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DYNAMIC DECOUPLING OF VOLTAGE FREQUENCY CONTROLLED INDUCTION MOTORS

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Abstract. A new nonlinear control scheme for voltage frequency controlled (VFC) induction motors is presented, based on dynamic state-feedback. The proposed approach allows to design an input-output decoupling controller for motor torque and flux, using as inputs the amplitude and the frequency of the supply voltage. The closed-loop system contains an unobservable sink. The dynamics of this part is stable and is related to the sinusoidal steady state behavior of the motor. Simulation tests are included which validate the control scheme.

1. Introduction

The control of drives using an induction motor as actuator is a long standing and thoroughly investigated problem [1,2,3]. The control system has to be designed in order to produce a torque output which tracks a given reference profile while keeping limited, even during fast transients, both the machine flux and the current sinked from the inverter. In fact, when the modulus of the flux exceeds some threshold value, which depends on the machine characteristics, the motor operates in an improper way; besides, the usual mathematical model of the machine does not hold anymore. On the other hand, an inverter cannot source a current value which is higher than its rated one, even for short time intervals. Therefore, limiting current transients in the motor has also a direct influence on the size of the inverter.

The stated control problem has an inherent smooth nonlinearity due to the fact that the output torque of an induction motor is a nonlinear function of the motor "physical" state variables, i.e. currents and fluxes. Moreover, different nonlinearities may arise in the dynamic behavior depending on the particular choice of the input variables.

Among the many solutions proposed for this problem, most of them are based on schemes which control separately the motor flux and the produced torque. In some of them, known as "field oriented" or "vector" control methods [1-4], it is possible to obtain, under certain hypotheses, an approximately linear and decoupled relation between input and output variables. Other approaches have been described which take explicitly into account the nonlinear nature of the model, based on the application of optimal [5,6] or adaptive [7,8] control. Recently, the use of differential-geometric concepts and control techniques for nonlinear systems has proven to be effective for the exact linearizing and decoupling control of reluctance motors [9] and induction machines [10,11].

Generally, these approaches heavily rely on a proper representation of the vector variables of the system in a reference frame which rotates at a suitable speed. Measurable quantities and control variables are instead inherently expressed in terms of a fixed reference frame. Therefore, the actual implementation of these techniques requires several coordinate trasformations, which represent a relevant overhead for the control task.

Moreover, all these methods use as control variables the two projections on a fixed frame of the representative vector of the supply voltage. This limits the available choices for the supply system (inverter and modulation device). When the AC machine is supplied by power devices which are driven by pulse-width modulating (PWM) techniques, the amplitude and the frequency (rotating speed) of the voltage vector are the most appealing inputs. In fact, based on these inputs, optimal PWM techniques exist which minimize some suitable performance index which takes into account e.g. the harmonic contents of the driving signal.

This paper proposes a nonlinear feedback control approach leading to a decoupling scheme for a VFC model of the induction motor which is based on a fixed frame description and uses the the amplitude and the frequency of the supply voltage as inputs.

The definition of the system outputs is typically connected with the control objectives. Usually, in drives the most important mechanical variable is the torque produced by the machine. To ensure correct motor operation, another controlled variable should be related to the motor flux and may be defined in terms of either stator or rotor fluxes, because of the tight couplings between the two windings.

The paper is organized as follows. The nonlinear dynamic model of the motor is described in Section 2. The synthesis of an input-output linearizing and decoupling feedback controller for the VFC induction motor is described in detail in Section 3. This result is an application of the theory of input-output decoupling of nonlinear systems via *dynamic* state-feedback [12]. Finally, simulation tests are reported and discussed.

2. Modeling of the induction machine

The dynamic behavior of a voltage fed induction motor can be described by a set of four differential equations, based on the two-phase equivalent machine representation [3]. Standard simplifying hypotheses are made, i.e. iron losses and magnetic circuits' saturation are neglected and an isotropic structure is assumed.

