Robotics 1

September 11, 2023

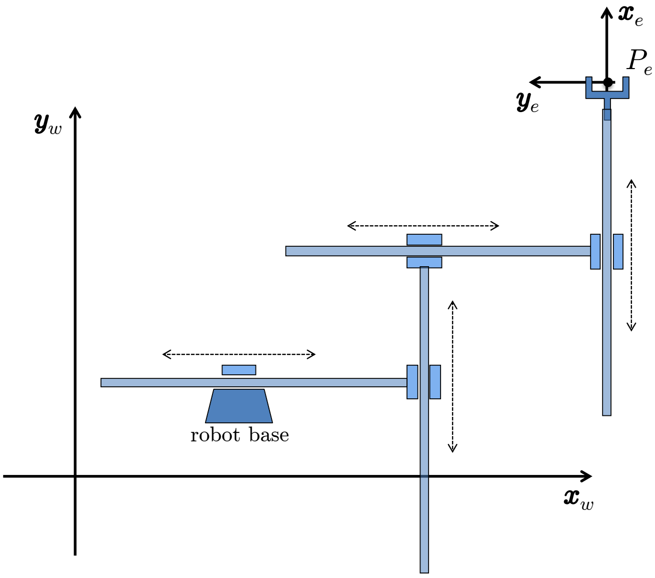

Figure 1: A 4P planar robot, with the world frame RF_w and of the end-effector frame RF_e.

Consider the 4-dof planar robot with a fixed base shown in Fig. 1. All robot joints are prismatic.

1. Draw the Denavit-Hartenberg (DH) frames and fill in the corresponding table of DH parameters.
2. Provide the two constant homogeneous transformations ${}^w\boldsymbol{T}_0$ and ${}^4\boldsymbol{T}_e$, relating respectively the world frame RF_w to the 0-th DH frame and the 4-th DH frame to the end-effector frame RF_e.
3. Compute the direct kinematics as expressed by the homogeneous transformation matrix

$$ {}^w\boldsymbol{T}_e(\boldsymbol{q}) = \begin{pmatrix} {}^w\boldsymbol{R}_e(\boldsymbol{q}) & {}^w\boldsymbol{p}_{we}(\boldsymbol{q}) \\ \boldsymbol{0}^T & 1 \end{pmatrix}, \qquad {}^w\boldsymbol{p}_{we}(\boldsymbol{q}) = \begin{pmatrix} p_x(\boldsymbol{q}) \\ p_y(\boldsymbol{q}) \\ p_z(\boldsymbol{q}) \end{pmatrix}. $$

4. Let the task vector be $\boldsymbol{r} = \boldsymbol{f}_r(\boldsymbol{q}) = (p_x(\boldsymbol{q}), p_y(\boldsymbol{q})) \in I\!R^2$. Compute the associated task Jacobian $\boldsymbol{J}(\boldsymbol{q}) = \partial \boldsymbol{f}_r / \partial \boldsymbol{q}$ and find its singularities.
5. At a given nonsingular $\boldsymbol{q}$, compute a basis for each of the two subspaces $\mathcal{N}(\boldsymbol{J})$ and $\mathcal{R}(\boldsymbol{J}^T)$.
6. Determine the joint velocity $\dot{\boldsymbol{q}} \in I\!R^4$ with minimum norm that realizes a desired $\dot{\boldsymbol{r}} = (3, -2)$ [m/s].
7. Determine a joint torque $\boldsymbol{\tau} \in I\!R^4$ that statically balances a Cartesian force $\boldsymbol{F} = (2, 1)$ [N] applied at the robot end-effector. Is this $\boldsymbol{\tau}$ unique in the present case?
8. Plan a linear Cartesian trajectory between $\boldsymbol{r}_{\text{in}} = (-1, 1)$ and $\boldsymbol{r}_{\text{fin}} = (3, 7)$ and determine the minimum rest-to-rest motion time T when the joint velocity and acceleration limits are

$$ |\dot{q}_i| \leq 2 \text{ [m/s]}, \qquad |\ddot{q}_i| \leq 5 \text{ [m/s}^2\text{]}, \qquad \text{for all } i \in \{1, 2, 3, 4\}. $$

Motion of all joints should be coordinated. Discontinuous joint accelerations are admissible.

[150 minutes, open books]