Proceedings of the 35th
Conference on Decision and Control
Kobe, Japan • December 1996

Control of Underactuated Mechanical Systems: Application to the Planar 2R Robot

Alessandro De Luca   Raffaella Mattone   Giuseppe Oriolo
Dipartimento di Informatica e Sistemistica
Università degli Studi di Roma "La Sapienza"
Via Eudossiana 18, 00184 Roma, Italy
{deluca,oriolo}@giannutri.caspur.it   mattone@labrob.ing.uniroma1.it

Abstract

We consider the problem of stabilizing a 2R robot which moves in the horizontal plane by using a single actuator at the base. This system is representative of the class of underactuated mechanical systems that are not controllable in the first approximation. The presence of a drift term in the dynamic equations makes the application of most existing control techniques impossible. The proposed stabilization method makes use of three basic tools, namely *(i)* partial feedback linearization of the dynamic equations, *(ii)* computation of a nilpotent approximation of the system, and *(iii)* iterative application of an open-loop control designed on the nilpotent system. Although the procedure is presented for the 2R robot case, it provides guidelines for devising a method of general applicability.

1 Introduction

The control of *underactuated* mechanical systems, i.e., with less control inputs than generalized coordinates, is receiving increasing attention in robotic applications. The possibility of building a mechanism that can perform complex tasks with a small number of actuators is indeed appealing, for it allows to reduce cost, weight as well as the occurrence of failures. However, the synthesis of effective control strategies requires a special effort, often calling for innovative tools and approaches.
The first interests in underactuated systems trace back to the study of *nonholonomic* robots, such as wheeled mobile robots under the rolling without slipping condition [1], dextrous hands with rolling fingers contact [2], and satellite-mounted manipulators under angular momentum conservation [3]. While nonholonomy is somewhat native to these systems, researchers are now trying to induce nonholonomic behaviors in order to reduce the complexity of the actuation system. For example, Bicchi and Sorrentino [4] have addressed the problem of designing a minimum-complexity robotic hand able to perform dextrous manipulation through rolling. In [5], a 6-joint nonholonomic manipulator has been presented whose configuration can be completely controlled by means of only two velocity input commands at the robot base. In the same spirit, in [6] we have determined conditions for choosing one (of the many) inverse kinematic maps of a redundant manipulator so that full accessibility of the configuration space is guaranteed by using only $m < n$ task velocity commands. Finally, Lynch and Mason [7] have addressed the problem of arbitrarily positioning an object in the plane by pushing it along a limited set of directions.
In all the above cases, the underlying differential constraints are in the first-order *Pfaffian* form $A(q)\dot{q} = 0$, where q are the system generalized coordinates. A fundamental property of this model is the absence of a *drift* term, which simplifies the controllability analysis as well as the control design. In particular, it has been shown that the system is completely controllable in spite of its reduced number of inputs [1], but smooth stabilization to a single equilibrium point is not possible [8]. Hence, one must either resort to open-loop controllers [9] or to time-varying [10, 11] and/or discontinuous feedback [12, 13].
In [14, 15] it has been shown that *dynamic models* (i.e., with generalized forces as inputs) of systems with first-order nonholonomic constraints inherit the same structural properties (and controllers). In fact, although a drift term is present in these models, it is possible to put the system via feedback in a form in which such term represents a trivial dynamic extension.
However, there are cases in which the underlying differential constraints appear directly in *second-order* form $R(q)\ddot{q} + s(q, \dot{q}) = 0$. Examples in this class are robot manipulators with passive joints, for which Oriolo and Nakamura [16] have presented a detailed analysis, and redundant robots driven through end-effector generalized forces [17].
The main difference between second-order and first-order models is the presence of a non-trivial drift term in the system equations. Therefore, the accessibility of the system does not imply controllability, which must be tested by using more sophisticated tools. Moreover, the negative result on smooth stabilizability stands and the control law synthesis is even more difficult than in the first-order case, due to the presence of drift.
To solve the stabilization problem for underactuated

