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MPC-Based Gait Generation for Humanoids: from Walking to Running

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***Abstract*—We present a Model Predictive Control (MPC) algorithm for 3D walking and running in humanoids. The scheme makes use of the Variable Height Inverted Pendulum (VH-IP) as prediction model, and generates a Center of Mass (CoM) trajectory and footstep positions online. The MPC works with the nonlinear dynamics by decomposing the problem into a vertical and a horizontal component. The vertical is solved first making the horizontal dynamics linear time-varying and therefore solvable in real-time. A stability constraint is incorporated to ensure internal stability. The algorithm is validated with dynamic simulations in DART.**

I. INTRODUCTION

Legged humanoid robots can perform locomotion using a range of different techniques, from walking on flat ground, to traversing slopes and stairs, and even running which involves very dynamic motions.

For walking on flat ground, it is common to use a simplified model in which the Center of Mass (CoM) height is assumed to be constant, in order to make the dynamics linear. This model is known as the Linear Inverted Pendulum (LIP), and it relates the dynamics of the CoM to the position of the Zero Moment Point (ZMP), which must be kept inside the support polygon in order to guarantee that the robot does not fall. Model Predictive Control (MPC) is a very effective way of generating gaits for the LIP [1] as it is capable of enforcing constraints, such as those required on the ZMP.

Despite the simplicity of the LIP model, CoM height variations are often desirable and even necessary, such as when we want to perform crouching motions or in order to climb a staircase. A natural extension of the LIP is known as Variable Height Inverted Pendulum (VH-IP), which essentially introduces a variable stiffness in the pendulum equation. Several works testified the effectiveness of adopting the VH-IP for 3D walking [2], where a nonlinear MPC is proposed, and running [3], where the vertical motion is generated independently and horizontal CoM/ZMP are obtained by solving the dynamics in order to meet proper boundary conditions.

A common issue in many models of humanoid dynamics is their inherent instability. In [4] we proposed an Intrinsically Stable MPC (IS-MPC) for humanoid gait generation, which makes use of the LIP model for state prediction, and enforces a *stability constraint*, ensuring internal stability of the scheme. In this paper we reformulate IS-MPC so as to be able to generate 3D walking and running, by making use of the VH-IP model.

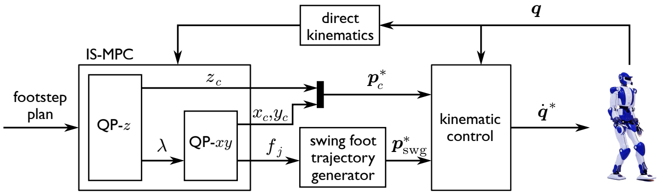

Fig. 1. Block scheme of the proposed approach.

II. FORMULATION AND APPROACH

The goal of the scheme is to replicate as closely as possible a given footstep plan. An overview of the general architecture can be seen in Fig. 1, where this plan is given as input, in the form of a sequence of candidate footstep positions and orientations, with specified timings. For each footstep, the plan also indicates an associated reference CoM height and whether the robot should reach it by walking or running. We assume this sequence to be generated by a footstep planner, either offline or online. The footstep plan is used by the IS-MPC block to generate a CoM trajectory and actual footstep positions. These will match the candidates whenever possible and will be modified when necessary to satisfy the enforced constraints. The output of the trajectory is used as the input to a kinematic controller, together with an appropriate trajectory for the swing foot. The kinematic controller generates joint commands that are applied to the humanoid.

The result will be in accordance with the sequence of gait phases: steps that are performed by walking will be constituted by a single support phase and a double support phase, in which respectively one or both feet are in contact with the ground; steps that are performed by running are constituted by a support and a flight phase, where the robot is either in contact or free-falling. The sequence and duration of these phases is determined by information present in the footstep plan, and is thus predefined.

The IS-MPC block uses a prediction model in order to forecast the state evolution along a *control horizon* T_c. The prediction model is different depending on whether the contact with the ground is present or not. In particular, during all support phases the adopted model is the VH-IP, while during flight phases the model is simply given by the dynamics of a free-falling mass.

The VH-IP equations are

$$\begin{aligned} \ddot{x}_c &= \lambda(x_c - x_z) && (1) \\ \ddot{y}_c &= \lambda(y_c - y_z) && (2) \\ \ddot{z}_c &= \frac{f_z}{m} - g && (3) \end{aligned}$$

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where $\boldsymbol{p}_c = (x_c, y_c, z_c)$ is the CoM position, $\boldsymbol{p}_z = (x_z, y_z, z_z)$ is the ZMP position, m is the robot mass, $\lambda = (\ddot{z}_c + g)/(z_c - z_z)$ is the variable stiffness (the square of the natural frequency) of the pendulum and f_z is the vertical Ground Reaction Force (GRF). The free-falling model adopted for flight phases can be stated as $\ddot{\boldsymbol{p}}_c = \boldsymbol{g}$ where $\boldsymbol{g} = (0, 0, -g)^T$ is the gravity acceleration. Here, the motion of the CoM is entirely controlled by gravity.

To solve the VH-IP equations, we exploit the fact that the vertical equation (3) is independent of x and y. This allows us to generate the vertical motion first, and then use the solution to generate a trajectory for $\lambda(t)$, that can be used to solve (1-2) as Linear Time-Varying (LTV) system. Both these steps are tackled by solving quadratic programs.

