Trajectory Tracking in Flexible Robot Arms

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Abstract. *We consider the problem of controlling the end-effector motion of flexible robot arms. Main difficulties arise both from the typical nonlinearities of robot dynamics and from the non-minimum phase nature of the flexible arm when tip position is chosen as output. This bars the straight application of inversion control solving the trajectory tracking problem in robots with rigid links. Three alternative strategies are presented: inversion control based on a suitable minimum phase output, nonlinear regulation of the tip output, and iterative learning control. The control laws are discussed from the point of view of system requirements and complexity, while their performance is compared by simulation on a simple but significative example. Finally, we report on the experimental activity in controlling flexible robots carried out in the Robotics Laboratory at DIS.*

1. Introduction

The adoption of lightweight materials and the development of very large manipulators have recently given an impulse to the modeling and control of robotic systems with flexible elements. From the application point of view, robots with flexible links are found in space, as in the Space Shuttle manipulator [1], and on earth, e.g. in automated crane devices for advanced building costruction. Flexibility plays a major role whenever long and dextrous arms are needed to have access to hostile environments, such as in nuclear sites for maintenance, in underground waste deposits for inspection, or in deep sea for exploration.

Robotic tasks require two types of motion: point-to-point moves for repositioning the end-effector and path following. However, it is always convenient to specify an interpolating trajectory so to guarantee smooth transition between initial and final configurations, impose an explicit time constraint on the overall motion, and eventually simplify the synthesis of the control gains. Moreover, when the task involves contact with the environment, the accuracy in executing the planned motion becomes quite crucial. As a result, trajectory tracking is considered to be the reference problem for evaluating advanced robotic control law.

In the case of robots with fully rigid links, straightforward solutions are offered by inversion control schemes, which use nonlinear state feedback to compensate for couplings and nonlinearities of the system equation [2].

In the presence of link flexibility, both the trajectory specification and the control design should take care also of the relevant induced vibrations in the robot arm. On one hand, fast trajectories are usually associated to large deformations that may excessively stress the mechanical structure and result in long oscillation settling times at the final point. Conversely, slow motions that do not excite the characteristic eigenfrequencies of the arm would limit the operational performance. As for the controller, two main objectives are of interest: exact or asymptotic reproduction of a conveniently defined output trajectory and stabilization of the arm flexible dynamics.

When inversion control is applied to a flexible robot arm for tracking an end-effector trajectory, serious instability problems occur. This phenomenon emerges already in a linear setting. In fact, the nature of the mapping between input torque and tip location of each link is non-minimum phase, i.e. the transfer function contains right-half plane zeros [3]. Sometimes this corresponds to the practical observation of an 'opposite' output motion in response to a given torque input command. Since inversion control tends to impose a prescribed linear input-output behavior with no zeros, this suggests that cancellation with unstable poles is attempted. In a nonlinear setting, the generalization of the concept of zero dynamics allows to draw analogous conclusions about instability due to cancellations [4]. Nonlinear systems with unstable zero dynamics are often called *non-minimum phase* systems, similarly to the linear case.

Recently, some control alternatives have been proposed that achieve accurate tracking in flexible robots, complying with the non-minimum phase characteristics of the end-effector output. These can be classified as follows.

- *Inversion control for an associated minimum phase output* [5–8]. This involves the definition of a suitable output for which inversion control yields a stable closed-loop system.
- *Output regulation of the end-effector output* [9–11]. A bounded evolution of the whole state can be associated to any given bounded reference trajectory and used as a reference for a state feedback stabilizing law, without involving cancellations. This reference state trajectory produces exactly the desired time profile at the system output.
- *Iterative learning control of repetitive trajectories* [12,13]. Provided that repeated trials are allowed, it is possible to learn the input command that imposes the desired output trajectory, processing error data at previous trials with a limited knowledge of the system dynamics.

The purpose of this paper is to present a comparison of the above control methodologies. As a matter of fact, several interesting relationships

exist among the different approaches: the application to a physical example, as the flexible robot arm, will help to enlighten them.

The first two classes of methods lead essentially to model-based controllers and will thus perform best when a good dynamic model of the robot arm is available. The third class uses instead a nominal model only for an efficient selection of the learning functions. Several approaches exist for modeling articulated flexible structures, ranging from finite- to infinite-dimensional descriptions and from linear to nonlinear dynamics [14,15].

Due to the overall complexity of modeling and controlling flexible manipulators, the analysis and the synthesis stages asks for an adequate experimental verification. To this aim, a two-link planar robot with a very flexible forearm has been designed and built in our Laboratory [16,17], and the robotic system is now available for control experiments. Such an apparatus allows to evaluate theoretical control findings, with respect to several real world aspects which are often neglected at the level of control design. However, it should be emphasized that the end-effector trajectory tracking in a flexible robot arm can be viewed as a prototype control problem for the larger class of nonlinear systems with unstable zero dynamics.

The paper is organized as follows. Dynamic modeling issues are summarized in Section 2. The three approaches to trajectory control based on inversion, regulation, and learning are presented in Section 3, directly for the case of multi-link flexible robot arms. In Section 4 a more specific comparison is performed on the basis of the most simple arm structure that inherits the non-minimum phase characteristic. Preliminary experimental results are then reported, obtained on the flexible manipulator available at DIS. Finally, on-going research activities and future developments are outlined.