0-7803-3590-2/96 $5.00 © 1996 IEEE

manipulators, we propose the following scheme: devise an open-loop control for the system which can steer its state closer to the desired equilibrium point in finite time, and apply it in an iterative fashion (i.e., from the state attained at the end of the previous iteration). Under appropriate hypotheses [18], such a strategy provides robust stabilization for a wide class of controllable systems. To simplify the computation of a suitable open-loop control, one can try to approximate the system equations by a nilpotent form, which can be easily integrated and, at the same time, preserves the fundamental properties of the original system (in particular, controllability). Approximate nilpotentization has already been used in nonholonomic motion planning by Laumond *et al.* [19].

The paper is organized as follows. In the next section, a general discussion is given about the problem of controlling an underactuated manipulator, outlining our stabilization approach. In Sect. 3, the main features of the used nilpotent approximation are briefly illustrated. We apply in Sect. 4 the proposed approach to a 2R planar robot with a single actuator at the base and provide some simulation results.

2 The control problem for underactuated robots

Consider a robotic manipulator with n joints having only m actuated joints. Denote by $q \in I\!R^n$ the joint coordinates vector, and by $\tau \in I\!R^m$ the vector of generalized forces.

2.1 Partial feedback linearization

Partition the joint vector as $q = (q_a, q_b)$, where $q_a \in I\!R^m$ is the subvector of *controlled* joints and $q_b \in I\!R^{n-m}$ are the *passive* joints. Following the Lagrangian approach, the dynamic model of the system can be written as

$$\begin{bmatrix} B_{aa} & B_{ab} \\ B_{ab}^T & B_{bb} \end{bmatrix} \begin{bmatrix} \ddot{q}_a \\ \ddot{q}_b \end{bmatrix} + \begin{bmatrix} h_a \\ h_b \end{bmatrix} = \begin{bmatrix} \tau \\ 0 \end{bmatrix},$$

with the corresponding partitions of the $n \times n$ inertia matrix $B(q)$ and of the n-vector $h(q, \dot{q})$, which collects centrifugal, Coriolis and gravitational terms. Note that the last $n - m$ equations provide directly the second-order differential constraint that is always satisfied by the robot during its motion.

The generalized forces τ can be chosen as a partially linearizing and decoupling feedback law so as to obtain a closed-loop system of the form

$$\ddot{q}_a = u, \quad (1)$$

$$\ddot{q}_b = f_b(q, \dot{q}) + G_b(q)u, \quad (2)$$

with $u \in I\!R^m$ an auxiliary input vector.

2.2 Controllability

A fundamental question that must be addressed is whether the robot can be completely controlled in spite of its reduced number of actuators. The standard accessibility property, which may be tested by means of the Lie algebra rank condition [20], cannot be used in this case. In fact, for nonlinear systems with drift, accessibility does not imply controllability.

However, one may resort to the concept of *small-time local controllability* (STLC), introduced in [21] by Sussmann, who gave sufficient conditions subsequently refined by Bianchini and Stefani [22]. Roughly speaking, a STLC system can reach any point near x^0 in arbitrarily small time with trajectories remaining arbitrarily close to x^0.

Since only sufficient conditions exist for the STLC property, their violation does not imply that the system is not STLC. However, the latter possibility must be taken into account. In this case, no general criteria can be used for testing controllability, which must be established through a constructive procedure.

2.3 Stabilization

We now address the problem of determining a sequence of input commands so as to transfer the system from an initial equilibrium point $x^0 = (q^0, 0) = (q_a^0, q_b^0, 0)$ to a desired equilibrium point $x^d = (q^d, 0) = (q_a^d, q_b^d, 0)$. Such a sequence certainly exists if the system is controllable. However, it is easy to prove that an underactuated robot moving in the horizontal plane is not smoothly stabilizable at an equilibrium point x^e via time-invariant feedback control laws. This result, which is a consequence of a well-known theorem due to Brockett [8], indicates that there is no *simple way* to design feedback commands u so as to move the underactuated robot between two joint configurations. In order to overcome the limitations of smooth controls, one may use time-varying or discontinuous feedback. However, while systematic approaches exist for controllable driftless systems—e.g., see [10, 12]—the case of systems with drift has received much less attention. In particular, systems that are not STLC have been studied so far on a case-by-case basis.