Different choices of two-dimensional vector variables may be used, describing the motor dynamics in terms of rotor and/or stator fluxes and/or currents. The projections of the stator current and flux vectors on a reference frame (α,β) which is *fixed* to the stator windings are taken as state variables. The i_α and i_β components of the stator current are obtained on the basis of direct measurements, while the φ_α and φ_β flux components can be reconstructed by means of an asymptotic observer of reduced order, as shown in [13,14]. Usually, the projections v_α and v_β of the supply voltage are assumed as input variables. Therefore, setting

$$\bar{x} = \begin{bmatrix} i_\alpha & i_\beta & \varphi_\alpha & \varphi_\beta \end{bmatrix}^T \qquad \bar{u} = \begin{bmatrix} v_\alpha & v_\beta \end{bmatrix}^T$$

the dynamic equations describing the motor are:

$$\dot{\bar{x}} = A\bar{x} + B\bar{u}$$

where

$$A = \begin{bmatrix} -(\alpha+\beta) & -\omega & \frac{\beta}{L_s} & \frac{\omega}{\sigma L_s} \\ \omega & -(\alpha+\beta) & -\frac{\omega}{\sigma L_s} & \frac{\beta}{L_s} \\ -\alpha\sigma L_s & 0 & 0 & 0 \\ 0 & -\alpha\sigma L_s & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} \frac{1}{\sigma L_s} & 0 \\ 0 & \frac{1}{\sigma L_s} \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and $\alpha = R_s / \sigma L_s$, $\beta = R_r / \sigma L_r$, $\sigma = 1 - (M^2 / L_s L_r)$. The parameters R_s and R_r are the stator and rotor resistances, L_s and L_r are the stator and rotor self-inductances and M is the mutual inductance. The speed ω can be considered as a slowly varying parameter, due to the large separation of time-scales between the mechanical and the electromagnetic dynamics. Treating ω as a state variable would require the inclusion of a model of the load, which is typically very poorly known and possibly even not smooth.

To obtain a voltage-frequency control scheme for the induction motor (see Figure 1), the voltage input vector should be expressed as

$$\bar{u} = \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \begin{bmatrix} V\cos\vartheta \\ V\sin\vartheta \end{bmatrix} \qquad \text{where} \quad \vartheta = \int_0^t \omega_a(\tau)\, d\tau$$

being ϑ its angular position and V the amplitude. ω_a is the voltage supply frequency. In order to use $\mathbf{u} = (V, \omega_a)$ as the new control input, the model has to be augmented with another state variable, $x_5 = \vartheta$. Setting

$$\mathbf{x} = \begin{bmatrix} \bar{x}^T & x_5 \end{bmatrix}^T, \qquad \mathbf{u} = \begin{bmatrix} V & \omega_a \end{bmatrix}^T$$

the motor state equations are written in their final form as

$$\dot{\mathbf{x}} = f(\mathbf{x}) + g(\mathbf{x})\,\mathbf{u}$$

with

$$f(\mathbf{x}) = \begin{bmatrix} A\bar{x} \\ 0 \end{bmatrix} \qquad g(\mathbf{x}) = \begin{bmatrix} \frac{\cos x_5}{\sigma L_s} & 0 \\ \frac{\sin x_5}{\sigma L_s} & 0 \\ \cos x_5 & 0 \\ \sin x_5 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} g_1(x_5) & \mathbf{0} \\ 0 & 1 \end{bmatrix}$$

This modeling approach has some nice features: i) the angular motion $\vartheta(t)$ of the voltage representative vector is smooth; ii) the model becomes nonlinear but is still linear in the new input $\mathbf{u}$; iii) in the steady state, the value of this new input is constant.

Suitable outputs for the systems are defined in terms of the stator flux and the torque. Hence, the following nonlinear output functions will be used

$$y_1 = \Phi_s^2 = x_3^2 + x_4^2 = h_1(\mathbf{x})$$

$$y_2 = T_m = x_2 x_3 - x_1 x_4 = h_2(\mathbf{x})$$

where a motor with only one pole pair is considered.

3. Decoupling control of induction motor

Starting from the motor triple $\{f(\mathbf{x}), g(\mathbf{x}), h(\mathbf{x})\}$ one may first check whether the condition for input-output decoupling via a static state-feedback of the form $\mathbf{u} = \alpha(\mathbf{x}) + \beta(\mathbf{x})\mathbf{v}$ is satisfied. It is well-known that this possible if and only if the decoupling matrix of the system is nonsingular [15]. It is easy to see that in this case the decoupling matrix becomes

$$A(\mathbf{x}) = L_g h = \begin{bmatrix} 2x_3\cos x_5 + 2x_4 \sin x_5 & 0 \\ (\frac{x_3}{\sigma L_s} - x_1)\sin x_5 - (\frac{x_4}{\sigma L_s} - x_2)\cos x_5 & 0 \end{bmatrix}$$

resulting structurally singular. Here, $L_g h$ denotes in a matrix compact form the Lie derivatives of the functions $h_j(\mathbf{x})$ w.r.t. the vector fields $g_i(\mathbf{x})$.

