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Control of Redundant Robots on Cyclic Trajectories

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Abstract

The majority of local methods for solving kinematic redundancy in manipulators are not repeatable, i.e. non-cyclic joint trajectories are generated when the end-effector is required to trace a closed path. This results in a joint motion with unpredictable or unstable characteristics. Previous researchers have used a differential geometric approach to identify a condition for a strategy to be repeatable. For a particular resolution method, even if such condition does not hold in general, there may exist specific initial joint configurations from which a cyclic motion is obtained. In this paper, the problem of achieving asymptotic cyclicity on a periodic end-effector task is considered. Starting from a generic initial condition, we obtain convergence of the joint trajectory towards a cyclic evolution generated by the chosen kinematic inversion scheme, initialized at a repeatable configuration. This is achieved through a series of different on-line kinematic control schemes producing either asymptotic or exact end-effector tracking.

1. Introduction

Kinematic redundancy provides robot manipulators with greater dexterity and improved capabilities over conventional structures. As a counterpart, the synthesis of joint trajectories realizing a given end-effector task implies a choice among an infinity of inverse kinematic solutions. The difficulty of this problem is increased by the nonlinearity of the kinematic mapping. Therefore, most redundancy resolution schemes are local in nature, in that they incrementally specify joint displacements by inverting the linear relationship between end-effector and joint velocities at a sequence of points along the path.

A drawback of local resolution methods is that most of them are not repeatable, i.e. non-cyclic joint trajectories are generated when the end-effector is required to trace a closed path. This is of course undesirable, because unpredictable joint motions may override the potential advantages of a redundant structure. In [1], a differential geometric condition was identified for a strategy to be repeatable, which implies the integrability of the inversion scheme. Even if such condition does not hold for all arm configurations, there may exist initial joint settings from which a cyclic motion is obtained.

In this paper, we tackle the problem of achieving asymptotic cyclicity for a given resolution strategy on a periodic task. In particular, starting from a generic joint configuration, convergence of joint evolution towards a cyclic trajectory is seeked. This reference trajectory is generated by the chosen kinematic inversion scheme, initialized at a joint configuration that satisfies the aforementioned condition. The problem is solved by introducing on-line kinematic control schemes which guarantee asymptotic convergence to the reference joint trajectory while the end-effector follows a given path. A number of alternative laws are obtained, which differ for the structure of the feedforward and feedback terms, yielding exact or asymptotic end-effector tracking.

In our analysis, we will consider simple Jacobian pseudoinversion as resolution method, and modify this basic scheme to attain asymptotic cyclicity; however, the developments apply to any generalized inversion technique. Also, the problem will be addressed at a purely kinematic level. In fact, dynamic nonlinearities may be cancelled by using a computed torque control.

An overview of the cyclicity problem is given in the next section. Possible ways to achieve repeatability are then discussed, motivating the need for on-line feedback laws. Several control schemes are introduced, providing stability proofs as well as simulation results.

2. The cyclicity property

The differential kinematics of a manipulator with n joints executing an m-dimensional task is expressed as

$$\dot{\mathbf{p}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}, \qquad \mathbf{q} \in I\!R^n, \mathbf{p} \in I\!R^m, \tag{1}$$

where $\mathbf{q}$ is the joint variables vector, and $\mathbf{p}$ specifies the end-effector location with respect to the particular task. Denoting by $\mathbf{k}(\mathbf{q})$ the direct kinematic mapping, $\mathbf{J}(\mathbf{q}) = \partial\mathbf{k}/\partial\mathbf{q}$ is the manipulator Jacobian matrix. For a kinematically redundant robot, $n > m$ and there exists an infinity of joint velocities solving the linear system (1) at a given $\mathbf{q}$.

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Consider instantaneous resolution laws of the form

$$\dot{\mathbf{q}} = \mathbf{G}(\mathbf{q})\dot{\mathbf{p}}, \qquad (2)$$

where $\mathbf{G}(\mathbf{q})$ is any *generalized inverse* of $\mathbf{J}(\mathbf{q})$, i.e. a matrix satisfying $\mathbf{JGJ} = \mathbf{J}$ for any $\mathbf{q}$. If $\mathbf{J}$ is full row rank (or equivalently, if $\mathbf{q}$ is a nonsingular configuration), then necessarily $\mathbf{JG} = \mathbf{I}$. A common choice for $\mathbf{G}$ in (2) is the Moore-Penrose pseudoinverse matrix $\mathbf{J}^{\dagger}$, which provides the minimum norm solution.

The non-cyclicity phenomenon was first discussed in [2] for the pseudoinverse solution. With the end-effector tracing a closed path, a *drift* in the joint position was observed at each cycle completion. Such drift converged to zero in some cases, but in general no limit behavior was observed. A mathematical explaination of this was given for a 3R planar robot, in terms of the non-integrability of a Pfaffian constraint associated with the pseudoinverse solution.