Our proposed method for the stabilization of the underactuated robot in the partially linearized form (1–2) prescribes the execution of two phases:

1. Drive in finite time T_1 the controlled joint variables q_a to their desired values q_a^d by a proper choice of u. Therefore, at the end of this phase we obtain $q_a(T_1) = q_a^d$ and $\dot{q}_a(T_1) = 0$. Correspondingly, we have $q_b(T_1) = q_b^I$ and $\dot{q}_b(T_1) = \dot{q}_b^I$, being in general $q_b^I \neq q_b^d$ and $\dot{q}_b^I \neq 0$.
2. Obtain asymptotic convergence of the passive joint positions q_b to their desired values q_b^d while guaranteeing that q_a returns to q_a^d.

The first phase, which we shall refer to as *alignment*, can be performed in feedback using a standard terminal controller for the decoupled chains of two integrators represented by eq. (1). For example, for $i = 1, \ldots, m$, one may set

$$u_i = -\gamma_i \operatorname{sign}(q_{a_i} - q_{a_i}^d + 2\gamma_i \dot{q}_{a_i} |\dot{q}_{a_i}|),$$

where γ_i is an arbitrary positive constant [15]. The final time T_1 will depend on γ_i as well as on the initial conditions for q_a.

As for the second phase, a possibility is to adopt the *iterative state steering* approach [18]. The main ingredient of this technique is an open-loop control that steers the system *closer* to the desired equilibrium point x_d in a finite time T_2. The iterated application of such a control (starting from the state attained at the end of the previous iteration) yields exponential convergence to x_d under the assumption that the open-loop control is continuous with respect to the initial conditions. Moreover, non-persistent perturbations are rejected, while ultimate boundedness of the system state is guaranteed in the presence of persistent perturbations [18]. Note that the resulting control is a time-varying law whose expression depends on a sampled feedback action.

In order to apply this method, we must devise an open-loop control law that produces a *cyclic* motion of duration T_2 on the q_a variables (i.e., a motion such that $q_a^{II} = q_a(T_1 + T_2) = q_a(T_1)$ and $\dot{q}_a^{II} = \dot{q}_a(T_1 + T_2) = 0$) while giving a final position $q_b(T_1 + T_2)$ for the passive joints that is closer to q_b^d than the initial condition q_b^I, with final velocity smaller than $\dot{q}_b^I$. To this end, it is convenient to select the control input u within a *parameterized* class, in order to simplify the computation of the required command. In some cases (e.g., when the system can be put in second-order triangular or Čaplygin form [17]), this computation can be directly performed by forward integration of the passive joints equation (2). In general, however, one can resort to an approximation of the dynamic equations, in order to obtain an easily integrable form. At the same time, the approximation should resemble the original system as closely as possible, and in particular should preserve controllability. An effective solution to this problem is provided by approximate nilpotentization.

3 Approximate nilpotentization

Nilpotent approximations [23] of control systems are an example of high-order approximation that can prove particularly useful when the linearized system does not preserve the original controllability properties. In this work, we use in particular the approximate nilpotentization technique proposed in [19], that we do not recall here for the sake of brevity. This technique can be applied to any system of the form

$$\dot{x} = f(x) + \sum_{i=1}^{m} g(x) u_i, \quad x \in I\!R^n, \tag{3}$$

satisfying the accessibility property.

The computation procedure is essentially based on the existence of a suitable set of *privileged* coordinates z, locally defined around any point x^0 where the system is accessible. With the system in these coordinates, the nilpotent approximation is obtained by expanding in Taylor series the components of the system vector fields, and truncating them to the terms of a proper order.

