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Relevance of dynamic models in analysis and synthesis of control laws for flexible manipulators

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Abstract

Dynamic modelling of multilink flexible manipulators poses a number of interesting issues, due to the distributed nature of link elasticity and to the inherent complexity of the interaction between rigid and flexible components. Obtaining a model in explicit form plays a significant role both for the analysis of simple control laws and for the synthesis of effective model-based strategies. We show how to suitably exploit dynamic model properties in order to prove asymptotic stability of joint collocated Proportional-Derivative (PD) control and to design inverse dynamics controllers for trajectory tracking. In particular, we investigate the implications on the control of using different joint boundary conditions (clamped vs. pinned) in connection with a Lagrangian-assumed modes model of the system.

1. INTRODUCTION

Lightweight materials are increasingly being used in the construction of manipulators with the advantage of a higher payload-to-structure weight ratio and of faster motion. The major drawback is the vibration induced by the structural flexibility distributed along the links.

As opposed to the case of rigid arms, where dynamic issues are often of limited importance, the availability of *dynamic models* is quite relevant for flexible manipulators. The Lagrangian approach provides a natural framework for deriving the equations of motion of mechanical systems undergoing structural deformations [1]. One critical point in modelling flexibility is the method used to derive a finite-dimensional model from an inherently distributed parameter description. In practice, this approximation is unavoidably needed for simulation and control purposes.

Computer simulation of flexible manipulators, aimed at forecasting the behaviour of the structure under various operative conditions, will benefit by the knowledge of an accurate dynamic model. In particular, the generation of reference trajectories that do not excite the vibratory modes of the arm could be satisfactorily investigated at this level; this also provides a way to calculate convenient feedforward motion commands, see e.g. [2].

On the other hand, *motion control* of flexible manipulators should address not only the classical joint position regulation and trajectory tracking problems but also the active suppression of link vibrations. This is complicated by the nonlinear and coupled nature of the equations of motion.

Several control strategies can be devised for flexible manipulators which attempt to extend well established results for rigid arms [3]. In this respect, model completeness is helpful for a rigorous analysis of the performance that can be achieved with simple control laws, not requiring explicit knowledge of model parameters for their implementation. Nonetheless, more advanced model-based control strategies rely for their synthesis not only on a complete model but also on its accuracy for a successful execution of fast trajectories.

In the present work, we focus our attention on the use of a complete nonlinear dynamic model based on assumed modes [4] for approximating link deflections. Some important aspects are analyzed that arise in conjunction with the choice of different boundary conditions for the deformation modes at the actuator side: clamped vs. pinned. The implications on the design of Proportional-Derivative (PD) controllers [5] as well as of inversion-based controllers [6,7] are evaluated in terms of the structure and complexity of the resulting control laws.

2. DYNAMIC MODELLING

Consider a robotic manipulator composed of a serial chain of links, some of which are flexible. The Lagrangian technique can be used to derive the dynamic model, through the computation of global kinetic and potential energy of the system [1]. Due to link flexibility, the dynamic model is of distributed nature. Slender links can be modelled as Euler-Bernoulli beams satisfying proper boundary conditions for the actuated joint and the link tip. While a linear model is in general sufficient to capture the dynamics of a single flexible link, the interplay of rigid body motion and flexible deflection in the multilink case gives rise to fully nonlinear dynamic equations.

In order to obtain a finite-dimensional model for convenient analysis and synthesis of control laws, basis functions for describing link deformation shapes are to be chosen with an associated set of generalized coordinates. Let θ denote the $n \times 1$ vector of joint coordinates, and δ the $m \times 1$ vector of link coordinates of an assumed modes description of link deflections.

For simplicity, we suppose to include only bending deformations limited to the plane of horizontal motion (no gravity is considered). The closed-form dynamic equations of the manipulator can be written as $n+m$ second-order nonlinear differential equations in the general form [8]

$$\begin{pmatrix} B_{\theta\theta}(\theta,\delta) & B_{\theta\delta}(\theta,\delta) \\ B_{\theta\delta}^{\mathrm{T}}(\theta,\delta) & B_{\delta\delta}(\theta,\delta) \end{pmatrix} \begin{pmatrix} \ddot{\theta} \\ \ddot{\delta} \end{pmatrix} + \begin{pmatrix} h_{\theta}(\theta,\delta,\dot{\theta},\dot{\delta}) \\ h_{\delta}(\theta,\delta,\dot{\theta},\dot{\delta}) \end{pmatrix} + \begin{pmatrix} 0 \\ K\delta + D\dot{\delta} \end{pmatrix} = \begin{pmatrix} I \\ Q_{\delta} \end{pmatrix} u. \qquad (1)$$