2. Dynamic modeling of flexible robot arms

Accurate and complete dynamic models are mandatory for simulation purposes, where efficient numerical formats are more convenient. On the other hand, when control design is of concern, the availability of the system equations in an explicit symbolic form allows to trade off between the complexity of a model-based derivation and the expected control performance [18].

In robots with flexible links, deflection is always distributed in nature. This supports the usual argument that infinite-dimensional dynamic equations describe at best the dynamic behavior of flexible manipulators [19]. Typically, each link is represented in the limit as an Euler beam with proper boundary conditions at the two ends [20]. When small deformations are assumed, an eigenvalue problem has to be solved for each link or, more correctly, for the entire system. However, for all but the most simple structures, it is quite difficult to solve for the time evolution of the arm deformation. In particular, time-varying boundary conditions appear whenever the robot is constituted by two or more flexible links [21]. As a result, several approximations have been introduced, all leading to the derivation of finite-dimensional dynamic models. In describing the arm flexibility, separability in time and space is a common hypothesis, e.g. for the bending deformation at time t of a point x along the ith link one has

$$p_i(x,t) = \sum_{i=1}^{n_i} \phi_{ij}(x)\delta_{ij}(t). \tag{1}$$

The n_i spatial components $\phi_{ij}(x)$ satisfy geometric and/or dynamic conditions, while time dependency is resumed within the generalized coordinates $\delta_{ij}(t)$.

Once the arm kinematics is properly described in terms of a set of N rigid variables (usually associated to the robot joints) and of $N_e = \sum_{i=1}^{N} n_i$ deformation variables, the dynamic equations of motion follows in a standard fashion using a Lagrangian approach [22]. Consider a robot arm with an open kinematic chain structure, rotational joints and flexible links. The coupling of N rigid motion equations with N_e flexible ones will result in a nonlinear dynamic model of the general form

$$\begin{bmatrix} B_{11}(\theta,\delta) & B_{12}(\theta,\delta) \\ B_{12}^{\mathrm{T}}(\theta,\delta) & B_{22}(\theta,\delta) \end{bmatrix} \begin{bmatrix} \ddot\theta \\ \ddot\delta \end{bmatrix} + \begin{bmatrix} n_1(\theta,\delta,\dot\theta,\dot\delta) \\ n_2(\theta,\delta,\dot\theta,\dot\delta) \end{bmatrix} + \begin{bmatrix} D_1\dot\theta \\ K\delta + D_2\dot\delta \end{bmatrix} = \begin{bmatrix} u \\ 0 \end{bmatrix}, \tag{2}$$

where θ is the vector of rigid joint variables and δ collects the generalized coordinates associated to deformations. B_{ij} are blocks of the positive definite inertia matrix B, partitioned according to the rigid and flexible components; vectors n_i contain the Coriolis, centrifugal and gravity terms. The diagonal matrix D_1 represents joint viscous friction, while the positive definite, symmetric (and typically diagonal) matrices K and D_2 are, respectively, the modal stiffness and the structural damping of the arm links. Accordingly, the input torques u appear only in the first set of N equations.

The model structure (2) holds for any finite-dimensional approximation of distributed flexibility, as long as the assumed modes of deformations satisfy the geometric *clamped* boundary conditions at the base of each link [14]. However, simplifications are obtained for the blocks in (2) whenever use is made of deformation modes $\phi_{ij}(x)$ that automatically satisfy orthonormal conditions. Moreover, relevant reductions follow from evaluating robot kinetic energy only in correspondence to the undeformed arm configuration ($\delta = 0$). Other couplings between rigid body motion and flexible deformations are also often neglected.

We point out that consistent dynamic modeling choices retain the same physical feature, relevant for control purposes: in the presence of uniform mass distribution for each link, all flexible robot models share the non-minimum phase property of the end-effector output.

3. Control strategies for trajectory tracking

We present here the basic steps involved in the synthesis of three trajectory control laws for general flexible manipulators. The common problem is to determine an input torque which leads to the reproduction of a desired smooth end-effector trajectory. This can be restated as finding ways to extend the well-known computed torque method [2] to flexible robot arms.

In the following, some simplifying assumptions are introduced: *(i)* for each link, deflections are limited to the plane of rigid motion; *(ii)* gravity effects are not included in the analysis. As a result, the terms n_i in (2) are purely quadratic in the velocity $(\dot\theta,\dot\delta)$ and any undeformed rest configuration will be an equilibrium state for the robot arm. Typical examples of this situation are the motion in a horizontal plane and robotic operations in space.

3.1 Joint and link-point based inversion control

Let the output of the system be defined in a parametric form as

$$y = \theta + \lambda C\delta, \qquad \lambda \in [0,1]. \tag{3}$$

When $\lambda = 0$, (3) is just the set of rigid joint variables. More in general, the component y_i is θ_i modified by a linear combination of the variables δ_{ij} relative to the ith link. When $\lambda = 1$, each output is an angle that points from the joint to the end of the associated flexible link.