In order to achieve decoupling consider the use of a dynamic state-feedback compensator of the form

$$\dot{\mathbf{z}} = a(\mathbf{x},\mathbf{z}) + b(\mathbf{x},\mathbf{z})\,\mathbf{v} \qquad \mathbf{u} = c(\mathbf{x},\mathbf{z}) + d(\mathbf{x},\mathbf{z})\,\mathbf{v}$$

where the compensator state $\mathbf{z}$ has a dimension not specified a priori. Following [12], the system is dynamically extended by adding one integrator to the first input:

$$u_1 = z \qquad \dot{z} = w_1 \qquad u_2 = w_2$$

To avoid a burdening of notation, the extended state is redefined as

$$\mathbf{x} = \begin{bmatrix} \bar{x}^T & z \end{bmatrix}^T = \begin{bmatrix} x_1 & \dots & x_6 \end{bmatrix}^T$$

and the state equations of the extended system become

$$\dot{\mathbf{x}} = f(\mathbf{x}) + G\mathbf{w}$$

with

$$f(\mathbf{x}) = \begin{bmatrix} A\bar{x} + x_6\, g_1(x_5) \\ 0 \\ 0 \end{bmatrix} \qquad G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Computing the decoupling matrix $A(\mathbf{x})$ for this system leads to relative degrees $r_1 = r_2 = 2$ associated to the outputs and

$$A(\mathbf{x}) = L_G L_f h(\mathbf{x}) = \begin{bmatrix} a_{11}(\mathbf{x}) & a_{12}(\mathbf{x}) \\ a_{21}(\mathbf{x}) & a_{22}(\mathbf{x}) \end{bmatrix} =$$

$$= \begin{bmatrix} 2(x_3\cos x_5 + x_4\sin x_5) & 2x_6(x_4\cos x_5 - x_3\sin x_5) \\ (\frac{x_3}{\sigma L_s} - x_1)\sin x_5 - (\frac{x_4}{\sigma L_s} - x_2)\cos x_5 & x_6\,[(\frac{x_3}{\sigma L_s} - x_1)\cos x_5 + (\frac{x_4}{\sigma L_s} - x_2)\sin x_5] \end{bmatrix}$$

which is a nonsingular matrix in all points of the extended state space where

$$\det A(\mathbf{x}) = 2\,x_6\,[\frac{x_3^2 + x_4^2}{\sigma L_s} - (x_1x_3 + x_2x_4)\,] \neq 0$$

It can be shown that the term inside the square brackets is proportional to the scalar product of the stator and rotor flux vectors and hence is nonzero during normal mode of motor operation. Moreover, x_6 is always different from zero being the amplitude of the supply voltage. Hence, the static decoupling feedback law from the extended state is a feasible one and yields

$$\mathbf{w} = \alpha(\mathbf{x}) + \beta(\mathbf{x})\,\mathbf{v}$$

where

$$\beta(\mathbf{x}) = A^{-1}(\mathbf{x}) = \frac{1}{\det A(\mathbf{x})}\begin{bmatrix} a_{22}(\mathbf{x}) & -a_{12}(\mathbf{x}) \\ -a_{21}(\mathbf{x}) & a_{11}(\mathbf{x}) \end{bmatrix}$$

and

$$\alpha(\mathbf{x}) = \beta(\mathbf{x})\, L_f^2 h(\mathbf{x})$$

$$L_f^2 h_1 = \frac{\partial}{\partial \mathbf{x}}\{2[x_6(x_4\sin x_5 + x_3\cos x_5) - \alpha\sigma L_s(x_1x_3 + x_2x_4)]\}\cdot f(\mathbf{x})$$

$$L_f^2 h_2 = \frac{\partial}{\partial \mathbf{x}}\{\, \omega(x_1x_3+x_2x_4) - (\alpha+\beta)(x_2x_3 - x_1x_4) - \frac{\omega}{\sigma L_s}(x_3^2+x_4^2)$$

$$+ x_6\,[(\frac{x_3}{\sigma L_s} - x_1)\sin x_5 - (\frac{x_4}{\sigma L_s} - x_2)\cos x_5]\,\}\cdot f(\mathbf{x})$$