The approximating vector fields $\hat{f}, \hat{g}_1, \ldots, \hat{g}_m$ can be given a coordinate-free expression on the tangent space of the state manifold. Moreover, they generate a nilpotent Lie algebra which is full rank around x^0, so that the approximating system is locally accessible. Finally, the i-th component of $\hat{g}_j$ is such that only the first $i-1$ variables $z_{1,\ldots,i-1}$ may appear in it. Hence, the approximating system is polynomial and has the *triangular* form

$$\dot{z}_i = \hat{f}_i + \sum_{j=1}^{m} \hat{g}_{j_i} u_j, \quad i = 1, \ldots, \nu, \tag{4}$$

$$\dot{z}_k = \hat{f}_k(z_{1,\ldots,k-1}) + \sum_{j=1}^{m} \hat{g}_{j_k}(z_{1,\ldots,k-1}) u_j, \quad k = \nu + 1, \ldots, n, \tag{5}$$

where ν is the rank of $f, g_1, \ldots, g_m$, and $\hat{f}_i, \hat{g}_{1_i}, \ldots, \hat{g}_{m_i}$, $i = 1, \ldots, \nu$, (the first ν components of $\hat{f}, \hat{g}_1, \ldots, \hat{g}_m$) are constant. In particular, it can be readily proven that the approximating system is partially decoupled and linearized. This suggests to perform a partial decoupling and linearization by feedback on the original system *before* proceeding with the approximation procedure, so that the decoupled dynamics is exactly recovered by subsystem (4).

4 Application to a planar 2R robot

Consider the 2R planar robot of Fig. 1, having two revolute joints and a single actuator at the base. We assume that no friction is present at the joints. The same mechanism was considered by Nakamura *et al.* [24], while Arai [25] analyzed the control problem for a 3R manipulator with a passive joint.

After the partial feedback linearization of Sect.2.1, and with the state vector $Q = (\dot{q}_1, q_1, \dot{q}_2, q_2) \in I\!R^4$, the first order dynamic model of the robot is

$$\dot{Q} = \begin{bmatrix} 0 \\ \dot{q}_1 \\ -K s_2 \dot{q}_1^2 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ -1 - K c_2 \\ 0 \end{bmatrix} u = f(Q) + g(Q) u,$$

being $s_2 = \sin(q_2)$, $c_2 = \cos(q_2)$, and K a constant depending on the geometric and inertial properties of the robot.

It can be readily verified that $\{g, [f,g], [g,[f,g]], [f,[g,[f,g]]]\}$ spans $I\!R^4$ at any

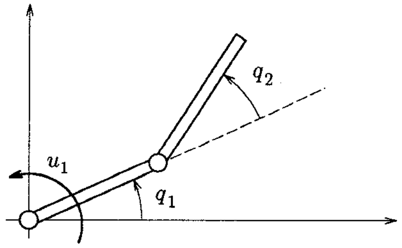

Figure 1: A 2R planar robot with a single actuator at the base

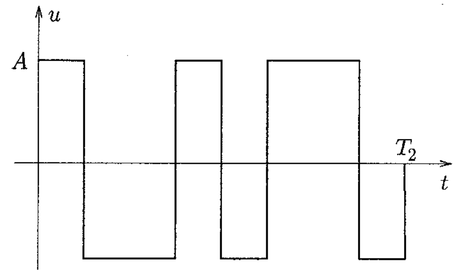

Figure 2: The profile of the cyclic open-loop control u in each iteration

Q such that $q_2 \neq k\pi/2$, $k = 0, 1 \ldots$. Hence, the system is accessible (in fact, *strongly* accessible). However, the sufficient conditions for STLC are not satisfied.

Assume that we wish to steer the 2R robot from $q^0 = (q_1^0, q_2^0)$ to $q^d = (q_1^d, q_2^d)$, with initial and final zero velocity. According to the control strategy proposed in Sect. 2.3, at the end of the alignment phase it will be $q_1(T_1) = q_1^d$ and $\dot q_1(T_1) = 0$. Correspondingly, $q_2(T_1) = q_2^I$ and $\dot q_2(T_1) = \dot q_2^I$. At this point, we must specify a cyclic open-loop controller for the second phase which brings the passive joint q_2 to a final position q_2^{II} closer to q_2^d than the initial condition q_2^0. To this end, we compute the nilpotent approximation of the system at points Q^I such that $\dot q_1^I = 0$ and $\dot q_2^I \neq 0$.