Under the assumption that the so-called decoupling matrix is nonsingular, the inversion algorithm requires to differentiate each output component until the input appears explicitly and then solve for the input u [23]. Define the inverse of the inertia matrix $H = B^{-1}$ and denote by H_{ij} the blocks of its partitioned form. The generic *relative degree* for system (2) with (3) is uniformly equal to two. Then, dropping dependencies,

$$\begin{aligned} \ddot y = \ddot\theta + \lambda C\ddot\delta &= (H_{11} + \lambda C H_{12}^{\mathrm{T}})(u - n_1 - D_1\dot\theta) \\ &\quad - (H_{12} + \lambda C H_{22})(n_2 + K\delta + D_2\dot\delta). \end{aligned} \tag{4}$$

Setting $\ddot y = v$ and solving for u yields, after some manipulations, the nonlinear feedback controller

$$\begin{aligned} u &= n_1 + D_1\dot\theta + B_{11}v \\ &\quad - (B_{12} - \lambda B_{11}C)(B_{22} - \lambda B_{12}^{\mathrm{T}}C)^{-1}(B_{12}^{\mathrm{T}}v + n_2 + K\delta + D_2\dot\delta). \end{aligned} \tag{5}$$

The closed-loop dynamics

$$\ddot y = v \tag{6a}$$

$$\ddot\delta = (B_{22} - \lambda B_{12}^{\mathrm{T}}C)^{-1}(B_{12}^{\mathrm{T}}v + n_2 + K\delta + D_2\dot\delta) \tag{6b}$$

should be asymptotically stable. The external input v is designed as a linear stabilizing feedback for (6a), while the choice of the output parameter λ will affect directly the stability of (6b).

A simplification occurs when $\lambda = 0$, i.e. when a joint-based inversion strategy is pursued. In this case, the control law becomes

$$u = n_1 + D_1\dot{\theta} - B_{12}B_{22}^{-1}(n_2 + K\delta + D_2\dot{\delta}) + (B_{11} - B_{12}B_{22}^{-1}B_{12}^{\mathrm{T}})v, \tag{7}$$

and the closed-loop unobservable dynamics is

$$\ddot{\delta} = -B_{22}^{-1}(B_{12}^{\mathrm{T}}v + n_2 + K\delta + D_2\dot{\delta}). \tag{8}$$

It can be shown that the zero dynamics associated to (8) (i.e. when $y \equiv 0$) is exponentially stable [24]. Thus, inversion at the joint level is always feasible. By continuity, there exists an interval $[0, \lambda_0]$ for λ that guarantees closed-loop stability while providing exact tracking of the associated output trajectory. Indeed, if $\lambda_0 < 1$ end-effector exact tracking will never be realized. However, the closer is the output to the tip of each link, the better the end-effector will approximate the same trajectory.

3.2 Nonlinear regulation of end-effector motion

In this case the output is chosen directly at the end of each flexible link, i.e. setting $\lambda = 1$ in (3) or

$$y = \theta + C\delta. \tag{9}$$

Under the hypothesis of small link deformation, this output is one-to-one related to the cartesian position and orientation of the end-effector through the standard direct kinematics of the arm.

Following [25], the design of a regulator requires the computation of a reference *state* trajectory associated to the *output* evolution, together with a nominal *input* needed to keep the state along its reference trajectory. The output reference trajectory is generated by a dynamic autonomous exosystem that can be chosen, without loss of generality, as a linear system in observable canonical form with

$$Y_d = \{y_{d,i},\, \dot{y}_{d,i},\, \ddot{y}_{d,i}, \ldots,\, y_{d,i}^{r_i-1};\ i = 1, \ldots, N\} \tag{10}$$

taken as its state. Here, r_i is the smoothness degree of the ith output reference trajectory.

Using (9), the robot dynamics (2) can be rewritten in terms of the new coordinates (y, δ) as

$$\begin{aligned} &B_{11}(y - C\delta, \delta)\ddot{y} + \left[B_{12}(y - C\delta, \delta) - B_{11}(y - C\delta, \delta)C\right]\ddot{\delta} \\ &\qquad + n_1(y - C\delta, \delta, \dot{y} - C\dot{\delta}, \dot{\delta}) + D_1(\dot{y} - C\dot{\delta}) = u, \end{aligned} \tag{11a}$$

$$\begin{aligned} &B_{12}^{\mathrm{T}}(y - C\delta, \delta)\ddot{y} + \left[B_{22}(y - C\delta, \delta) - B_{12}^{\mathrm{T}}(y - C\delta, \delta)C\right]\ddot{\delta} \\ &\qquad + n_2(y - C\delta, \delta, \dot{y} - C\dot{\delta}, \dot{\delta}) + K\delta + D_2\dot{\delta} = 0. \end{aligned} \tag{11b}$$

The reference state evolution is then equivalently specified in terms of y_d and δ_d, and of their derivatives $\dot{y}_d$ and $\dot{\delta}_d$. In particular, it is easy to see that the problem reduces to determining only the N_e functions $\delta_d = \pi(Y_d)$, with $\dot{\delta}_d = (\partial\pi/\partial Y_d)\,\dot{Y}_d$ as a direct consequence.