The resulting overall dynamic compensator is

$$\dot{z} = w_1 = \alpha_1(\bar{\mathbf{x}},z) + \beta_{11}(\bar{\mathbf{x}},z)\, v_1 + \beta_{12}(\bar{\mathbf{x}},z)\, v_2$$

$$u_1 = z$$

$$u_2 = \alpha_2(\bar{\mathbf{x}},z) + \beta_{21}(\bar{\mathbf{x}},z)\, v_1 + \beta_{22}(\bar{\mathbf{x}},z)\, v_2$$

where $\alpha_i(\mathbf{x})$ and $\beta_{ij}(\mathbf{x})$ are elements of the (α,β) pair defined above.

The closed-loop equations may be written in terms of new coordinates ξ defined as

$$\xi = T(\mathbf{x}) = \begin{bmatrix} h_1(\mathbf{x}) & L_f h_1(\mathbf{x}) & h_2(\mathbf{x}) & L_f h_2(\mathbf{x}) & x_1 & x_2 \end{bmatrix}^T$$

This state-space smooth transformation is a diffeomorphism wherever the Jacobian of $T(\mathbf{x})$ is nonsingular i.e. where

$$\det\left[\frac{\partial T}{\partial \mathbf{x}}\right] = -2\,(x_1x_3 + x_2x_4)\cdot \det A(\mathbf{x}) \neq 0$$

Hence, the set of coordinates ξ is a feasible one in the subregion of definition of the control law ($\mathbf{x}$ s.t. $\det A(\mathbf{x}) \neq 0$) where the stator current vector is not orthogonal to the stator flux. In this singular situation a different description may be used. In any case, this does not affect the control law.

Figure 2 shows the equivalent structure of the closed-loop system. The input-output relation from $\mathbf{v}$ to $\mathbf{y}$ is given by two double integrators and an unobservable part of dimension $n - (r_1 + r_2) = 6 - 4 = 2$ arises. The stability of this subsystem is a crucial issue in the whole design procedure. Extensive simulations have shown that the behavior of this sink

$$\dot{\xi}_5 = \Psi_5(\xi) \qquad \dot{\xi}_6 = \Psi_6(\xi)$$

is indeed a stable one, although not asymptotically. The functional form of Ψ_5 and Ψ_6 is rather complex, but can be derived straightforward from $T(\mathbf{x})$ and its inverse transformation.

The analysis of this dynamics in the steady state is of special interest. When the outputs, i.e. the square of the stator flux modulus and the torque, are constant then

$$\xi_1 = \Phi_d^2 \quad \xi_2 = 0 \quad \xi_3 = T_d \quad \xi_4 = 0$$

and $v_1 = v_2 = 0$. Substituting these values into the dynamics of ξ_5 and ξ_6, yields the so-called zero-dynamics of the system [16], under a constant shift of the outputs. In the present case, it is possible to verify that in the dynamic controller α_1 = constant and $\alpha_2 = 0$, so that the input $\mathbf{u}$ to the VFC motor is itself constant. This means that the motor is supplied by sinusoidal voltages of constant amplitude and frequency. Hence, the zero-dynamics can be computed using the standard steady-state analysis of the induction machine. Therefore, the closed-loop stator currents satisfy the following linear equations

$$\begin{bmatrix} \dot{\xi}_5 \\ \dot{\xi}_6 \end{bmatrix} = \begin{bmatrix} 0 & -\omega_a \\ \omega_a & 0 \end{bmatrix} \begin{bmatrix} \xi_5 \\ \xi_6 \end{bmatrix}$$

where ω_a is the supply frequency.

4. Simulation results

The proposed control approach has been simulated on a high power induction motor, having as model parameters:

$$\alpha = 27.232 \text{ sec}, \quad \beta = 17.697 \text{ sec}, \quad \sigma = 0.064, \quad L_s = 0.179 \text{ H}.$$

These constants result in a very fast dynamics of the electromagnetic circuits. The rated value of the stator flux is equal to 7.3 V sec while the maximum torque is 1000 Nm.

Starting from a steady-state situation with $T_d = 100$ and $\omega = 300$ rad/sec, the motor torque undergoes two step changes at t = 30 msec and 90 msec, respectively to the positive and negative maximum values.