In (1), the $(n+m) \times (n+m)$ positive definite symmetric inertia matrix B is partitioned into blocks according to the joint (rigid) and link (flexible) coordinates. The $(n+m) \times 1$ vector h contains Coriolis and centrifugal forces, and can be computed via the Christoffel symbols, i.e. via differentiation of the inertia matrix elements; it can be shown that a factorization of h exists

$$h(\theta, \delta, \dot{\theta}, \dot{\delta}) = \begin{pmatrix} h_\theta \\ h_\delta \end{pmatrix} = \begin{pmatrix} S_{\theta\theta} & S_{\theta\delta} \\ S_{\delta\theta} & S_{\delta\delta} \end{pmatrix} \begin{pmatrix} \dot{\theta} \\ \dot{\delta} \end{pmatrix} = S(\theta, \delta, \dot{\theta}, \dot{\delta}) \begin{pmatrix} \dot{\theta} \\ \dot{\delta} \end{pmatrix} \tag{2}$$

such that the matrix $\dot{B} - 2S$ is skew-symmetric, similarly to the rigid case [9]. Further, the positive definite —typically diagonal— matrices K and D in (1) describe modal stiffness and damping of flexible links, respectively.

Several simplifying assumptions can be introduced affecting the actual dependence of terms in the inertia matrix. Typically, $B_{\theta\delta}$ may become a function of θ only and $B_{\delta\delta}$ a constant matrix.

In any case, the terms in (1) assume different analytical expressions and numerical values, depending on the choice of the *assumed modes* of link deflection and of the related geometric/dynamic *boundary conditions*. Assume that a complete set of correct mode shapes has been obtained. For each link, the kinematic description of bending deformation is usually given in terms of two alternative frames; namely, the *clamped* frame, aligned with the direction of the undeformed link at the joint location, and the *pinned* frame, pointing at the instantaneous center of mass of the deformed link. Accordingly, the joint and link coordinates attain different meanings and the terms in the dynamic equations (1) assume different expressions; however, it is always possible to transform one set of coordinates into the other [10]. Eigenfrequencies of the system are the same in both representations, but simplifications may occur in the model. From the model structure point of view, the main difference resides in the $m \times n$ matrix Q_δ that weights the $n \times 1$ vector of joint input torques u in the lower equations. In particular, using the principle of virtual work, it can be shown that $Q_\delta = O$ in the *clamped* case, while in the *pinned* case Q_δ will be a constant matrix depending on the mode shapes.

3. PD CONTROL

To perform a task of robot arm positioning, simple control laws can be derived. For a rigid manipulator in the absence of gravity, it is well known that a PD control based on joint feedback errors ensures asymptotic stability of any desired constant arm posture [9]. Below we show that a similar result holds also for flexible manipulators, with different implications on the controller structure in the pinned and clamped cases.

Consider the *linear* feedback law

$$u = K_P(\theta_{\text{des}} - \theta - Q_\delta^{\text{T}}\delta) - K_D(\dot{\theta} + Q_\delta^{\text{T}}\dot{\delta}), \tag{3}$$

where θ_{des} is the desired arm configuration with no deformation involved, and K_P, K_D are positive definite symmetric matrices. It must be pointed out that the model structure is highly important for the analysis, although the control law is practically model-independent (except for Q_δ).

Theorem 1. *The equilibrium state* $(\theta, \delta, \dot{\theta}, \dot{\delta}) = (\theta_{\mathrm{des}}, 0, 0, 0)$ *of system (1) is asymptotically stable under the control (3).* ∎

Proof. Let $q = (\theta^{\mathrm{T}} \;\; \delta^{\mathrm{T}})^{\mathrm{T}}$ denote the arm configuration and $e = \theta_{\mathrm{des}} - \theta$ the joint error. Consider the energy-based Lyapunov function candidate

$$V = \frac{1}{2}\dot{q}^{\mathrm{T}} B \dot{q} + \frac{1}{2}\delta^{\mathrm{T}} K \delta + \frac{1}{2}(e - Q_\delta^{\mathrm{T}}\delta)^{\mathrm{T}} K_P (e - Q_\delta^{\mathrm{T}}\delta) \geq 0, \tag{4}$$

vanishing only at the desired equilibrium state. The time derivative of (4) along the trajectories of system (1) is

