Reprinted from

NONLINEAR CONTROL SYSTEMS DESIGN

Selected Papers from the IFAC Symposium, Capri, Italy
14–16 June 1989

Edited by

A. ISIDORI

Università di Roma, "La Sapienza", Rome, Italy

Published for the

INTERNATIONAL FEDERATION OF AUTOMATIC CONTROL

by

PERGAMON PRESS

Member of Maxwell Macmillan Pergamon Publishing Corporation

OXFORD · NEW YORK · BEIJING · FRANKFURT
SÃO PAULO · SYDNEY · TOKYO · TORONTO

THE DESIGN OF LINEARIZING OUTPUTS FOR INDUCTION MOTORS

A. De Luca and G. Ulivi

Dipartimento di Informatica e Sistemistica, Università di Roma "La Sapienza", Via Eudossiana 18, 00184 Roma, Italy

Abstract. Nonlinear control schemes based on inversion techniques are presented for voltage-frequency controlled (VFC) induction motors. The selection of different output sets is discussed and the properties of the resulting closed-loop systems are investigated. When the two components of the rotor flux vector are chosen as controlled variables, the system is invertible and has no zero-dynamics. Therefore, the VFC induction motor can be fully linearized and input-output decoupled by means of a dynamic state-feedback law. Two other pairs of outputs are then considered: the torque and the stator or the rotor flux modulus, respectively. An input-output linearizing and decoupling dynamic compensator can be designed for both cases. The associated closed-loop systems contain an unobservable part whose dynamics is shown to be stable. The comparison among the obtained results provides a deeper insight into the structural control properties of the induction motor.

Keywords. Nonlinear control systems, Linearization, Decoupling, Motor control, Induction machines.

Introduction

A common driving mode for an induction motor is to use the amplitude and the frequency of the supply voltage vector as control variables (Voltage-Frequency Control, VFC). This is particularly appealing when the machine is supplied by an inverter since most of the pulse-width modulation techniques used in such devices assume these quantities as inputs (Leonhard, 1985). When the induction motor is driven in a VFC mode, its dynamic behavior becomes nonlinear and smooth.

Several objectives have to be considered in the design of a control system for an induction machine. If the motor is used as an actuator, controlling the produced torque is the major concern. In any case, both the machine flux and the current sinked from the inverter have to be kept limited even during fast transients, so that the motor can operate properly. As a matter of fact, the relevant quantities to be controlled are nonlinear functions of the motor "physical" state variables, which are currents and fluxes.

The system outputs are typically defined in connection with the above specified control objectives. If inversion-based schemes are used for control (Singh, 1981, Isidori *et al.*, 1986), the exact tracking of output time profiles becomes feasible. By inverting the plant, the chosen outputs have a direct influence on the characteristics of the resulting closed-loop system which may contain an unobservable and possibly nonlinear part. This internal dynamics should be stable in order to validate the overall nonlinear control design. When several alternative sets of outputs are considered, these should be evaluated in terms of the induced closed-loop characteristics. With respect to this analysis, a relevant role is played by the so-called zero-dynamics of the system defined as its internal behavior when the inputs and the initial state are chosen so that the outputs are constrained on a given manifold (Byrnes and Isidori, 1988).

The purpose of this paper is to study some possible choices of outputs for a VFC induction motor. Each choice specifies a nonlinear state-feedback control scheme and leads to a different closed-loop system. In particular, it will be shown that using the two components of the rotor flux vector as outputs yields an invertible system without zero-dynamics; this system can be fully linearized by means of a nonlinear dynamic state-feedback (Isidori *et al.*,1986). Two other pairs of outputs are then considered: the torque and the squared norm of the stator flux, and the torque and the squared norm of the rotor flux. It turns out that a nonlinear dynamic feedback is again required for input-output linearization and decoupling in both cases; each resulting closed-loop system has an unobservable part which is of dimension two and, respectively, one. It will be shown that this internal dynamics is always stable in the operating region.

The steps that will be followed to get these results lead to a better understanding of the intrinsic control structure of a VFC induction motor. In particular, the zero-dynamics has a nice physical interpretation and may provide in a single framework also the standard steady-state conditions of the machine.

Induction motor model

Figure 1 shows the block diagram of a voltage-fed induction motor which is controlled using the amplitude V and the frequency ω_a of the supply voltage. The standard model relies on the two-phase equivalent vector representation of the motor current, flux and voltage vectors. Let $i_{s\alpha}$, $i_{s\beta}$ be the components of the stator current and $\varphi_{s\alpha}$, $\varphi_{s\beta}$ the ones of the stator flux, as projected on a reference frame (α,β) which is fixed to the stator windings. A set of four differential equations can be derived to describe the dynamic behavior of the motor in terms of these quantities and of the projections v_α and v_β of the supply voltage which is applied to the stator windings. These are related to the control inputs by

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \begin{bmatrix} V\cos\vartheta \\ V\sin\vartheta \end{bmatrix}, \qquad \vartheta = \int_0^t \omega_a(\tau)\,d\tau + \vartheta_0$$