4.1 Nilpotent approximating model

After applying the technique of Sect. 3, the vector fields characterizing the approximate model of the system are (see [26] for details)

$$\hat f = \begin{bmatrix} 1 \\ 0 \\ -z_2 \\ \frac{1}{2Kc_2^I} z_2^2 \end{bmatrix}, \quad \hat g = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -\frac{(\dot q_2^I)^2}{4Ks_2^I} z_1^2 - \frac{\beta}{2Kc_2^I} z_3 \end{bmatrix},$$

where $\beta = 1 + K\cos(q_2^I)$, $\gamma = K\sin(q_2^I)\dot q_2^I$, $\delta = K^2 \sin(2q_2^I)$, and the change of coordinates $Q = Q(z)$ is given by

$$\begin{aligned} \dot q_1 &= z_2, \\ q_1 &= q_1^I - z_3, \\ \dot q_2 &= \dot q_2^I - \beta z_2 + \gamma z_3 - \delta z_4 + \gamma z_1 z_2, \\ q_2 &= q_2^I + \dot q_2^I z_1 + \beta z_3. \end{aligned} \tag{6}$$

Note that the dynamics of the q_1 and $\dot q_1$ variables, which correspond to z_3 and z_2, is *exactly* recovered, thanks to the partial feedback linearization performed in advance on the original system. Instead, the use of the nilpotent form for q_2 and $\dot q_2$ will induce an approximation error whose magnitude can be made arbitrarily small by reducing T_2. Besides, the expression of $\hat f$ shows that $z_1 = t$, which is true in general for nilpotent approximations when a drift term is present.

4.2 Design of the control strategy

As a preliminary step, we highlight the peculiar behavior of the considered control system. The assumption that u is cyclic of period T_2, and the use of eqs. (6) imply

$$\begin{aligned} \dot q_1^{II} = \dot q_1^I = 0 \quad &\Rightarrow \quad z_2(T_2) = 0, \\ q_1^{II} = q_1^I \quad &\Rightarrow \quad z_3(T_2) = 0. \end{aligned}$$

Hence,

$$\Delta q_2 = q_2^{II} - q_2^I = \dot q_2^I z_1(T_2) = \dot q_2^I T_2, \tag{7}$$

showing that the variation of the passive joint position along the cycle does not depend on the particular control input, but only on its period and on the initial velocity $\dot q_2^I$. Moreover, we have

$$\Delta \dot q_2 = \dot q_2^{II} - \dot q_2^I = -\delta z_4(T_2).$$

Performing some computations (see [26]) we obtain

$$\Delta \dot q_2 = K^2 s_2^I c_2^I \int_0^{T_2} z_2^2(t)\, dt - K c_2^I (\dot q_2^I)^2 \int_0^{T_2} z_3(t)\, dt. \tag{8}$$

In the right hand side of the above expression, the sign of the first term does not depend on the choice of the specific cyclic input, but only on q_2^I, while the second term is $o((\dot q_2^I)^2)$. Hence, the approximating system is *not* STLC at equilibrium points[1]. In spite of this, the system is controllable, as we will show constructively.

At this point, we choose a specific class of cyclic inputs. In particular, assume that u is the piecewise-constant function shown in Fig. 2, with duration T_2 and amplitude A. For such input, we have

$$\int_0^{T_2}\int_0^{\tau}\int_0^{\theta} u(t)\, dt\, d\theta\, d\tau = 0 \;\Rightarrow\; \int_0^{T_2} z_3(t)\, dt = 0,$$

and eq. (8) implies

$$\Delta \dot q_2 = \frac{{T_2}^3 K^2 s_2^I c_2^I}{192} A^2, \tag{9}$$

which shows that, at each control iteration, we can obtain only $\Delta \dot q_2$ of the same sign of $\sin(2q_2^I)$, i.e., positive for q_2^I in the first and third quadrant and negative in the second and the fourth. As for the variation Δq_2, eq. (7) must be applied.