The quadratic program for the vertical dynamics is called QP-z, and uses the vertical GRF as decision variable

$$\left\{\begin{array}{l} \quad\min_{F_z^k} \;\; \alpha_z \|Z_c^k - \hat{Z}_c^k\|^2 + \beta_z \|\dot{Z}_c^k\|^2 + \gamma_z \|\Delta F_z\|^2 \\ \text{subject to:} \\ \quad \bullet\ \text{support phase GRF constraints} \\ \quad \bullet\ \text{flight phase GRF constraints} \end{array}\right.$$

where variables over the horizon are collected in vectors: F_z^k for the vertical GRF, Z_c^k for the predicted CoM heights, $\hat{Z}_c^k$ for the reference CoM height, ΔF_z for the input increment. α_z, β_z and γ_z are suitable weights. Two constraints are enforced: support phase GRF constraints impose a minimum value of f_z to avoid slipping and flight phase GRF constraints ensure $f_z = 0$ when contact is absent.

Having computed $z(t)$ and thus $\lambda(t)$, the dynamics (1-2) can be treated as an LTV system. An additional quadratic program QP-xy is set up and solved. For simplicity of exposition, we assume that x and y can be decoupled (which is the case when footsteps have the same orientation), splitting QP-xy into two separate QP-x and QP-y, with similar structure. QP-x is given by

$$\left\{\begin{array}{l} \min_{X_z^k, X_f^k} \;\; \alpha_x \|X_z^k - X_{mc}^k\|^2 + \beta_x \|\Delta X_z^k\|^2 + \mu_x \|X_f^k - \hat{X}_f^k\|^2 \\ \text{subject to:} \\ \quad \bullet\ \text{ZMP constraints} \\ \quad \bullet\ \text{kinematic constraints} \\ \quad \bullet\ \text{swing foot constraints} \\ \quad \bullet\ \text{stability constraint} \end{array}\right.$$

where variables over the horizon are collected in vectors: the ZMP and footstep positions X_z^k and X_f^k constitute the decision variables, while $\hat{X}_{mc}^k$ collects the reference ZMP positions, ΔX_z^k the ZMP increments over sampling intervals, $\hat{X}_f^k$ the candidate footsteps. α_x, β_x and μ_x are weights.

QP-x enforces the following constraints: ZMP constraints keep the ZMP inside the support polygon, kinematic constraints ensure kinematically feasible footstep positions and limit the adapted footsteps to the same ground patch as planned, and swing foot constraints account for the actuation limits of the legs. QP-y is identical with the exception of the kinematic constraints, which are modified to avoid self-collision of the legs.

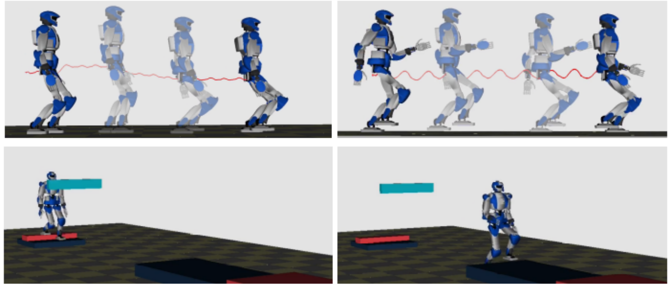

Fig. 2. Snapshots from the DART simulations. Walking with variable height (top left), running (top right), climbing stairs and running in a complex environment (bottom left and right).

The stability constraint ensures bounded CoM/ZMP trajectories, avoiding internal instability. On the LIP, this constraint is derived by isolating the unstable dynamics with an appropriate change of coordinates [4]. This is not possible in the VH-IP, as there is no time-invariant coordinate change that fully decouples the LTV dynamics (1-2). However, we can perform a simplifying assumption by posing that the dynamics become time invariant after the end of the control horizon. In this way, the dynamics can be decoupled at the end of the horizon, and a stability condition can be formulated in the form of a terminal constraint.

III. RESULTS AND CONCLUSIONS

We present dynamic simulations on an HRP-4 humanoid in DART. The results are in the accompanying video, while snapshots are shown in Fig. 2. In the first simulation HRP-4 must walk forward while varying its CoM height. The range of variation of the height is between 0.58 m and 0.77 m. In the second simulation it must perform a running gait. The resulting trajectories are consistent with the trajectories observed in human running, reaching the apex during flight phases. In the last simulation, the robot alternates between different motions while moving in a complex environment: it climbs and descends stairs, lowers its CoM to clear an obstacle and then starts running. The scheme runs at a frequency of 100 Hz, and thus it is suitable for real-time implementation on a physical humanoid robot.

Future work will focus on experiments, robustness extensions and the development of a footstep planner specifically designed for running gaits.

REFERENCES

[1] P.-B. Wieber, "Trajectory free linear model predictive control for stable walking in the presence of strong perturbations," in *6th IEEE-RAS Int. Conf. on Humanoid Robots*, 2006, pp. 137–142.

[2] S. Caron, A. Escande, L. Lanari, and B. Mallein, "Capturability-based pattern generation for walking with variable height," *IEEE Transactions on Robotics*, vol. 36, no. 2, pp. 517–536, 2020.

[3] J. Englsberger, C. Ott, and A. Albu-Schäffer, "Three-dimensional bipedal walking control based on divergent component of motion," *IEEE Transactions on Robotics*, vol. 31, no. 2, pp. 355–368, 2015.

[4] N. Scianca, D. De Simone, L. Lanari, and G. Oriolo, "MPC for humanoid gait generation: Stability and feasibility," *IEEE Transactions on Robotics*, vol. 36, no. 4, pp. 1171–1188, 2020.