For $J = 100$ [rad/s^3] and $T = 2$ [s], provide the numerical value of Δ. If we wish to have a displacement $\Delta = -2$ [rad] in $T = 4$ [s], what should be the numerical value of J?

[180 minutes, open books]