[1] This can be also verified by directly constructing the nilpotent approximation at an equilibrium point.

In order to meet the iterative steering paradigm, we must guarantee that the final error contracts, i.e.,

$$|q_2^d - q_2^{II}| \leq \eta_1 |q_2^d - q_2^I|, \quad (10)$$
$$|\dot{q}_2^{II}| \leq \eta_2 |\dot{q}_2^I|, \quad (11)$$

with $\eta_{1,2} < 1$. However, in view of the lack of the STLC property for our approximating system, entailed by eqs. (7) and (9), the above conditions can be satisfied only in particular situations.
For example, assume that q_2^d belongs to the first quadrant $\mathcal{Q}_1$. If $(q_2^I, \dot{q}_2^I)$ verify

$$\dot{q}_2^I < 0, \quad q_2^I > q_2^d, \quad q_2^I \in \mathcal{Q}_1, \quad (12)$$

one can directly apply the proposed iterative steering technique using the open-loop control u of Fig. 2 in which:

1. The duration T_2 is chosen so as to satisfy eq. (10) according to eq. (7).
2. The amplitude A is chosen so as to satisfy eq. (11) with the aid of eq. (9).

Note that the resulting control law is a continuous function of the initial conditions $(q_2^I, \dot{q}_2^I)$. Hence, the *contraction* phase yields exponential convergence to the desired equilibrium point $(q_2^d, 0)$.
If, on the other hand, any of the (12) does not hold, one can easily verify that it is not possible to satisfy both the conditions (10–11) while approaching the desired configuration. Therefore, before we can apply the iterative steering technique it is in this case necessary to attain an initial condition $(q_2^I, \dot{q}_2^I)$ satisfying eqs. (12). This *transition* phase can be executed in finite time as follows: if the initial velocity of the second joint is negative, keep it constant until q_2 enters $\mathcal{Q}_1$, else keep it constant until q_2 enters the $\mathcal{Q}_1$ or $\mathcal{Q}_3$, where the sign of $\dot{q}_2$ can be made negative. Note that, in order to keep $\dot{q}_2$ constant one simply sets $u = 0$.
The above procedure can be improved by minimizing the duration of the transition phase by a clever choice of the arm 'maneuvers'. The cases $q_2^d \in \mathcal{Q}_2$, $\mathcal{Q}_3$ or $\mathcal{Q}_4$ can be treated in a similar way.

4.3 Simulation Results

In order to illustrate the performance of the proposed control strategy, we present some simulation results for a 2R robot with $K = 0.5$. We have assumed that, at the end of the alignment phase, $q_2^I = \pi/8$ rads and $\dot{q}_2^I = 0.23$ rads/s, while the desired configuration of the passive joint is $q_2^d = \pi/4$ rads.
Being $q_2^d \in \mathcal{Q}_1$ and $\dot{q}_2^I > 0$, the control strategy of Sect. 4.2 prescribes the execution of a transition phase, in which $\dot{q}_2$ is kept constant until q_2 enters $\mathcal{Q}_2$, where $\dot{q}_2$ can be made negative. As q_2 returns in $\mathcal{Q}_1$, the contraction phase takes over. By properly tuning the contraction rates on q_2 and $\dot{q}_2$, it has been possible to use a constant duration $T_2 = 1$ s for all iterations.