The generalization of this result was obtained by Shamir and Yomdin [1], who established a condition for a generalized inverse $\mathbf{G}$ to yield a repeatable resolution scheme. In particular, let $\mathbf{g}_1, \ldots, \mathbf{g}_m$ be the columns of matrix $\mathbf{G}$, and $\Delta(\mathbf{q})$ be the distribution identified by the m vector fields $\mathbf{g}_1(\mathbf{q}), \ldots, \mathbf{g}_m(\mathbf{q})$ [3]. Assume that a closed path is to be executed in a simply-connected region of the cartesian workspace. Then, a necessary and sufficient condition for cyclicity is that Δ is involutive[1]. We will refer to this as the involutivity condition (IC). The IC implies the existence of an invariant m-dimensional integral manifold M for Δ, to which all joint trajectories are confined. Since the kinematic mapping is one-to-one on M, the produced joint motion is cyclic.

The above integrability condition can be analytically checked, once a mechanical structure and a generalized inversion strategy have been chosen. For most choices of $\mathbf{G}$ in (2), such condition does not hold in general, i.e. for any value of the joint position $\mathbf{q}$. The corresponding resolution method must then be considered non-repeatable. However, there may exist a set $\mathbf{Q}^r_{\mathbf{G}}$ of joint configurations where the IC holds, and which is invariant under $\mathbf{G}$. From these configurations, cyclicity is propagated throughout the motion: initializing (2) at any $\mathbf{q}(0) = \mathbf{q}_r \in \mathbf{Q}^r_{\mathbf{G}}$, the joint posture $\mathbf{q}_r$ is recovered after each end-effector cycle. $\mathbf{Q}^r_{\mathbf{G}}$ will be called the set of *repeatable* configurations for $\mathbf{G}$. Starting from a $\mathbf{q}(0) \notin \mathbf{Q}^r_{\mathbf{G}}$ results in a nonzero drift.

Further studies on the characteristics of the joint drift have been conducted in [4], where numerical simulations showed that the drift has predictable properties in some situations. A more rigorous investigation is carried out in [5]. By defining a drift density measure, it is possible to identify initial configurations producing a stable drift under pseudoinverse control, with the aid of a Lyapunov analysis.

Indeed, the fulfillment of all these mathematical conditions for a generalized inverse matrix $\mathbf{G}$ depends also on the robotic structure under consideration. In fact, the same strategy may be repeatable for some manipulators and not for others. As an example, pseudoinverse control is always repeatable for planar structures with only prismatic joints, but is not in general for other robotic structures. Even for the same manipulator, different choices of joint generalized coordinates will yield different sets of repeatable configurations, being a change of coordinates equivalent to the use of a weighted generalized inverse $\mathbf{G}' \neq \mathbf{G}$. As an example, we derived in [6] the repeatable configurations of a 3R planar arm under pseudoinversion performed in relative coordinates; these are different from the repeatable configurations obtained in [1] using absolute coordinates.

Equation (2) is not the general solution to system (1), because a joint velocity vector belonging to the null space $\mathcal{N}(\mathbf{J})$ could be added, or

$$\dot{\mathbf{q}} = \mathbf{G}(\mathbf{q})\dot{\mathbf{p}} + \big(\mathbf{I} - \mathbf{G}(\mathbf{q})\mathbf{J}(\mathbf{q})\big)\mathbf{v}, \quad \mathbf{v} \in I\!R^n. \qquad (3)$$

However, we show next that in general such a choice rules out repeatability. For this, a slightly different argument will be used to derive the conditions of [1]. When joint motions generated by a resolution strategy on closed end-effector paths are cyclic, this strategy defines an *inverse kinematic function* [7]. As a consequence, there exists an $(n-m)$-dimensional *holonomic* constraint on generalized coordinates and the joint evolution is restricted to an m-dimensional manifold M. In view of this, repeatability of the resolution scheme implies that the corresponding dynamic system is *not* accessible[2], since its accessible state space must have dimension $m < n$, coinciding in fact with M itself. A necessary condition for k-accessibility (i.e. to a k-dimensional set) is that the distribution $\mathcal{A}(\mathbf{q}) = \text{span}\{\mathbf{f}(\mathbf{q}), \mathbf{g}_1(\mathbf{q}), \ldots, \mathbf{g}_m(\mathbf{q}),$ *and all repeated Lie brackets*$\}$ has dimension k in an open and dense subset of the state space [8].