The following PD law is used for the external inputs v_i, $i = 1,2$:

$$v_i = -K_v\,\xi_{2i} + K_p\,(y_{d,i} - \xi_{2i-1}) = -K_v\,L_f h_i(\mathbf{x}) + K_p\,(y_{d,i} - h_i(\mathbf{x}))$$

with the gains $K_p = 10^4$ and $K_v = 140$.

Figure 3 shows the response of the two system outputs in the first 200 msec. The sampling time used is 100 μsec. The torque follows the desired profile while the stator flux is kept constant even during the transients, thus confirming the achieved decoupling. Small deviations are due only to discretization effects. The 5% overshooting in the torque is the one expected from the chosen PD gains.

In Figure 4, the associated control inputs $u_1 = V$ and $u_2 = \omega_a$ are reported while the two components of the closed-loop stator current (i.e. ξ_5 and ξ_6) are depicted in Figure 5. Stator fluxes are not shown but have indeed a sinusoidal profile. It is worth to point out the specific behavior of the current at the instant of torque inversion; a phase shift of about π is produced between stator current and flux and this is accomplished in only one period.

Robustness with respect to variations in the machine parameter β has been tested. A 50% lower value gives rise only to small steady state-errors and overshooting. Finally, the same control law performs well also when applied up to every 5 msec, as can be expected from the smoothness of the obtained control profiles.

5. Conclusions

It has been shown that the use of nonlinear control techniques based on the differential-geometric approach is effective for solving the control problem of voltage-frequency controlled induction motors. Input-output decoupling is possible by means of a dynamic nonlinear state-feedback. Tuning of the control parameters can be easily made on the linear side of the problem. A major aspect of the presented approach is the ability of controlling the electrical transients without explicitly introducing a rotating frame description and the related field-oriented quantities. The chosen system inputs are the ones which are directly available on common industrial inverters used for supplying the induction machine. Finally, it should be mentioned that the whole control law derivation was performed using a program written by the Authors in a symbolic manipulation language and running on a personal computer.

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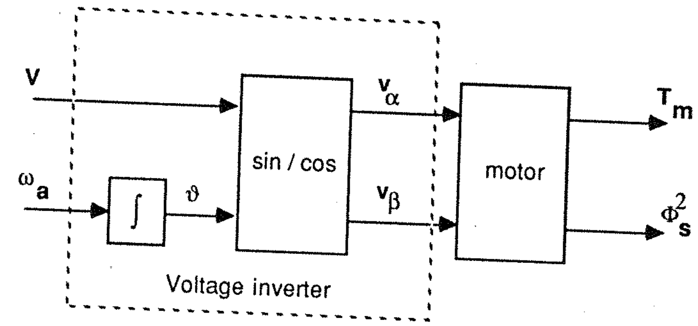