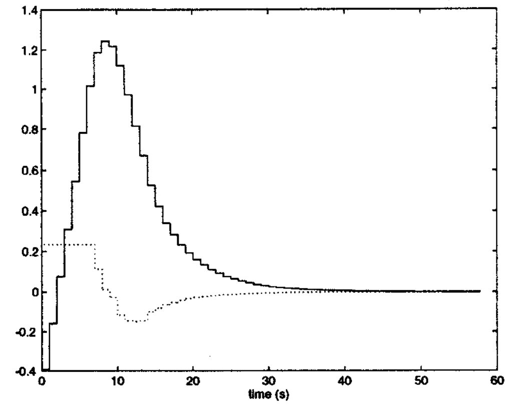

Figure 3: Errors samples for q_2 (rads, solid) $\dot{q}_2$ (rads/sec, dotted) during the second control phase

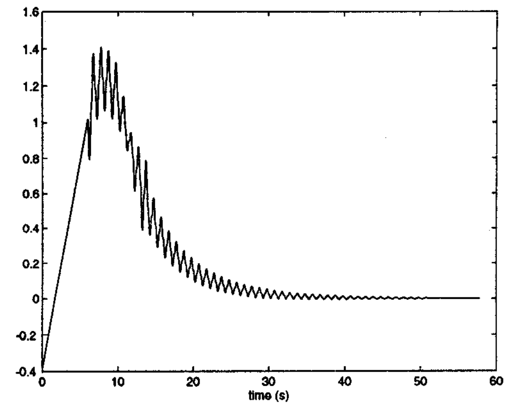

Figure 4: Error on q_2 (rads) during the second control phase

Figure 3 shows the error samples for q_2 (solid) and $\dot{q}_2$ (dotted) during the second control phase (transition+contraction) with sampling time of 1 s, while in Fig. 4 the complete time history of the q_2 error is reported. Note the constant velocity of the second joint at the beginning of the transition phase and the exponential convergence rate during the contraction phase.

5 Conclusions

We have presented a solution approach for the stabilization problem of a planar horizontal 2R robot with a single actuator at the base. Such system, which is subject to a second-order nonholonomic constraint, is not smoothly stabilizable; moreover, the presence of a drift term in the dynamic equations complicates remarkably the control synthesis. The stabilization strategy consists of three phases, namely *(i)* alignment, in which the first joint is brought to its desired position, *(ii)* transition, where simple maneuvers are executed to obtain the correct initial condition for *(iii)* contraction, based on the iterative application of a suitable open-

loop control designed for a nilpotent approximation of the system.
The proposed method can be extended to the large class of underactuated mechanical systems, which are often found in robotic applications. We are currently implementing the controller on a prototype 2R robot available in our laboratory.

References

[1] J.-P. Laumond, "Nonholonomic motion planning versus controllability via the multibody car system example," Tech. Rep. STAN-CS-90-1345, Stanford University, Dec. 1990.

[2] R. M. Murray, Z. Li, and S. S. Sastry, *A Mathematical Introduction to Robotic Manipulation*, CRC Press, Boca Raton, 1994.

[3] Y. Umetani and K. Yoshida, "Resolved motion rate control of space manipulators with generalized jacobian matrix," *IEEE Trans. on Robotics and Automation*, vol. 5, pp. 303–314, 1989.

[4] A. Bicchi and R. Sorrentino, "Dexterous manipulation through rolling," *1995 IEEE Int. Conf. on Robotics and Automation*, Nagoya, J, pp. 452–457, 1995.

[5] O. J. Sørdalen, Y. Nakamura, and W. J. Chung, "Design of a nonholonomic manipulator," *1994 IEEE Int. Conf. on Robotics and Automation*, San Diego, CA, pp. 8–13, 1994.

[6] A. De Luca and G. Oriolo, "Nonholonomy in redundant robots under kinematic inversion," *4th IFAC Symp. on Robot Control*, Capri, I, pp. 179–184, 1994.

[7] K. M. Lynch and M. T. Mason, "Controllability of pushing," *1995 IEEE Int. Conf. on Robotics and Automation*, Nagoya, J, pp. 112–119, 1995.

[8] R. W. Brockett, "Asymptotic stability and feedback stabilization," in *Differential Geometric Control Theory* (R. W. Brockett, R. S. Millman, and H. J. Sussmann, Eds.), pp. 181–191, Birkhäuser, Boston, 1983.