$$\begin{aligned} \dot{V} &= \dot{q}^{\mathrm{T}}(B\ddot{q} + \frac{1}{2}\dot{B}\dot{q}) + \delta^{\mathrm{T}} K \dot{\delta} - (e - Q_\delta^{\mathrm{T}}\delta)^{\mathrm{T}} K_P (\dot{\theta} + Q_\delta^{\mathrm{T}}\dot{\delta}) \\ &= \dot{q}^{\mathrm{T}}\left(-\begin{pmatrix} 0 \\ K\delta + D\dot{\delta} \end{pmatrix} + \begin{pmatrix} I \\ Q_\delta \end{pmatrix} u\right) + \delta^{\mathrm{T}} K \dot{\delta} - (e - Q_\delta^{\mathrm{T}}\delta)^{\mathrm{T}} K_P (\dot{\theta} + Q_\delta^{\mathrm{T}}\dot{\delta}), \end{aligned} \tag{5}$$

where identity (2) and the skew-symmetry of the matrix $\dot{B} - 2S$ have been used. Plugging the control (3) into (5) and simplifying terms yields

$$\dot{V} = -(\dot{\theta}^{\mathrm{T}} \;\; \dot{\delta}^{\mathrm{T}}) \begin{pmatrix} K_D & K_D Q_\delta^{\mathrm{T}} \\ Q_\delta K_D & D + Q_\delta K_D Q_\delta^{\mathrm{T}} \end{pmatrix} \begin{pmatrix} \dot{\theta} \\ \dot{\delta} \end{pmatrix} \leq 0, \tag{6}$$

where the last inequality follows from the factorization of the core matrix in (6) as

$$\begin{pmatrix} I & O \\ Q_\delta & I \end{pmatrix} \begin{pmatrix} K_D & O \\ O & D \end{pmatrix} \begin{pmatrix} I & Q_\delta^{\mathrm{T}} \\ O & I \end{pmatrix}.$$

When $\dot{V} = 0$, it is $\dot{\theta} = 0$ and $\dot{\delta} = 0$, so that the closed-loop system (1,3) collapses into

$$B\ddot{q} = \begin{pmatrix} K_P & -K_P Q_\delta^{\mathrm{T}} \\ O & -K \end{pmatrix} \begin{pmatrix} e \\ \delta \end{pmatrix}, \tag{7}$$

showing that $\ddot{q} = 0$ if and only if $e = 0$ and $\delta = 0$. Invoking LaSalle invariance set theorem [9], asymptotic stability of the desired state follows. Q.E.D.

Notice that, in the general pinned case ($Q_\delta \neq O$), *full state* feedback would be required. On the other hand, in the clamped case, the above proof goes through similarly by setting $Q_\delta = O$. The PD control law reduces to

$$u = K_P(\theta_{\mathrm{des}} - \theta_c) - K_D\dot{\theta}_c, \tag{8}$$

which uses only joint measurements and no information about the mode shapes; since the effects are the same, *partial state* feedback is enough for guaranteeing asymptotic stability in a flexible robot arm. Remarkably, the clamped joint variables θ_c and $\dot{\theta}_c$ are directly measurable by means of ordinary actuator sensors, e.g. encoders and tachometers.

4. INVERSION CONTROL

Trajectory tracking in nonlinear systems is usually achieved by input-output *inversion control* techniques [11]. For rigid manipulators, this approach yields the so-called *computed torque* method that provides *exact* reproduction of smooth desired trajectories in nominal conditions; the output can be taken either at the joint or at the end-effector level [12]. The extension of this result to flexible manipulators is not trivial, since the stability of the resulting closed-loop system is not always guaranteed [6,7].

In what follows, only the case of joint output trajectory is considered and the effects of clamped vs. pinned representations on the synthesis of the control law are assessed. For the purpose of control derivation, it is convenient to extract the flexible accelerations from (1) as

$$\ddot{\delta} = B_{\delta\delta}^{-1}(Q_\delta u - (h_\delta + K\delta + D\dot{\delta}) - B_{\theta\delta}^{\mathrm{T}}\ddot{\theta}) \tag{9}$$

which, substituted into the upper part of (1), gives

$$(B_{\theta\theta} - B_{\theta\delta}B_{\delta\delta}^{-1}B_{\theta\delta}^{\mathrm{T}})\ddot{\theta} + h_\theta - B_{\theta\delta}B_{\delta\delta}^{-1}(h_\delta + K\delta + D\dot{\delta}) = Fu, \tag{10}$$

with

$$F = I - B_{\theta\delta}B_{\delta\delta}^{-1}Q_\delta. \tag{11}$$

Notice that Eq. (10) describes the modification that undergoes the rigid body dynamics obtained by imposing $\delta \equiv 0$ in (1), $B_{\theta\theta}\ddot{\theta} + h_\theta = u$, due to the effects of link flexibility.