As for system (2), if $\dot{\mathbf{p}}$ is interpreted as input vector, the m columns of the generalized inverse matrix $\mathbf{G}$ are the input vector fields, while no free evolution term is present. Since $\mathbf{G}$ is full column rank away from singularities, distribution $\mathcal{A}$ has at least dimension m. For

[1] The distribution Δ is *involutive* if and only if the Lie bracket $[\mathbf{g}_i, \mathbf{g}_j] \in \text{span}\{\mathbf{g}_1, \ldots, \mathbf{g}_m\}$, for any i, j, where $[\mathbf{g}_i, \mathbf{g}_j] = (\partial \mathbf{g}_j / \partial \mathbf{x})\mathbf{g}_i - (\partial \mathbf{g}_i / \partial \mathbf{x})\mathbf{g}_j$.

[2] A dynamic system $\dot{\mathbf{q}} = \mathbf{f}(\mathbf{q}) + \sum_{i=1}^{m} \mathbf{g}_i(\mathbf{q})u_i$ evolving on a n-dimensional smooth manifold Q is said to be *locally accessible* if the set of reachable states from any point contains a non-empty open (hence of dimension n) set of Q. The vector field $\mathbf{f}(\mathbf{q})$ is also termed the system *free evolution*.

system (2) to be m-accessible, the involutivity of the columns of $\mathbf{G}$ is then required. Thus, the repeatability condition of [1] is recovered. Coming to (3), assume that the null space vector is a function of the joint variables only, i.e. $\mathbf{v} = \mathbf{v}(\mathbf{q})$. In this case, $(\mathbf{I}-\mathbf{GJ})\mathbf{v} = \mathbf{f}(\mathbf{q})$ is a free evolution vector in the null space of $\mathbf{J}$. Since $\mathcal{R}(\mathbf{G}) + \mathcal{N}(\mathbf{J}) = I\!R^n$, the dimension of the distribution $\mathcal{A}(\mathbf{q})$ is at least $m+1$, and repeatability cannot be achieved in any case. If instead $\mathbf{v} = \mathbf{L}(\mathbf{q})\dot{\mathbf{p}}$, where $\mathbf{L}$ is a generic $n \times m$ matrix, (3) may be rewritten as

$$\dot{\mathbf{q}} = \big(\mathbf{G} + (\mathbf{I} - \mathbf{GJL})\big)\dot{\mathbf{p}}. \tag{4}$$

This is equivalent to (2) with a different generalized inverse $\mathbf{G}' = \mathbf{G} + (\mathbf{I} - \mathbf{GJL})$. Again, the involutivity of the columns of $\mathbf{G}'$ is needed for (4) to be cyclic. We will come back to this in Section 3.

The above reasoning shows that *no null space velocity is allowed* in (3) if a repeatable scheme is desired, unless it is chosen as a linear term in $\dot{\mathbf{p}}$. Thus, there is no loss of generality in studying the cyclicity problem restricted to resolution schemes (2).

3. Ways to achieve cyclicity

Suppose that a particular strategy (2) used to generate joint trajectories is not repeatable for the given initial arm configuration $\mathbf{q}_0$. Our objective is to modify or complete the resolution strategy in order to achieve a cyclic behavior on closed end-effector paths. Henceforth, we will refer for illustration to the case $\mathbf{G} = \mathbf{J}^\dagger$, and denote the corresponding set of repeatable configurations simply by $\mathbf{Q}^r$. All the following can be indeed repeated for any generalized inverse.

In principle, there are two alternative approaches to obtain cyclic redundancy resolution from a given $\mathbf{q}_0 \notin \mathbf{Q}^r$:

(i) modify the structure of the resolution scheme (2) so to render $\mathbf{q}_0$ repeatable for the new strategy, or *exact cyclicity*;

(ii) design a kinematic feedback control law to produce convergence of the joint variables towards a cyclic trajectory (one starting from a configuration in $\mathbf{Q}^r$), or *asymptotic cyclicity*.

In turn, two options are available for case *(i)*.

A first possibility is to start with the general solution of equation (1) as in (3), or

$$\dot{\mathbf{q}} = \mathbf{J}^\dagger(\mathbf{q})\dot{\mathbf{p}} + \big(\mathbf{I} - \mathbf{J}^\dagger(\mathbf{q})\mathbf{J}(\mathbf{q})\big)\mathbf{v}. \tag{5}$$

In (5) it is necessary to specify $\mathbf{v}$ as a linear term in $\dot{\mathbf{p}}$ if cyclicity is desired, as previously shown. Being this choice equivalent to using a different generalized inverse (see (4)), the linear mapping $\mathbf{v} = \mathbf{L}(\mathbf{q})\dot{\mathbf{p}}$ has to be chosen in such a way that the set of repeatable configurations includes $\mathbf{q}_0$. This is strongly related to the *Extended Jacobian* method [9], which in fact gives the expression of $\mathbf{v}$ so to make $\mathbf{q}_0$ repeatable, as shown in the following.