Figure 1 - Voltage Frequency Control (VFC) scheme for an induction motor drive

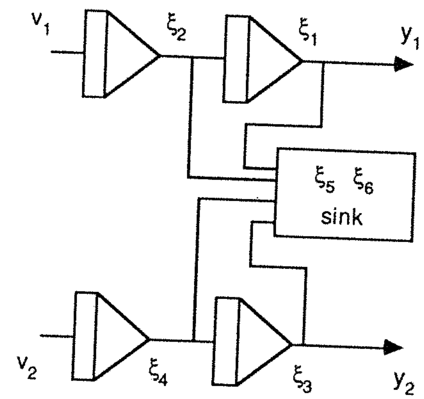

Figure 2 - Structure of the closed-loop decoupled system

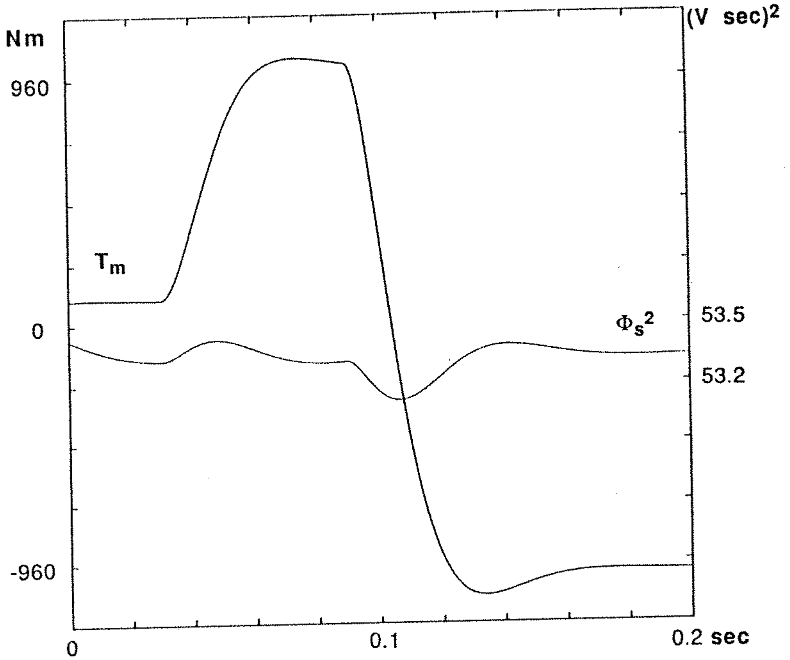

Figure 3 - System outputs: torque and stator flux

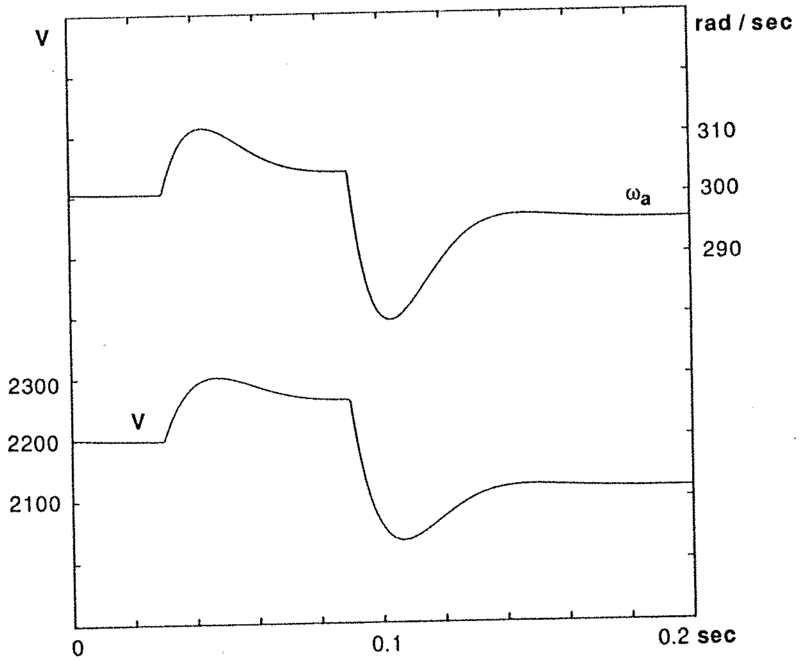

Figure 4 - Control inputs: frequency and amplitude of stator voltage

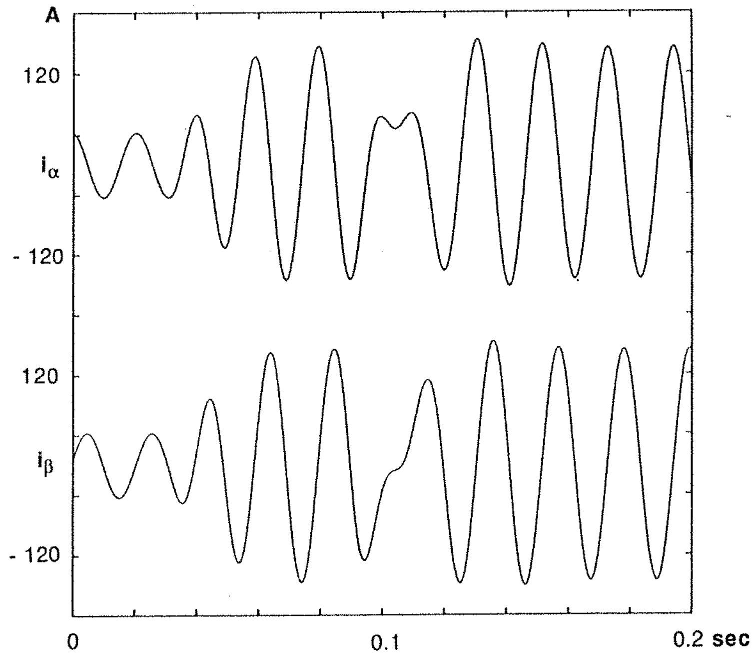

Figure 5 - Closed-loop stator currents