Robot Control

Dynamics, Motion Planning, and Analysis

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Part 10
Redundant Robots

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ROBOT MANIPULATORS are said to be *kinematically redundant* when the number n of degrees of freedom (dof) owned by the mechanism is larger than the number m of variables strictly needed for accomplishing a given task. The difference $n - m$ characterizes the *degree of redundancy*. Redundancy is therefore a relative concept for a robot, depending on the particular type of task to be executed. Typically, six joints (e.g., all revolute) are necessary in a robot arm for arbitrary positioning and orienting the end-effector within the workspace. If only the positioning task is of concern, the same arm becomes a redundant one with degree of redundancy equal to three. Similarly, tasks like welding or pointing do not require the full capabilities of six-dof robots, because the final rolling motion around the approach vector is not specified.

Therefore, when the task is of reduced dimensions, exploiting the available degrees of freedom of a conventional arm is already a matter of redundancy utilization. As a result, robots with redundant capabilities already exist in both industrial and research environments. On the other hand, the presence of unavoidable *kinematic singularities*, internal to the robot workspace, restricts feasible motions of a conventional six-dof arm. Thus, its overall functionality and ease of use, for instance in programming realizable end-effector trajectories, is considerably reduced.

This drawback, and in general the need for dexterous manipulation with increased skills, motivated the introduction of a new generation of manipulators with seven or more degrees of freedom. Actual production of redundant robots (i.e., with definitely more than six dof) is in fact growing quite rapidly. Examples are the Cybotech P-15, the K/B-1207 of Robotics Research Corporation [1], or the Cesar manipulator developed at Oak Ridge National Labs [2]. Indeed, efforts in the mechanical design of redundant arms with optimal kinematics [3] have to be complemented by the realization of advanced robot controllers—intended in a broad sense in their supervisory, trajectory planning, and servoing functions—that can make best use of the increased dexterity.

General methodologies for planning and controlling motion of redundant robots are presented in Part 10, rather than specific approaches or solutions. It is easy to find out that most of the reported material applies in a direct way to a variety of robotic systems:

- Single robot arms with seven or more dof, and in particular with a very large degree of redundancy—as in spine-type robots having in practice all continuous configurations;
- Two-robot systems, including also dual-arm robots, or multirobot systems performing strictly cooperative tasks, such as holding together a heavy object;
- Dexterous multifingered hands needed for fine manipulation of complex objects that common grippers cannot grasp firmly;
- Multilegged locomotion systems, where a redundant number of supporting legs is used to improve stability and to allow different possible gaits.

In these illustrative situations, one or more of the following advantages are obtained through redundancy:

- Collision with obstacles in a crowded workspace can be avoided while keeping the motion of selected points (mostly, the end-effector) along prespecified paths;
- Tasks are executed with full utilization of the available joint range, in particular without reaching the geometric limits in the robot configuration space;
- The robotic system is able to assume the best posture for the given task, exerting compatible forces in selected directions and enhancing Cartesian velocity in others;
- Automatic singularity avoidance can be performed so that the feasible workspace, in which the robotic system has full manipulability, may coincide with the reachable one (except for boundaries);
- The same task effort can be distributed over the degrees of freedom, to reduce global joint motion, and/or over the actuators, to minimize total torque demand.

All achieved features can be summarized by the fact that a redundant robot acquires *self-motion* capabilities: A motion can be performed in the joint space, changing the internal arm configuration, without affecting task-space coordinates. This may happen also for conventional arms in correspondence to singular points, but the effective task space would be restricted in that case.

In the face of the above benefits, some factors which have limited the introduction of redundancy should be taken into account:

- The mechanical construction of a redundant arm is usually more complex and requires a larger number of actuating elements, so a more expensive design results;
- Kinematic control algorithms are certainly more sophisticated, because of the one-to-many nature of the inverse kinematic mapping.

Pros and cons have to be traded off, and the success of using redundant robots will depend on the value added to specific applications. On the other hand, many advanced tasks assume high dexterity, self-organized flexibility, and autonomous motion planning as robotic prerequisities. Tasks performed in hazardous environments, currently in a slow fashion and with an expert human operator in the loop (e.g., space servicing or nuclear-plant maintenance), may take advantage of redundant robotic systems capable of autonomous low-level motion adjustment in response to high-level commands. Within the remote manipulation field, an interesting application of redundancy is the use of a simple "master" arm to drive a redundant "slave" manipulator.