Thus, the vector function $\pi(Y_d)$ should satisfy equation (11b), evaluated along the reference evolution of the output

$$\begin{aligned} &B_{12}^{\mathrm{T}}(y_d, \pi(Y_d))\ddot{y}_d + \left[B_{22}(y_d, \pi(Y_d)) - B_{12}^{\mathrm{T}}(y_d, \pi(Y_d))C\right]\ddot{\pi}(Y_d, \dot{Y}_d, \ddot{Y}_d) \\ &\qquad + n_2(y_d, \pi(Y_d), \dot{y}_d, \dot{\pi}(Y_d, \dot{Y}_d)) + K\pi(Y_d) + D_2\dot{\pi}(Y_d, \dot{Y}_d) = 0. \end{aligned} \tag{12}$$

This equation is independent from the applied torque and should be considered as a dynamic constraint for $\pi(Y_d)$. Being (12) a *nonlinear time-varying* differential equation, it is almost impossible to determine a bounded solution in closed form. Note that this would be equivalent to finding proper initial values for $\pi(Y_d)$ and $\dot{\pi}(Y_d, \dot{Y}_d)$ at time $t = 0$ such that forward integration of (12) yields a bounded evolution.

A feasible approach is to build an approximate solution $\widehat{\pi}(Y_d)$ by using basis elements which are bounded functions of their arguments, e.g. polynomials in Y_d [10]. As long as each component $y_{d,i}(t)$ of the desired trajectory and its derivatives up to the $(r_i - 1)$ order are bounded, the approximation $\widehat{\pi}(Y_d)$ is necessarily a bounded function of time. The problem of determining the constant coefficients in the expansion is then solved through a recursive procedure based on the polynomial identity principle. Once a solution $\widehat{\pi}(Y_d) \approx \pi(Y_d)$ is obtained up to any desired accuracy, backsubstitution of the reference deformation δ_d, of the desired output trajectory y_d, and of their time derivatives into (11a) will give the nominal feedforward term $u_d = \gamma(Y_d, \dot{Y}_d, \ddot{Y}_d)$ of the regulation law.

If the initial state is not on the reference trajectory, a stabilizing term should be added in order to drive the state towards this solution and only *asymptotic* output reproduction can be guaranteed. Since the system linearization is controllable, this can be accomplished —at least locally— using a simple linear state error feedback characterized by a matrix F. The final regulation law takes on the form

$$u = \gamma(Y_d, \dot{Y}_d, \ddot{Y}_d) + \widehat{F}\begin{bmatrix} y_d - y \\ \dot{y}_d - \dot{y} \\ \delta_d - \delta \\ \dot{\delta}_d - \dot{\delta} \end{bmatrix} = \gamma(Y_d, \dot{Y}_d, \ddot{Y}_d) + F\begin{bmatrix} \theta_d - \theta \\ \dot{\theta}_d - \dot{\theta} \\ \delta_d - \delta \\ \dot{\delta}_d - \dot{\delta} \end{bmatrix}. \tag{13}$$

If a particular globally stabilizing law exists, then convergence to the reference state behavior associated to the desired output trajectory can be achieved from *any* initial state. For robot arms with flexible links it is known that, in the absence of gravity, proportional-derivative (PD) joint feedback is enough for stabilizing any arm configuration [8,26]. As a result, a simplified matrix F can be used in (13) and the overall regulator for the nonlinear system can be implemented using only *partial state* feedback.

3.3 Iterative learning of end-effector motion

Suppose that the task requires to execute the desired end-effector trajectory several times. We may then acquire from experiments (or from simulations using an accurate dynamic model) the input torque needed for accurate reproduction of that particular trajectory. At each trial, a feedforward command is applied to the robot and output error data are stored for subsequent off-line processing. The next feedforward term is computed by combining the previous command ('memory') with the error at the current trial ('update'). Provided that the robot arm is stabilized, a contraction mapping argument can be used to prove convergence of the learning process.

It has been shown [27] that for rigid robot arms one can design the learning algorithm based only on approximate information about the system dynamics. In the presence of flexibility, additional filtering of high-frequency signal components is needed to guarantee convergence. The price to pay is that tracking accuracy is preserved only within a bandwidth of interest (i.e. for $f < f_0$) [28]. However, this operation prevents other unmodeled effects from destroying learning convergence.

In the synthesis of a learning controller a linearized model can be used, e.g. obtained by expanding (2) around a static configuration $\theta = \theta_0$, $\delta = 0$. In order to better shape the system characteristics, a preliminary linear feedback is applied. Following the same arguments used in (13), a PD joint feedback law is sufficient and the closed-loop linearization yields

$$B(\theta_0, 0)\begin{bmatrix}\Delta\ddot{\theta}\\ \Delta\ddot{\delta}\end{bmatrix} + \begin{bmatrix}F_P\Delta\theta + (D_1 + F_D)\Delta\dot{\theta}\\ K\Delta\delta + D_2\Delta\dot{\delta}\end{bmatrix} = \begin{bmatrix}v\\ 0\end{bmatrix}, \tag{14}$$