Assume that an additional task is imposed through the $(n-m)$-dimensional constraint $\mathbf{J}_a(\mathbf{q})\dot{\mathbf{q}} = \mathbf{0}$. The velocity update can be computed as

$$\dot{\mathbf{q}} = \mathbf{J}_e^{-1}(\mathbf{q}) \begin{bmatrix} \dot{\mathbf{p}} \\ \mathbf{0} \end{bmatrix}, \quad \mathbf{J}_e(\mathbf{q}) = \begin{bmatrix} \mathbf{J}(\mathbf{q}) \\ \mathbf{J}_a(\mathbf{q}) \end{bmatrix} \tag{6}$$

provided that no singularities are encountered. Besides kinematic rank singularities of $\mathbf{J}$ and $\mathbf{J}_a$, also configurations where $\mathcal{N}(\mathbf{J}) \cap \mathcal{N}(\mathbf{J}_a) \neq \mathbf{0}$, commonly known as *algorithmic singularities*, must be avoided. It is possible to express solution (6) in the form (5), by properly choosing $\mathbf{v}$ as a linear term in $\dot{\mathbf{p}}$. In fact, let

$$\mathbf{v} = -\mathbf{Y}^\dagger \mathbf{J}_a \mathbf{J}^\dagger \dot{\mathbf{p}}, \qquad \text{with} \quad \mathbf{Y} = \mathbf{J}_a(\mathbf{I} - \mathbf{J}^\dagger \mathbf{J}), \tag{7}$$

and note that $\mathbf{Y}$ is full row rank whenever $\mathbf{J}_a$ is. By substituting (7) in (5), it is easy to see that the additional constraint $\mathbf{J}_a(\mathbf{q})\dot{\mathbf{q}} = \mathbf{0}$ will be satisfied. Thus, the Extended Jacobian method is equivalent to (5) where $\mathbf{v}$ is given by (7). At this stage, repeatability can be checked without the need of testing the IC. In fact, the obtained motion will be cyclic iff $\mathbf{J}_a$ is the Jacobian of a function $\mathbf{a}(\mathbf{q})$. It follows that every exactly cyclic strategy is equivalent to an Extended Jacobian method [10]. The occurrence of algorithmic singularities may however limit its applicability, especially for highly redundant structures.

A second —though actually equivalent— way to modify the structure of (2) for exact cyclicity is to use a non-constant matrix $\mathbf{W}(\mathbf{q})$ to perform a weighted pseudoinversion. However, the construction of a matrix $\mathbf{W}(\mathbf{q})$ providing an integrable weighted pseudoinverse $\mathbf{J}^\dagger_{\mathbf{W}}$ is quite cumbersome; a procedure based on artificial compliance functions associated to the arm is presented in [11].

For the asymptotic cyclicity approach *(ii)*, a simple solution might be to recover a repeatable initial condition $\mathbf{q}^r \in \mathbf{Q}^r$ by performing a self-motion in advance, i.e. without moving the end-effector. However, it can be shown that this approach may not be feasible[3]; in any case it implies a waste of time. A more efficient solution is to achieve convergence of the joint variables over time towards the cyclic trajectory that starts from a repeatable $\mathbf{q}^r$. This will result in a joint space drift converging to zero. In most cases, fast convergence can be achieved, so that the drift is already eliminated in the first cycle; on subsequent cycles the joint motion will be repetitive.

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[3] In general, it is not possible to perform a desired self-motion by using only smooth feedback.

4. Asymptotic cyclicity via kinematic control schemes

In this section, we introduce kinematic feedback control schemes which guarantee asymptotic convergence to cyclic joint motions. These schemes may be used for real-time control, since they generate joint trajectories which may be directly fed to a computed torque controller. Although reference accelerations are also needed to this aim, we will address here the problem at the velocity level, since the extension to second-order schemes is straightforward.

Let the initial configuration $\mathbf{q}_0$ be not repeatable with the pseudoinversion strategy. To obtain a reference trajectory $\mathbf{q}_r(t)$ associated to a repetitive closed path $\mathbf{p}_r(t)$, we impose the IC[4] to find a repeatable configuration $\mathbf{q}_{r0}$. The joint trajectory generated by

$$\mathbf{q}_r(t) = \int_0^t \mathbf{J}^\dagger(\mathbf{q}_r(\tau))\dot{\mathbf{p}}_r(\tau)d\tau, \qquad \mathbf{q}_r(0) = \mathbf{q}_{r0}, \quad (8)$$

is cyclic by definition. Our objective is to achieve convergence of the *actual* joint trajectory $\mathbf{q}(t)$, the one starting from $\mathbf{q}_0$, to the reference $\mathbf{q}_r(t)$. Note that an initial cartesian error is allowed, i.e. $\mathbf{p}_0 = \mathbf{k}(\mathbf{q}_0)$ may also not be on the desired path.