The starting point for the formal analysis of redundant robots is in the kinematic relationship between joint-space variables and task-space coordinates:

$$\boldsymbol{p} = \boldsymbol{f}(\boldsymbol{q}), \qquad \boldsymbol{p} \in \boldsymbol{R}^m, \boldsymbol{q} \in \boldsymbol{R}^n, n > m. \tag{1}$$

The inverse kinematic problem consists in finding one configuration $\boldsymbol{q}$ among the set of ∞^{n-m} solutions to (1), for a given instant value of the task-space vector $\boldsymbol{p}$. This is a highly nonlinear problem, with no general closed-form solution available. Therefore, kinematics is usually rewritten at a differential level as

$$\dot{\boldsymbol{p}} = \frac{d\boldsymbol{f}(\boldsymbol{q})}{d\boldsymbol{q}}\dot{\boldsymbol{q}} = \boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}}, \qquad \boldsymbol{J}: n \times m \text{ robot Jacobian.} \tag{2}$$

For each configuration $\boldsymbol{q}$ and assigned task velocity $\dot{\boldsymbol{p}}$, an underdetermined linear system results, with m equations in the n unknowns $\dot{\boldsymbol{q}}$. The choice of one possible joint velocity $\dot{\boldsymbol{q}}$ is simpler, owing to the linearity of this relation. It can be shown that all solutions to (2) can be set in the form

$$\dot{\boldsymbol{q}} = \boldsymbol{J}^{\dagger}(\boldsymbol{q})\dot{\boldsymbol{p}} + [\boldsymbol{I} - \boldsymbol{J}^{\dagger}(\boldsymbol{q})\boldsymbol{J}(\boldsymbol{q})]\boldsymbol{v}, \tag{3}$$

where $\boldsymbol{J}^{\dagger}$ is the (unique) pseudoinverse of $\boldsymbol{J}$, and $\boldsymbol{v}$ is an arbitrary joint-velocity vector. The second term on the right-hand side is the homogeneous contribution, where the projection matrix into the null space of $\boldsymbol{J}$ appears.

In order to select a specific solution, that is, a $\boldsymbol{v}$ in (3), either an *optimization* or a *task-augmentation* approach is followed. In the first case, several performance criteria have been proposed, reflecting some measure of the desired arm behavior. The most common are the weighted distance from the center of the available joint range [4], the distance from the closest obstacle [5], various kinematic manipulability indices [6], [7], [8] and their extensions to include inertial properties [9]. In particular, the definition of a suitable manipulability index to correctly measure distance from structural singularities is an important issue. The smallest singular value of the Jacobian matrix $\boldsymbol{J}$ has been indicated as a reliable one, but it is interesting to note that most definitions can be given a coordinate-free expression [10]. Once a proper criterion $H(\boldsymbol{q})$ is selected, the additional degrees of freedom are used for its local maximization or minimization, mainly based on the Projected Gradient method, yielding $\boldsymbol{v} = \pm\nabla_q H$. Here, local refers to an instantaneous solution that could be derived on-line, without knowledge of the future reference trajectory. This approach is indeed very convenient from the implementation point of view. Note that the optimization process is in principle a constrained one, because (1) has to be satisfied anyway. When using (3), however, the choice of the n-vector $\boldsymbol{v}$ is completely unconstrained.

The philosophy behind task augmentation is that the additional degrees of freedom of a redundant robot may be used for executing other parallel tasks, implicitly defined through a mechanism of constraint satisfaction. When the overall dimension m of the augmented task reaches n, no more redundancy will be left over. There is a strict connection between the two approaches, and usually one may be translated into the other and vice versa. For example, additional constraints could be introduced as the necessary conditions of optimality for an auxiliary problem [11]. Since optimization and task augmentation can also be combined, it is a matter of convenience whether to define a subtask as a hard constraint or to include it into an objective function. Finally, the above two approaches were developed initially at a kinematic (first-order) level, but reformulations at the dynamic level, including robot dynamics, have been considered later.

The four papers in this part discuss in detail the above issues. These contributions have been selected based also on the influence they have had on robotics research in this field.