The $n \times n$ matrix $B_{\theta\theta} - B_{\theta\delta}B_{\delta\delta}^{-1}B_{\theta\delta}^{\mathrm{T}}$ in (10) has full rank, as can be seen from the following identity

$$\begin{pmatrix} B_{\theta\theta} & B_{\theta\delta} \\ B_{\theta\delta}^{\mathrm{T}} & B_{\delta\delta} \end{pmatrix} \begin{pmatrix} I & O \\ -B_{\delta\delta}^{-1}B_{\theta\delta}^{\mathrm{T}} & I \end{pmatrix} = \begin{pmatrix} B_{\theta\theta} - B_{\theta\delta}B_{\delta\delta}^{-1}B_{\theta\delta}^{\mathrm{T}} & B_{\theta\delta} \\ O & B_{\delta\delta} \end{pmatrix} \tag{12}$$

and from the positive definiteness of the inertia matrix. Further, physical arguments can be used to show that the $n \times n$ matrix F in (11) has full rank. For instance, consider the arm in an undeformed rest configuration, i.e. $\delta = \dot{\delta} = 0$, $\dot{\theta} = 0$. If F were singular, a non-zero input torque u would exist that yields $\ddot{\theta} = 0$, thus keeping the arm at rest; this is clearly in contrast with the mechanical intuition.

Let a denote a given joint acceleration. Setting $\ddot{\theta} = a$ in (10) and solving for u yields the feedback law

$$u = F^{-1}\big((B_{\theta\theta} - B_{\theta\delta}B_{\delta\delta}^{-1}B_{\theta\delta}^{\mathrm{T}})a + h_\theta - B_{\theta\delta}B_{\delta\delta}^{-1}n_\delta\big), \tag{13}$$

where $n_\delta = h_\delta + K\delta + D\dot{\delta}$. In the pinned case, a computationally efficient expression for F^{-1} is [13]

$$F^{-1} = I + B_{\theta\delta}(B_{\delta\delta} - Q_\delta B_{\theta\delta})^{-1}Q_\delta. \tag{14}$$

The control (13) transforms the closed-loop system into the input-output linearized form

$$\ddot{\theta} = a \tag{15}$$

$$\ddot{\delta} = -(I + Q_\delta F^{-1}B_{\theta\delta})B_{\delta\delta}^{-1}(B_{\theta\delta}^{\mathrm{T}}a + n_\delta) + B_{\delta\delta}^{-1}Q_\delta F^{-1}u_r, \tag{16}$$

where $u_r = B_{\theta\theta}a + h_\theta$ is the computed torque control for the equivalent rigid system.

At this point, we observe that in the clamped case Eq. (13) becomes

$$u_c = (B_{\theta_c\theta_c} - B_{\theta_c\delta_c}B_{\delta_c\delta_c}^{-1}B_{\theta_c\delta_c}^{\mathrm{T}})a + h_{\theta_c} - B_{\theta_c\delta_c}B_{\delta_c\delta_c}^{-1}n_{\delta_c}, \tag{17}$$

where subscript c denotes quantities in the clamped model. Notice that only the inversion of the $m \times m$ block relative to the flexible variables is required for implementation of the control law (17). Therefore, the complexity of this nonlinear feedback strategy increases only with the number of flexible variables; in particular, whenever $B_{\delta_c\delta_c}$ is constant, the inversion can be conveniently performed off-line once for all. Using (17), Eqs. (15,16) remarkably simplify to

$$\ddot{\theta}_c = a \tag{18}$$

$$\ddot{\delta}_c = -B_{\delta_c\delta_c}^{-1}(B_{\theta_c\delta_c}^{\mathrm{T}}a + n_{\delta_c}). \tag{19}$$