The basic idea is to generate a feedforward term on the basis of the reference trajectory $\mathbf{q}_r$, while adding a feedback action that smoothly drives the manipulator arm towards the cyclic behavior. Different control laws can be derived, depending on the chosen structure for the feedforward and feedback terms. As the most natural implementation of this approach, we propose

scheme I

$$\dot{\mathbf{q}} = \mathbf{J}^\dagger(\mathbf{q}_r)\dot{\mathbf{p}}_r + \mathbf{K}(\mathbf{q}_r - \mathbf{q}), \quad (9)$$

where $\mathbf{K}$ is a positive definite matrix. The feedforward term is simply the reference velocity $\dot{\mathbf{q}}_r$, while the stabilizing feedback is linear. The following result holds.

Proposition 1. *The joint trajectory $\mathbf{q}_r(t)$ is asymptotically stable for (9).*

Proof. Define the error vector $\mathbf{e} = \mathbf{q}_r - \mathbf{q}$, and consider the Lyapunov candidate

$$V = \frac{1}{2}\mathbf{e}^T\mathbf{e} \geq 0. \quad (10)$$

Its time derivative along the system trajectories is

$$\dot{V} = \mathbf{e}^T[\dot{\mathbf{q}}_r - \mathbf{J}^\dagger(\mathbf{q}_r)\dot{\mathbf{p}}_r - \mathbf{K}(\mathbf{q}_r - \mathbf{q})] = -\mathbf{e}^T\mathbf{K}\mathbf{e} \leq 0, \quad (11)$$

where (8) has been used. $\dot{V}$ is zero only for $\mathbf{e} = \mathbf{0}$, implying asymptotic convergence. ∎

[4] If no repeatable configurations exist, like for a PPR manipulator [6], this approach cannot be applied. A different generalized inverse should then be used.

In [6] it is shown how to design an acceleration scheme with the same characteristics.

Remark 1. Scheme I will guarantee asymptotic joint cyclicity also when a nonzero initial cartesian error is present. As a counterpart, even if the initial cartesian error is zero, it will increase in the transient phase before converging to zero at steady-state. Note that this error behavior is intrinsically related to the nonlinearity of the equations. The feedforward term in (9) will introduce an end-effector transient error because the pseudoinverse is computed along the *reference* joint trajectory, not the actual one. Moreover, the feedback term is clearly not contained in $\mathcal{N}(\mathbf{J})$.

Remark 2. The simple kinematic scheme (9) can be recasted in the framework of *nonlinear regulation* [3] for the input-state-output system $\dot{\mathbf{q}} = \mathbf{u}$, $\mathbf{y} = \mathbf{k}(\mathbf{q})$, where the nonlinearity is concentrated in the output map. This general theory allows, under suitable hypotheses, to design controllers achieving asymptotic output tracking while preserving internal stability, even in the presence of unstable zero-dynamics [12,13].

In view of Remark 1, some suitable modifications of scheme I are proposed for the case of zero initial cartesian error. In this situation we would like to achieve asymptotic joint cyclicity while ensuring *exact* end-effector tracking, i.e. maintaining zero cartesian error at all instants. We argue that it should be possible to reconfigure the arm while moving along the cartesian path, as a 'dynamic' generalization of the concept of self-motion.

One possibility is given by

scheme II

$$\dot{\mathbf{q}} = \mathbf{J}^\dagger(\mathbf{q}_r)\dot{\mathbf{p}}_r + \left(\mathbf{I} - \mathbf{J}^\dagger(\mathbf{q})\mathbf{J}(\mathbf{q})\right)\mathbf{K}(\mathbf{q}_r - \mathbf{q}). \quad (12)$$

The filtering action of the Jacobian null space projection matrix (computed on the *actual* joint trajectory) prevents the end-effector to be perturbed by the joint error term. The resulting feedback is now truly nonlinear. The feedforward term is still a source of cartesian error, but this error magnitude is often negligible, as will be shown in the numerical simulations. However, only simple stability can be proven for this scheme.