The first paper (C. A. Klein and C. H. Huang, "Review of Pseudoinverse Control for Use with Kinematically Redundant Manipulators," is a pioneering, short, and handy review on the use of the Jacobian pseudoinverse $\boldsymbol{J}^{\dagger}$ for velocity control of redundant robots. The authors list the properties of pseudoinverses and the basic techniques for their computation. The inclusion of homogeneous solutions to improve performance is illustrated using the simple but ubiquitous 3R planar robot. For this arm, it is proven for the first time that noncyclic joint paths are obtained from cyclic Cartesian paths when redundancy is resolved by simple pseudoinversion (i.e., using the resolved motion method of [12]). The same technique is shown to generate different arm behaviors if a closed Cartesian path is executed clockwise or counterclockwise. Furthermore, pseudoinverse control, though simple and appealing, does not guarantee avoidance of singularities.

In the second paper, by Y. Nakamura et al., "Task-Priority Based Redundancy Control of Robot Manipulators," the very useful concept of priority of tasks is introduced. Arm redundancy is exploited for augmenting the number of tasks to be performed together with the primary one—usually a trajectory to be followed by the robot end-effector. The designer orders the additional list of tasks, each requiring m_i degrees of freedom, by their relative importance. In general, the sum m of dimensions of all tasks can be greater than, equal to or less than n. When their simultaneous satisfaction is impossible, execution of low-priority tasks will be relaxed without affecting high-priority ones. As opposed to the *Extended Jacobian* method [13] defined for $m = n$, computations are always organized to carefully handle the so-called *algorithmic singularities*. A proper use of projection operators avoids a global deterioration of performance, restraining it to the less-revelant tasks. The idea amounts to automatically solving an inconsistent set of linear equations, giving privilege to some of them and least-squaring on the others. Numerical simulations and experiments are presented, including also preliminary consideration of dynamic issues. An obstacle-avoidance task is successfully executed using the task-priority method. For this problem, a similar technique was presented in [5].

The third paper (J. M. Hollerbach and K. C. Suh, "Redundancy Resolution of Manipulators Through Torque Optimization,") focuses on the inclusion of dynamics in the redundancy resolution algorithm. Some of the results in this paper are outgrowths of initial research reported in [14]. Generalized inverses formulated at the acceleration level and used in conjunction with the robot dynamic model were first considered in [15]. Hollerbach and Suh, however, consider torque requirements in a more direct way: In particular, local minimization of weighted and unweighted norms of joint torques is pursued. These two methods are compared with the inertia-weighted and the unweighted minimum-norm acceleration solutions through extensive simulations. When norm weighting is chosen according to the allowable torque range, the obtained solution tends to stay within actuator limits. On the other hand, a rather unexpected instability problem arises for long end-effector trajectories. In fact, although torque is minimized at each instant, the actual torque demand may suddenly "explode" after a smooth initial behavior. The authors interpret this effect physically as a whiplash action, needed to keep the end-effector on the desired path against the induced high velocity of arm motion. This issue is further examined by Maciejewski [16]. In any case, this stability limitation is intrinsic to the local nature of the problem formulation, providing a strong motivation for exploring globally optimal, or, at least, overall-stable dynamic-resolution schemes. It should be noted that inclusion of dynamic aspects in the redundancy-resolution process does not necessarily imply the use of a dynamic (model-based) control law. For this, the reader may refer to the papers of Hsu et al. [17] and of Egeland [18].

The final paper (D. R. Baker and C. W. Wampler, "On the Inverse Kinematics of Redundant Manipulators,") investigates fundamental properties shared by most redundancy-resolution algorithms. The major concern is on local schemes, because these are the only ones executable on line in a sensor-driven motion. The basic concepts of *inverse kinematic function* and, accordingly, of *tracking algorithm* are formally stated. The problem of existence of such functions and algorithms and their characterization are then addressed. Using topological arguments, it is shown that no continuous inverse kinematic function exists for tasks like pointing over the whole sphere or orienting the end-effector in an arbitrary way (e.g., with a spherical wrist), no matter how large the arm-redundancy degree is. Moreover, the authors prove that a tracking algorithm corresponds naturally to an inverse kinematic function if and only if it maps cyclic end-effector paths into cyclic joint paths. The proof of sufficiency is constructive, showing how to obtain an inverse kinematic function from a cyclic tracking algorithm. It is also pointed out how the Extended Jacobian method fits into this general analysis. These results, although of some negative flavor, set precise limitations on what can be done and what cannot.