with obvious meaning of the variations Δ. Taking only the diagonal elements $w_{ii}(s)$ of the matrix transfer function associated to (14) and to the tip output (9), a decentralized synthesis can be performed in the frequency domain. Let v_k be the feedforward command used at the $k$trial (v_1 usually contains all a priori information). The next feedforward is derived as

$$v_{k+1,i}(s) = \alpha_i(s)e_{k,i}(s) + \beta_i(s)v_{k,i}(s), \qquad i = 1, \ldots, N, \tag{15}$$

where $e_{k,i} = y_{d,i} - y_{k,i}$ is the ith joint error at the kth trial, and α_i and β_i are the two learning filters to be designed. For rigid manipulators, it has been shown that the learning process is convergent provided that [28]

$$\left|\beta_i(s) - \alpha_i(s)w_{ii}(s)\right| < 1, \qquad i = 1, \ldots, N. \tag{16}$$

Although a similar argument has not yet been proven for the flexible case, this simple design approach can be attempted and shown to converge by simulation. Note that noncausal filtering is allowed in (15), as well as anticipative shifting of signals in time, since the whole processing is performed off-line.

The applied control law at the kth trial is then

$$u = v_k + F\begin{bmatrix}\theta_d - \theta\\ \dot{\theta}_d - \dot{\theta}\end{bmatrix}. \tag{17}$$

Also in this case, as for regulation, the stabilizing feedback used in (17) is not closed around the output for which trajectory tracking is desired. However, the tip tracking error is taken into account here, being fed back 'off-line' through the learning procedure. Accordingly, control effort will move gradually from feedback to feedforward along iterations.

3.4 Discussion of the methods

It is interesting to summarize assumptions and properties of the above three tracking control methods. Table 1 collects various aspects which should help in understanding system requirements and ease of implementation.

	Inversion control	Nonlinear regulation	Learning control
End-effector tracking	approximate	asymptotic	exact ($f < f_0$)
Feedback type	nonlinear	linear	linear
Partial state feedback	no	yes	yes
Trajectory known a priori	no	yes	yes
Repeated trials	no	no	yes
Anticipative action	no	no	yes
Model accuracy	high	high	low

Tab. 1 – Comparison of tracking controllers for flexible robot arms

The following additional remarks are in order.

- An inversion control approach may give stable results even when applied at the end-effector level, provided that the flexible arm is correctly initialized. This would mean to 'preload' the link deflection (and its velocity) at time $t = 0$ in a way that depends on the desired tip trajectory.
- When applying the final feedforward computed by learning, the system state at time $t = 0$ will be in the right 'preloaded' conditions, as a result

of the anticipative action of the input torque. Thus, learning iterations may be seen as a computational scheme for obtaining the reference state evolution that produces exact output tracking. In practice, some error will be left due to the frequency cut-off at $f = f_0$ (see Table 1).

- At steady operation, e.g. with a sinusoidal trajectory, the feedforward terms of the regulation and the learning control schemes will coincide.

4. Numerical results

4.1 Comparative simulations

In order to compare inversion, regulation, and learning control, we consider a one-link planar arm of 1 m and 0.2 kg, modeling flexibility by just one elastic spring ($k = 10$ Nm/rad, corresponding to an eigenfrequency of $\approx$ 5 Hz) located halfway along the link. Large deflections are allowed, resulting in two second-order weakly nonlinear dynamic equations that can be found in [5]. Damping coefficients (see (2)) are $d_1 = d_2 = 0.01$ Nm/rad. This simple model is still representative since the end-effector angular position is a non-minimum phase output.

The desired output trajectory has a sinusoidal velocity profile and moves 90° in 1 second, with zero initial and final velocity. Simulations were run in nominal conditions for 1.8 seconds, using a 2nd order Runge-Kutta integration routine. In all cases the robot initial state is zero (undeformed and at rest).

The results obtained with inversion control for an output point at $\lambda = 0.3$ of the distal half link are shown in Figs. 1–3. Note that in this case $\lambda_0 = 2/3$. Although satisfactory tip tracking is obtained (maximum error is 0.45 deg), the error is persistent while the control effort is highly oscillatory. Note that the small error drift after the trajectory completion is due to the absence of a stabilizing feedback, i.e. $v = \ddot{y}_d$ is used in (6a).

Nonlinear regulation results are shown in Figs. 4–6, using a feedback matrix F in (13) which sets the poles of the closed-loop system linearization at $(-20, -20, -30, -30)$. After a transient phase, which vanishes in 0.2 seconds, perfect tracking is obtained in this case. The second transient error at the final trajectory time ($T = 1$ sec in Fig. 5) is due to the switching of the reference trajectory from a sinusoidal one to a constant value, a situation that cannot be generated by any smooth autonomous exosystem. The required torque is similar to the one needed if the arm was rigid (dashed line in Fig. 6).