At this point, in order to track a twice-differentiable desired trajectory $\theta_{\mathrm{des}}(t)$, the given joint acceleration is chosen as

$$a = \ddot{\theta}_{\mathrm{des}} + K_D(\dot{\theta}_{\mathrm{des}} - \dot{\theta}_c) + K_P(\theta_{\mathrm{des}} - \theta_c), \tag{20}$$

where $K_P > 0$, $K_D > 0$ allow pole placement in the open left-hand complex half plane for the linear input-output behaviour in (18). However, the feasibility of this approach is based on the stability of (19). For, it is sufficient that the associated *zero-dynamics* [11] is asymptotically stable. This dynamics is obtained by forcing to zero (or to a constant value) the output θ_c of the nonlinear system. Accordingly, we have

$$\ddot{\delta}_c = -B_{\delta_c\delta_c}^{-1}(h_{\delta_c} + K\delta_c + D\dot{\delta}_c), \tag{21}$$

where a factorization of the type $h_{\delta_c} = S_{\delta_c\delta_c}(\delta_c, \dot{\delta}_c)\dot{\delta}_c$ exists, with $\dot{B}_{\delta_c\delta_c} - 2S_{\delta_c\delta_c}$ skew-symmetric, see also (2). The following result holds.

Theorem 2. *The equilibrium state* $(\delta_c, \dot{\delta}_c) = (0, 0)$ *of system (21) is asymptotically stable.* ■

Proof. Consider the energy-based Lyapunov function candidate

$$V = \frac{1}{2}\dot{\delta}_c^{\mathrm{T}}B_{\delta_c\delta_c}\dot{\delta}_c + \frac{1}{2}\delta_c^{\mathrm{T}}K\delta_c, \tag{22}$$

vanishing only at the desired equilibrium state. The time derivative of (22) along the trajectories of system (21) is

$$\dot{V} = \dot{\delta}_c^{\mathrm{T}}(B_{\delta_c\delta_c}\ddot{\delta}_c + \frac{1}{2}\dot{B}_{\delta_c\delta_c}\dot{\delta}_c) + \delta_c^{\mathrm{T}}K\dot{\delta}_c = -\dot{\delta}_c^{\mathrm{T}}D\dot{\delta}_c \leq 0. \tag{23}$$

Since $\ddot{\delta}_c = -B_{\delta_c\delta_c}^{-1}K\delta_c$ when $\dot{\delta}_c = 0$ (viz. $\dot{V} = 0$), asymptotic stability of the origin follows from LaSalle theorem.

Q.E.D.

Similar arguments can be used in the pinned case, although the developments are much more involved and omitted here for brevity.

From (23), the rate of asymptotic convergence to zero of the flexible variables is established by the arm damping matrix D, generally resulting in a poorly damped behaviour. This may still be satisfactory during the large maneuvering phase of the manipulator, but it may represent a major concern near the end of the trajectory. The standard remedy is to resort to an active linear stabilizer for the deflection variables, designed for a linearized version of the system around the final configuration. It is convenient, indeed, to superimpose such a stabilizing control to the nonlinear one (17,20); in this way, the synthesis can be advantageously performed on the system (18,19) rather than on the original system [14]. Alternatively, damping can be increased in a passive fashion by a mechanical treatment of the lightweight structure, e.g. attaching thin layers of viscoelastic material to the link surfaces.

It should be pointed out that, even in the clamped case, the inversion-based control (17,20) requires full state feedback, as opposed to the PD control (8). For measuring link deflection, different apparatus can be used ranging from strain gauges to accelerometers, or optical devices. In spite of the availability of these direct measurements, it may be convenient to avoid their use within the computation of the nonlinear part of the controller. The joint-based approach naturally lends itself to a cheap implementation in terms of joint variable measures only. In fact, one can preserve the robustifying *linear feedback* (20) and perform the *nonlinear compensation* (17) as a *feedforward* action [15].

5. CONCLUSIONS

The relevance of a Lagrangian-assumed modes dynamic model for the analysis and synthesis of control laws for multilink flexible robot arms has been explored in this work.

On the basis of the complete nonlinear dynamic equations, we have shown that a PD joint collocated linear controller leads to the asymptotic stability of any desired arm configuration. When using the pinned description, the resulting control law is still linear but requires feedback also from the deflection variables. This result has been obtained in the absence of gravity; its generalization requires a suitable compensation of the gravitational terms.

The design of inversion-based nonlinear control laws that allow exact reproduction of smooth joint trajectories has been investigated. The clamped case has been presented in detail, in view of the reduced complexity of the control law in this case. It has been proved that the associated zero dynamics is always asymptotically stable, thus guaranteeing bounded internal deformations of the flexible arm during motion. Inclusion of gravity for this type of controllers is straightforward, as shown in a preliminary version of this work [16].

The above laws handle the positioning and tracking problems at the joint level. If enough structural damping is present, satisfactory performance is obtained also for the end-effector motion. In any case, the design of robust and effective non-collocated controllers for the arm tip that do not violate the stability requirements is a challenging topic that deserves further investigation.

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