Proposition 2. *The joint trajectory $\mathbf{q}_r(t)$ is stable for the closed-loop system (12).*

Proof. Consider the Lyapunov candidate

$$V = \frac{1}{2}\mathbf{e}^T\mathbf{K}\mathbf{e} \geq 0. \quad (13)$$

Its time derivative is

$$\begin{aligned}\dot{V} &= \mathbf{e}^T\mathbf{K}[\dot{\mathbf{q}}_r - \mathbf{J}^\dagger(\mathbf{q}_r)\dot{\mathbf{p}}_r - \left(\mathbf{I} - \mathbf{J}^\dagger(\mathbf{q})\mathbf{J}(\mathbf{q})\right)\mathbf{K}(\mathbf{q}_r - \mathbf{q})] \\ &= -\mathbf{e}^T\mathbf{K}\left(\mathbf{I} - \mathbf{J}^\dagger(\mathbf{q})\mathbf{J}(\mathbf{q})\right)\mathbf{K}\mathbf{e} \leq 0,\end{aligned} \quad (14)$$

being the projection matrix positive semidefinite. ∎

Asymptotic stability cannot be proved since $\dot{V} = 0$ whenever $\mathbf{Ke} \in \mathcal{N}(\mathbf{I} - \mathbf{J}^{\dagger}\mathbf{J})$, or equivalently when $\mathbf{Ke} \in \mathcal{R}(\mathbf{J}^T)$ (recall that $\mathcal{N}(\mathbf{J}) \oplus \mathcal{R}(\mathbf{J}^T) = I\!R^n$). According to LaSalle's Theorem for periodic systems [14], all trajectories will converge to the set $S = \{\mathbf{q} : \dot{V}(\mathbf{q}) = 0\}$, but in this case does not provide any further insight.

From (12), it should be noted that $\dot{\mathbf{e}} = \dot{\mathbf{q}}_r - \dot{\mathbf{q}} = \mathbf{0}$ in S. Thus, if a joint error is present at steady-state motion, it is guaranteed to be constant. The invariance of joint error implies that cyclicity has been achieved, *although* on a lifted joint trajectory, resulting in a different end-effector path. If the cartesian error magnitude is small, the performance of scheme II can be considered satisfactory. This, however, suggests a more robust control law as

scheme III

$$\dot{\mathbf{q}} = \mathbf{J}^{\dagger}(\mathbf{q}_r)(\dot{\mathbf{p}}_r + \mathbf{K}_p\mathbf{e}_p) + \left(\mathbf{I} - \mathbf{J}^{\dagger}(\mathbf{q})\mathbf{J}(\mathbf{q})\right)\mathbf{K}(\mathbf{q}_r - \mathbf{q}), \tag{15}$$

with the end-effector error $\mathbf{e}_p = \mathbf{p}_r - \mathbf{k}(\mathbf{q})$, and a gain matrix $\mathbf{K}_p > 0$. The inclusion of this cartesian correction is effective also in reducing joint errors, as will be shown by simulation.

A more appropriate choice for the case of zero initial cartesian error is

scheme IV

$$\dot{\mathbf{q}} = \mathbf{J}^{\dagger}(\mathbf{q})\dot{\mathbf{p}}_r + \left(\mathbf{I} - \mathbf{J}^{\dagger}(\mathbf{q})\mathbf{J}(\mathbf{q})\right)\mathbf{K}(\mathbf{q}_r - \mathbf{q}). \tag{16}$$

The feedforward is now reduced to the cartesian term $\dot{\mathbf{p}}_r$, with the pseudoinverse evaluated on the *actual* joint trajectory. This control law will ensure exact end-effector tracking at all instants, if the initial end-effector error is zero. In particular, if multiple cyclic joint solutions exist, the control law (16) may allow to 'switch' between two of them while executing the assigned task. Similarly to (15), also scheme IV may be improved by cartesian error feedback, resulting in

scheme V

$$\dot{\mathbf{q}} = \mathbf{J}^{\dagger}(\mathbf{q})(\dot{\mathbf{p}}_r + \mathbf{K}_p\mathbf{e}_p) + \left(\mathbf{I} - \mathbf{J}^{\dagger}(\mathbf{q})\mathbf{J}(\mathbf{q})\right)\mathbf{K}(\mathbf{q}_r - \mathbf{q}). \tag{17}$$

The latter was extensively tested on several trajectories, providing exact end-effector tracking as well as *practical* joint stabilization.

5. Simulation Results

The proposed kinematic control schemes will be illustrated by simulation on a 3R planar robot arm with links of unit length. Pseudoinversion is performed using absolute joint angles (i.e. defined w.r.t. the x axis), so that the repeatable configurations are given by [1]

$$q_1 = q_2, \quad \text{or} \quad q_2 = q_3, \quad \text{or} \quad q_1 = q_3. \tag{18}$$

In all tests, feedback gains were chosen so to yield the best numerical results.