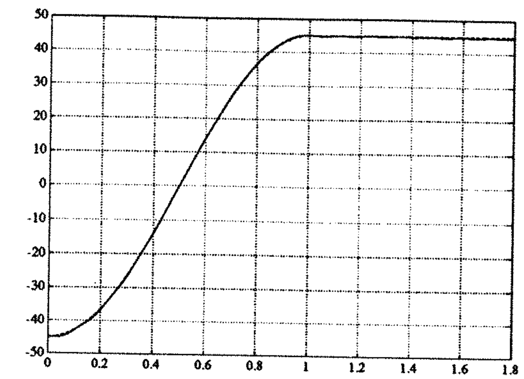

Fig. 1 – Tracking with inversion control ($\lambda = 0.3$)

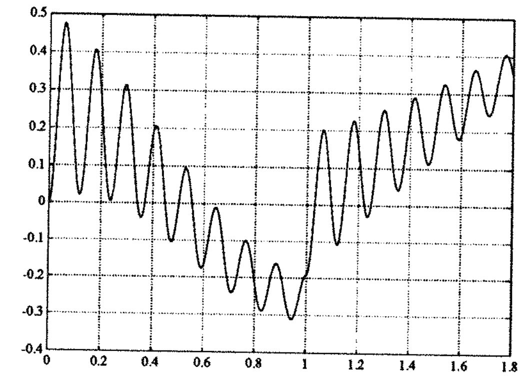

Fig. 2 – Trajectory error with inversion control ($\lambda = 0.3$)

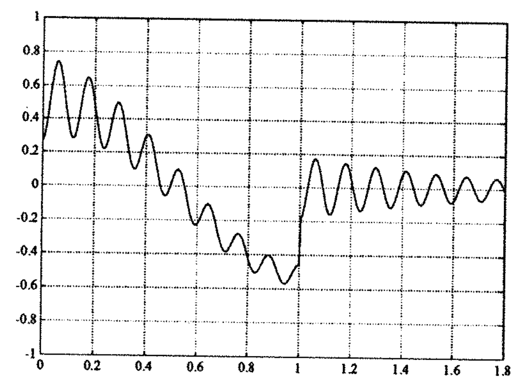

Fig. 3 – Input torque with inversion control ($\lambda = 0.3$)