[9] R. M. Murray and S. S. Sastry, "Nonholonomic motion planning: Steering using sinusoids," *IEEE Trans. on Automatic Control*, vol. 38, pp. 700–716, 1993.

[10] C. Samson, "Time-varying feedback stabilization of a car-like wheeled mobile robot," *Int. J. of Robotics Research*, vol. 12, no. 1, pp. 55–64, 1993.

[11] J.-M. Coron, "Links between local controllability and local continuous stabilization," *Nonlinear Control Systems Design Symposium 92 (NOLCOS'92)*, Bordeaux, F, pp. 477–482, 1992.

[12] I. V. Kolmanovsky, M. Reyhanoglu, and N. H. McClamroch, "Discontinuous feedback stabilization of nonholonomic systems in extended power form," *33rd IEEE Conf. on Decision and Control*, Lake Buena Vista, FL, pp. 3469–3474, 1994.

[13] R. M. Murray and R. T. M'Closkey, "Converting smooth, time-varying, asymptotic stabilizers for driftless systems to homogeneous, exponential stabilizers," *3rd European Control Conference (ECC 95)*, Rome, I, pp. 2620–2625, 1995.

[14] G. Campion, B. d'Andrea-Novel, and G. Bastin, "Modelling and state feedback control of nonholonomic mechanical systems," *30th IEEE Conf. on Decision and Control*, Brighton, UK, pp. 1184–1189, 1991.

[15] A. M. Bloch, M. Reyhanoglu, and N. H. McClamroch, "Control and stabilization of nonholonomic dynamic systems," *IEEE Trans. on Automatic Control*, vol. 37, pp. 1746–1757, 1992.

[16] G. Oriolo and Y. Nakamura, "Control of mechanical systems with second-order nonholonomic constraints: Underactuated manipulators," *30th IEEE Conf. on Decision and Control*, Brighton, UK, pp. 2398–2403, 1991.

[17] A. De Luca, R. Mattone and G. Oriolo, "Dynamic mobility of redundant robots using end-effector commands," *1996 IEEE Int. Conf. on Robotics and Automation*, Minneapolis, MN, April 1996.

[18] P. Lucibello and G. Oriolo, "Stabilization via iterative state steering," *35th Conf. on Decision and Control*, Kobe, J, Dec. 11–13, 1996.

[19] A. Bellaïche, J.-P. Laumond, and M. Chyba, "Canonical nilpotent approximation of control systems: Application to nonholonomic motion planning," *32nd IEEE Conf. on Decision and Control*, San Antonio, TX, pp. 2694–2699, 1993.

[20] A. Isidori, *Nonlinear Control Systems*, 3rd Edition Springer-Verlag, London, 1995.

[21] H. J. Sussmann, "A general theorem on local controllability," *SIAM J. on Control and Optimization*, vol. 25, pp. 158–194, 1987.

[22] R. M. Bianchini and G. Stefani, "Controllability along a trajectory: A variational approach," *SIAM J. on Control and Optimization*, vol. 31, no. 4, pp. 900–927, 1993.

[23] H. Hermes, "Nilpotent and high-order approximations of vector field systems," *SIAM Review*, vol. 33, no. 2, pp. 238–264, 1991.

[24] T. Suzuki, M. Koinuma, and Y. Nakamura, "Chaos and nonlinear control of a nonholonomic free-joint manipulator," *1996 IEEE Int. Conf. on Robotics and Automation*, Minneapolis, MN, pp. 2668–2675, 1996.

[25] H. Arai, "Controllability of a 3-DOF manipulator with a passive joint under a nonholonomic constraint," *1996 IEEE Int. Conf. on Robotics and Automation*, Minneapolis, MN, pp. 3707–3713, 1996.

[26] A. De Luca, R. Mattone, and G. Oriolo, "Control of underactuated mechanical systems: Application to the planar 2R robot," *Nonlinear Control and Robotics Preprints*, Dipartimento di Informatica e Sistemistica, Università di Roma "La Sapienza", Italy, 1996.