The performance of the various schemes has been compared on an example with zero initial cartesian error. The end-effector should trace periodically, with a cycle time of 1 sec, a circular path of radius 0.38 m centered at (2.43, 0.77), as in Fig. 1. The initial configuration of the manipulator is $\mathbf{q}_0 = (39°, 16°, 5°)$. The joint drift induced when solving redundancy from $\mathbf{q}_0$ by pure pseudoinversion (8) is evident from Fig. 2. The cyclic trajectory used as reference is generated from $\mathbf{q}_{r0} = (0°, 30°, 30°)$. This gives an initial error norm $\|\mathbf{q}_0 - \mathbf{q}_{r0}\|$ of approximately 48°.

Results obtained with scheme I are reported in Figs. 3–4, using $\mathbf{K} = 10 \cdot \mathbf{I}$. Convergence to the desired joint trajectory is obtained within the first cycle. Although the cartesian error is zero at the beginning, its norm increases in the transient up to a value of 0.155 m, approximately one seventh of one link length.

In order to reduce this cartesian error, we have applied schemes II and III. Figures 5–6 refer to scheme II with $\mathbf{K} = 150 \cdot \mathbf{I}$, and show that a non-negligible steady-state joint error is produced, although the maximum cartesian error is reduced in magnitude to 0.045 m. Note that a constant joint error gives rise to a periodic end-effector one: the obtained joint motion is cyclic along a trajectory which produces a slight oscillation with respect to the reference circle. The effects of the cartesian correction in scheme III are reported in Figs. 7–8, where besides convergence to the desired joint trajectory, rapid decay to zero of the cartesian error is shown, with a maximum transient error of 0.02 m in norm. Here, $\mathbf{K} = 150 \cdot \mathbf{I}$ and $\mathbf{K}_p = 10 \cdot \mathbf{I}$.

Finally, schemes IV and V have been used to accomplish exact end-effector tracking at all instants. As pointed out in Section 4, these schemes do not introduce any cartesian error even during the joint stabilization process. The result obtained at the joint level with scheme IV and $\mathbf{K} = 50 \cdot \mathbf{I}$ is displayed in Fig. 9, where the scale has been magnified. A small periodic error is still present, with a peak value of about 0.15°. Note that the oscillation has the same period of the task trajectory. The best performance is obtained by scheme V with $\mathbf{K} = 50 \cdot \mathbf{I}$ and $\mathbf{K}_p = 50 \cdot \mathbf{I}$, as shown in Fig. 10. Asymptotic convergence of the joint trajectory to the desired one is realized, always with zero end-effector tracking error. This satisfactory behavior was confirmed also in other simulations [6].

6. Conclusions

We have investigated the problem of how to achieve a cyclic joint behavior in redundant robots performing cyclic tasks, motivated by the fact that most singularity-free local resolution methods produce non-repeatable joint motions. A controllability analysis of the inverse kinematic system allows to recover the well-known repeatability conditions of [1], and to further conclude that no null space velocity can be specified if a repeatable scheme is seeked, unless it is chosen as a linear term in the end-effector velocity.

The problem of achieving asymptotic cyclicity for a given inversion strategy has been solved via suitable kinematic controls, which guarantee convergence to cyclic joint trajectories along the desired end-effector path. Depending on the structure of the feedforward and feedback terms in the control law, a number of different schemes were proposed, yielding exact or asymptotic end-effector tracking. The provided stability proofs as well as the satisfactory simulation results confirm the advantage of using these simple control strategies. Addressing the problem at a purely kinematic level is not restrictive, since dynamic nonlinearities are not essential and may be compensated for by using inverse dynamics, viz. computed torque control. Also, following the approach in this paper, asymptotically cyclic resolution schemes may be devised at the acceleration level [6,15].

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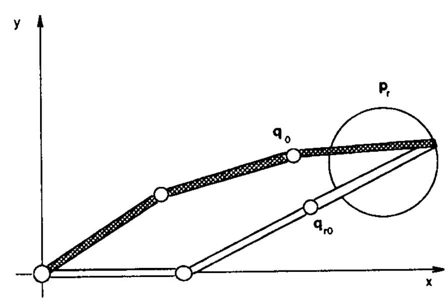

Fig. 1 - End-effector path $\mathbf{p}_r$, initial configuration $\mathbf{q}_0$, and repeatable configuration $\mathbf{q}_{r0}$

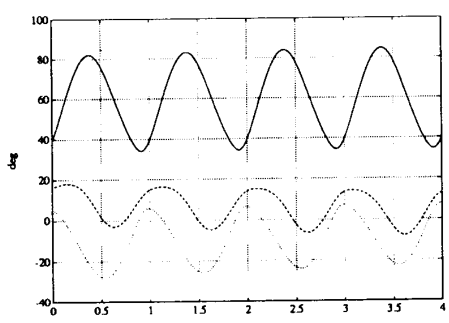

Fig. 2 - Joint drift with pure pseudoinversion (q_1 plain, q_2 dashed, q_3 dotted)