Because of the tutorial nature of this collection of papers, some topics have been left out. A list of problems is given next, reflecting current trends of investigation in the area of redundant robots and deserving the attention of active researchers.

Global Optimization. As already mentioned, most schemes resolving redundancy via optimization are defined locally. If the whole end-effector trajectory is known in advance, one may also look for global solutions, optimal along the path. The problem becomes much more difficult, requiring the minimization of an objective functional (an integral criterion) subject to two-point boundary-value conditions. Variational techniques or Pontryagin's principle apply to this formulation. Although the associated optimality conditions are easily stated, the bottleneck stands mainly in the numerical computations involved, for instance, with the multiple-shooting solution algorithm. Minimization of joint velocities or kinetic energy along the whole trajectory are two typical objectives. The problem of global torque optimization, which is roughly twice as hard to solve, has been addressed with two different but similar approaches in [19] and [20]. Boundary conditions, which are specified at the initial and final trajectory points, play a major role in all cases. Interestingly enough, cyclic solutions can be obtained for a closed Cartesian path by imposing equal arm configurations at the initial and final instants. Moreover, contrary to the local situation, no explosion of joint torque is reported in the globally minimizing solution. A different research direction is pursued in [21] and [22]: Cases are found where the solution to a global optimization problem coincides, under suitable conditions, with the solution to a related local problem. Then, numerical complexity is dramatically reduced to the simple integration of differential equations.

Computational Aspects. One common disadvantage in handling redundancy is the need for computing pseudoinverses. This is already apparent in (3), and occurs repeatedly in the task-priority method as well as in dynamic optimization schemes. In general, this matrix operation involves a singular value decomposition (SVD) of the robot Jacobian. Advances in the efficient computation of SVD in the robotic case are reported in [23]. In the full rank case for $\boldsymbol{J}$, ways to rearrange pseudoinverse derivation have been suggested in [24] and [2]. Still, the presence of $\boldsymbol{J}^{\dagger}$ and of the projection operator $\boldsymbol{I} - \boldsymbol{J}^{\dagger}\boldsymbol{J}$ complicates unnecessarily the solution, often obscuring its significance. The use of the simple Jacobian transpose $\boldsymbol{J}^T$ in a closed-loop scheme has been proposed in [25]. This method is also robust w.r.t. singularities, similar to [26] and [27]. On the other hand, full exploitation of the idea of decomposing joint variables to reduce the optimization task to the smaller space of the $n - m$ extra degrees of freedom leads to the fast and efficient Reduced Gradient method of [28].

Cyclicity. By letting the robot configuration be defined on a smooth manifold, the powerful tools of differential geometry can be used to investigate the problem of cyclic, that is, repeatable, motion. Shamir and Yomdin [29] have given one significant outcome of this connection, showing that cyclicity with pseudoinverse control is achieved if and only if the columns of $\boldsymbol{J}^{\dagger}$ are *involutive* vector fields. As a consequence, it is found for the 3R planar robot that cyclic Cartesian paths can still be mapped into cyclic joint paths via $\boldsymbol{J}^{\dagger}$, provided the arm starts from certain configurations. This involutivity result, which is in fact an integrability condition related to Frobenius Theorem, is general and applies to any other inverse mapping $\boldsymbol{K}$ in place of $\boldsymbol{J}^{\dagger}$. Its extension to second-order inverse functions (i.e., in terms of accelerations) would be interesting, together with the definition of a strategy for choosing $\boldsymbol{v}$ in (3) to force repeatability.

Time-optimality. The following planning task can be posed. For a redundant robot, let the desired end-effector path be specified in parametric form from source to destination. We would like to find the optimal timing along this path and the optimal sequence of arm configurations, to minimize the total traveling time under joint torque limits. A feasible initial arm configuration may or may not be assigned. This problem is a variant of the time-optimal motion on a given Cartesian geometric path: In the case of conventional robots, very efficient phase-plane solution techniques have been found (see Part 5). At present, the extended problem for redundant manipulators is still open.

As a final remark, it should be stressed that interest has been focused here on robots with redundant kinematics, not on redundant systems in robots. In the latter category, actuational redundancy [30] as well as sensor redundancy and fusion are very important topics, aimed at achieving a more robust robot behavior in the presence of failures or uncertainties.

A list of papers that give a rather complete picture of the area of redundant robots is included for further reading. Surveys on parts of these investigations have appeared recently, [31], [32], wherein more extensive references can be found.

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