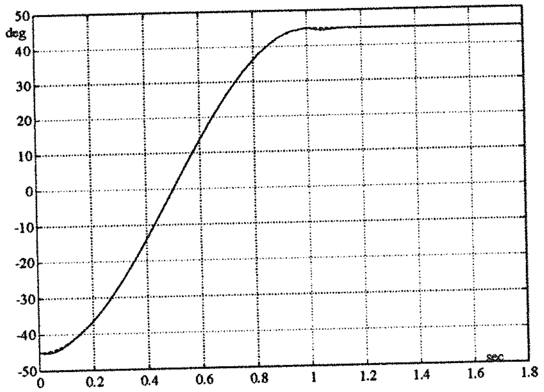

Fig. 4 – Tracking with nonlinear regulation

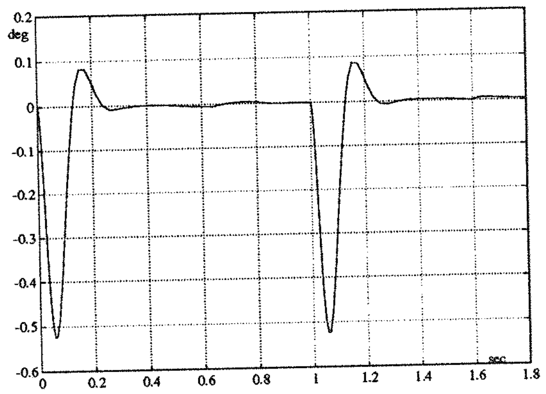

Fig. 5 – Trajectory error with nonlinear regulation

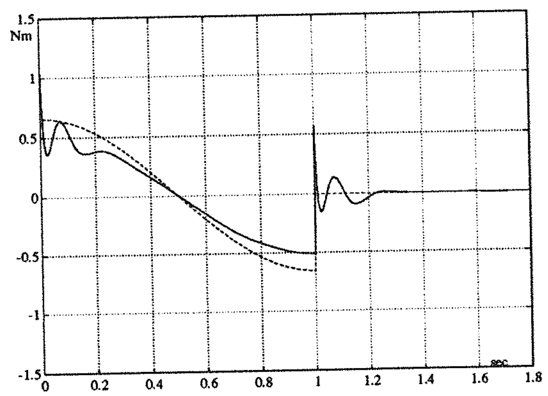

Fig. 6 – Input torque with nonlinear regulation

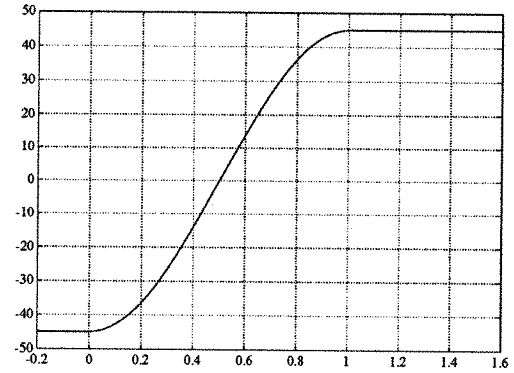

Fig. 7 – Tracking with learning control

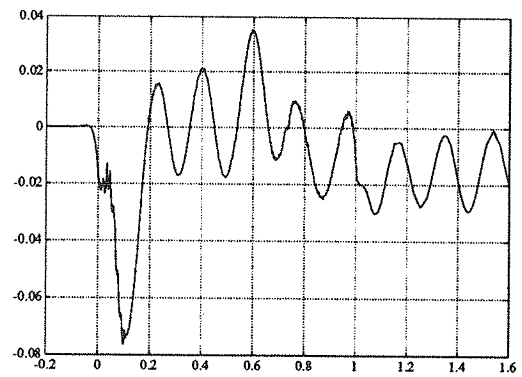

Fig. 8 – Trajectory error with learning control

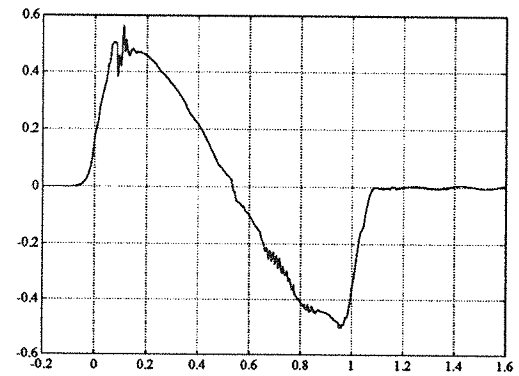

Fig. 9 – Input torque with learning control

Finally, learning control results are summarized in Figs. 7–9, where a time shift of 0.2 seconds is used to show that the input torque begins before the start and ends after the completion of the actual output trajectory. These results are obtained after 50 trials, using an underlying PD joint feedback with $F_P = 45$ Nm/rad and $F_D = 90$ Nm·sec/rad. The maximum error is almost reduced by a factor of 10, thanks to the anticipative torque action. This is realized by the use of a 10 msec time lead in the filter $\alpha(s)$. The very limited residual oscillations are due to the overall frequency cut-off at 13 Hz introduced by both $\alpha(s)$ and $\beta(s)$.

4.2 Experimental validation

The above control strategies are currently being implemented on the experimental testbed available in the Robotics Laboratory at DIS. The robot is a two-link planar manipulator with a very flexible forearm, driven by direct drive DC motors. The links are 0.3 and 0.7 meters long, with inertias of 0.447 and 0.303 kg·m^2 with respect to the joint axes. The first two clamped eigenfrequencies of the flexible link are at 4.7 and 14.4 Hz. A detailed description of the overall system is given in [16,29]. In particular, control laws are executed with 5 msec sampling time.

Figures 10 and 11, that refer to experiments performed keeping the first (rigid) link fixed, are obtained using learning control on a fast quintic polynomial trajectory with a rest-to-rest slew of 90° in 1 second. The typical lag and overshoot with respect to the desired tip trajectory of a pure PD joint feedback are practically eliminated over the iterations. After 27 trials, the maximum error is reduced to about 0.3°. Further results with learning control are presented in [13].

As a preliminary step towards end-effector regulation for the full two-link flexible arm, nonlinear regulation of desired *joint* trajectories has been implemented first [29].

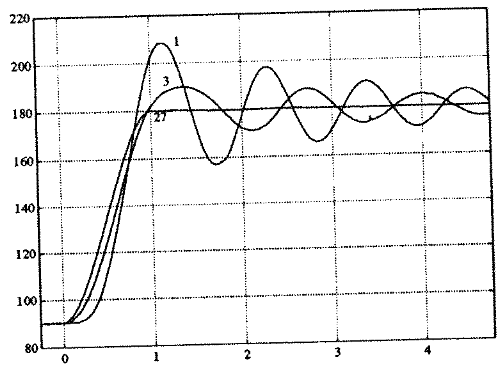

Fig. 10 – Tip tracking with learning control (*one-link experiment*)

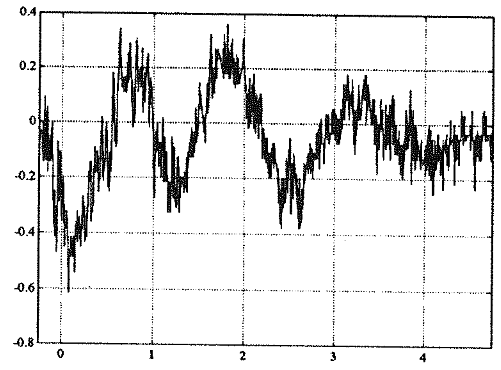

Fig. 11 – Tip error with learning control (*one-link experiment*)

Figures 12–14 illustrate the performance on a very demanding bang-bang acceleration trajectory; in particular, the second link undergoes a 4 seconds motion of 360°, with reversal at the midpoint. The stabilizing PD gains are $F_P = \text{diag}\{11.5, 6\}$ and $F_D = \text{diag}\{2, 0.8\}$. As expected, in contrast to the good accuracy obtained at the joint level (Fig. 12), the end-effector behavior is not satisfactory, with a maximum angular deflection of 7° (Fig. 13). Note that the deformation profile is similar to the one computed by simulation (dashed line) with an identified model of the robot arm. The torque oscillations in Fig. 14 compensate the effects of link deflections so to obtain a quasi-rigid joint motion. Only joint position and velocity measures are used in these experiments.