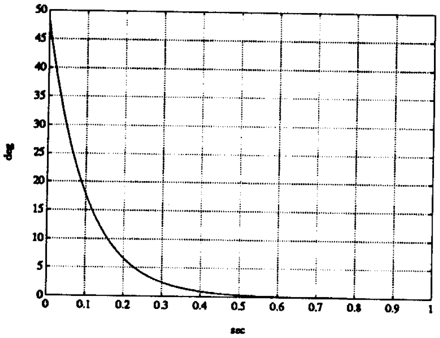

Fig. 3 - Scheme I: joint error norm

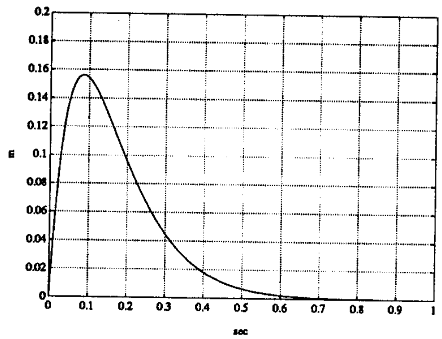

Fig. 4 - Scheme I: cartesian error norm

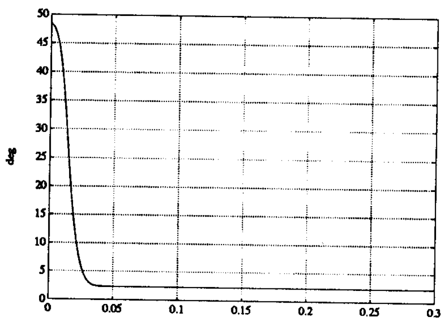

Fig. 5 - Scheme II: joint error norm

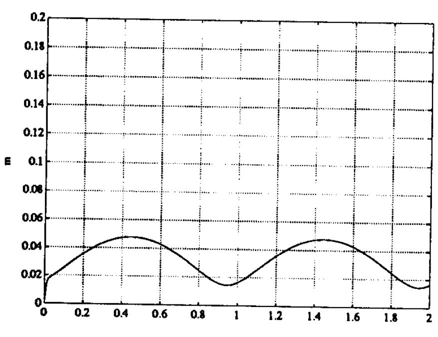

Fig. 6 - Scheme II: cartesian error norm

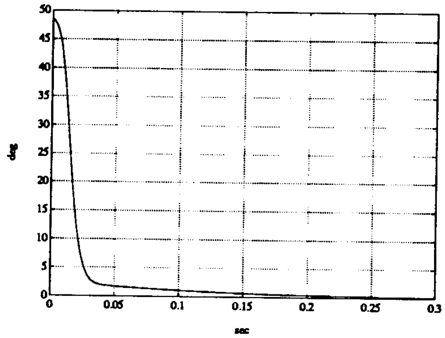

Fig. 7 - Scheme III: joint error norm

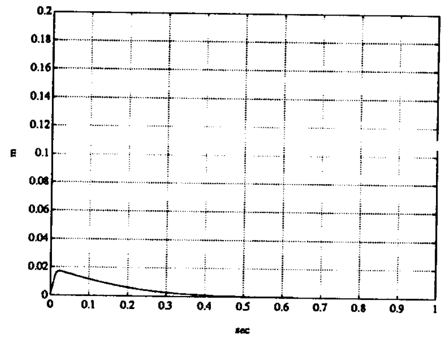

Fig. 8 - Scheme III: cartesian error norm

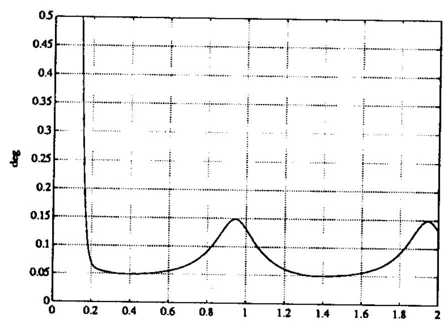

Fig. 9 - Scheme IV: joint error norm

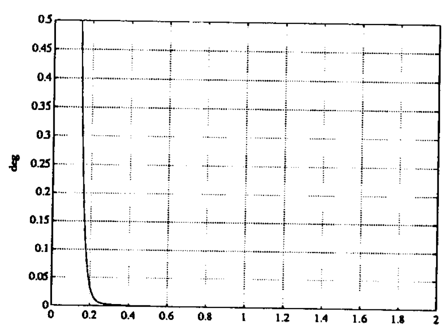

Fig. 10 - Scheme V: joint error norm