We point out that the same control hardware is ready to be used for end-effector regulation. In fact, once the joint trajectories associated to the desired tip motion are computed from the regulator equations (viz. $\theta_d = y_d - \delta_d$), these can be provided as reference for the joint controllers together with the proper feedforward.

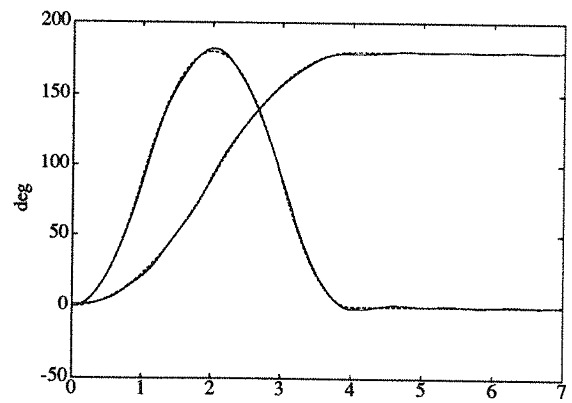

Fig. 12 – Joint trajectory regulation (*two-link experiment*)

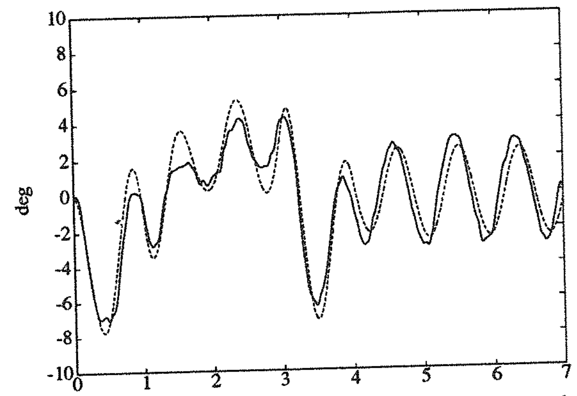

Fig. 13 – Tip angular error with joint regulation (*two-link experiment*)

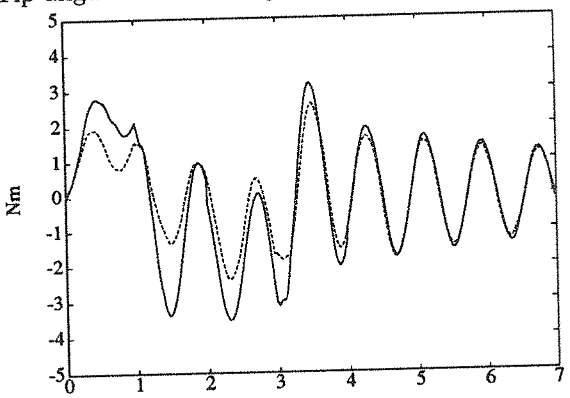

Fig. 14 – Input torques with joint regulation (*two-link experiment*)

5. Conclusions

Accurate control of the end-effector motion in flexible robot arms is a paradigmatic instance of the trajectory tracking problem for nonlinear systems with non-minimum phase outputs. The obtained results support the confidence in the proposed control methods as appealing solutions to this problem, in particular those based on nonlinear regulation and iterative learning theory. On-going research is devoted to the realization of robust (practical) tracking controllers for flexible robots. We are currently investigating the design of observers for the flexible deformation velocity. Within the same framework, we will evaluate the feasibility of end-effector trajectory regulation using only tip measurements and nonlinear dynamic output feedback. On-line identification of dynamic parameters (e.g. a tip payload) is also being studied, for inclusion in an adaptive regulation scheme.

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Variable Structure, Bilinear and Intelligent Control with Power System Application

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Dedication. On this 20th anniversary of our first U.S.-Italy Seminar on Variable-Structure Synthesis in Sorrento, it is an opportunity to express my gratitude to Professor Antonio Ruberti for this enlightening scientific and cultural collaboration.

Abstract. A brief background on variable-structure and bilinear systems relates this research to the first U.S.-Italy seminar on this topic. Sliding-mode control of a flexible ac transmission system (FACTS) is analyzed and shown to be quite effective for a simplified single-machine infinite-but model (SMIB). For cases where system complexities may cause robustness problems, a bilinear self-tuning control and a neural-network based control are introduced.

1. Introduction

An excellent tutorial on variable-structure control which emphasizes sliding modes is given in [1]. It is shown that such sliding-mode control can be designed so that system performance is largely independent of plant dynamics. About 30 years ago, Emelyanov and his associates investigated variable-structure systems in some detail and derived elaborate switching policies for second-order processes in particular. This work was finally published in book form in 1970 [2]. One interesting application of a sliding-mode (with chattering) to state-space (power) constrained optimal control for nuclear-powered rocket propulsion was independently developed about the same time [3,4].

Meanwhile, bilinear systems (BLS) research grew out of the Los Alamos project and was sponsored by the National Science Foundation. As a special form of variable-structure system (e.g., linear system with controlled damping), appropriately designed BLS were shown to exhibit superior performance and improved controllability over their linear subclass [5-7].

As a result of a cooperative research program with Ruberti at the University of Rome (La Sapienza), a number of publications evolved on variable structure systems and BLS. This created a substantial base of theory