As a consequence, another differential equation should be written for the angular position ϑ. Defining the state **x** and input

u as

$$\mathbf{x} = \begin{bmatrix} i_{s\alpha} & i_{s\beta} & \varphi_{s\alpha} & \varphi_{s\beta} & \vartheta \end{bmatrix}^T =: \begin{bmatrix} \bar{\mathbf{x}}^T & \vartheta \end{bmatrix}^T \qquad \mathbf{u} = \begin{bmatrix} V_a & \omega_a \end{bmatrix}^T$$

the VFC motor state equations become:

$$\dot{\mathbf{x}} = f(\mathbf{x}) + g(\mathbf{x})\,\mathbf{u} \tag{1}$$

with

$$f(\mathbf{x}) = \begin{bmatrix} -(\alpha+\beta) & -\omega & \frac{\beta}{\Lambda_s} & \frac{\omega}{\sigma\Lambda_s} & 0 \\ \omega & -(\alpha+\beta) & -\frac{\omega}{\sigma\Lambda_s} & \frac{\beta}{\Lambda_s} & 0 \\ -\alpha\sigma\Lambda_s & 0 & 0 & 0 & 0 \\ 0 & -\alpha\sigma\Lambda_s & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{x} = \begin{bmatrix} \bar{A} & \mathbf{0} \\ \mathbf{0}^T & 0 \end{bmatrix} \begin{bmatrix} \bar{\mathbf{x}} \\ x_5 \end{bmatrix} = A\mathbf{x}$$

$$g(\mathbf{x}) = \begin{bmatrix} \frac{\cos x_5}{\sigma\Lambda_s} & 0 \\ \frac{\sin x_5}{\sigma\Lambda_s} & 0 \\ \cos x_5 & 0 \\ \sin x_5 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{g}_{11}(x_5) & \mathbf{0} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} g_1(x_5) & g_2 \end{bmatrix}$$

where $\alpha = R_s / \sigma\Lambda_s$, $\beta = R_r / \sigma\Lambda_r$, $\sigma = 1 - (M^2/\Lambda_s\Lambda_r)$. R_s and R_r are the stator and rotor resistances, Λ_s and Λ_r are the stator and rotor self-inductances and M is the mutual inductance between the two windings.

In the following two sections, different outputs $\mathbf{y} = h(\mathbf{x})$ will be considered in association with the above state equations and their properties analyzed.

Full linearization via dynamic feedback

It will be shown here that a set of outputs exists for the induction motor (1) such that the resulting system is invertible and fully linearizable by means of a state-feedback of the dynamic type. Consider the following outputs:

$$y_1 = x_3 - \sigma\Lambda_s x_1 \tag{2a}$$

$$y_2 = x_4 - \sigma\Lambda_s x_2 \tag{2b}$$

These linear functions have a clear physical meaning, being proportional (by a factor Λ_r/M) to the (α,β) components of the rotor flux. The generalized inversion algorithm (Singh, 1981, Byrnes and Isidori, 1988) will be applied to the triple $\{f(\mathbf{x}), g(\mathbf{x}), h(\mathbf{x})\}$ formed by (1) and (2) in order to check whether the two inputs can be recovered from the knowledge of the outputs and of their time derivatives. The first input u_1 appears explicitly after two time derivatives:

$$\ddot{y}_1 = (a_3 - \sigma\Lambda_s a_1)A\mathbf{x} + (a_3 - \sigma\Lambda_s a_1)g_1(x_5)u_1 \tag{3a}$$

$$\ddot{y}_2 = (a_4 - \sigma\Lambda_s a_2)A\mathbf{x} + (a_4 - \sigma\Lambda_s a_2)g_1(x_5)u_1 \tag{3b}$$

where a_j denotes the j-th row of the system matrix A. To proceed with the algorithm a new "dummy" output function $\lambda(\mathbf{x})$ has to be defined by combining the above expressions so to cancel the dependence on the first input. This is achieved by solving for $\gamma(\mathbf{x})$

$$\begin{bmatrix} 1 & \gamma(\mathbf{x}) \end{bmatrix} \begin{bmatrix} (a_3 - \sigma\Lambda_s a_1)\, g_1(x_5) \\ (a_4 - \sigma\Lambda_s a_2)\, g_1(x_5) \end{bmatrix} = 0$$

which gives:

$$\gamma(x_5) = -\frac{(a_3 - \sigma\Lambda_s a_1)g_1(x_5)}{(a_4 - \sigma\Lambda_s a_2)g_1(x_5)} = -\frac{\cos x_5}{\sin x_5} \tag{4}$$

Therefore, the new function

$$\lambda = \begin{bmatrix} 1 & \gamma(x_5) \end{bmatrix} \begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} =$$

$$= [(a_3 - \sigma\Lambda_s a_1) - \gamma(x_5)(a_4 - \sigma\Lambda_s a_2)]\, A\mathbf{x} \tag{5}$$

is independent from the inputs. Note that the singularity $\sin x_5 = 0$ in (4) could have been easily avoided by using a different admissible definition for $\lambda(\mathbf{x})$. The derivative of λ w.r.t. time is

$$\dot{\lambda} = \dddot{y}_1 + \dot{\gamma}\ddot{y}_2 + \gamma\dddot{y}_2 =$$

$$= [(a_3 - \sigma\Lambda_s a_1) - \gamma(x_5)(a_4 - \sigma\Lambda_s a_2)]\,(A^2\mathbf{x} + Ag_1(x_5)u_1) +$$
$$- \frac{\partial\gamma}{\partial x_5}(a_4 - \sigma\Lambda_s a_2)\,A\mathbf{x}\,u_2 \tag{6}$$

As the coefficient which premultiplies u_2 does not vanish identically, the algorithm stops providing the following vector equation used for solving w.r.t. **u**:

$$\begin{bmatrix} \ddot{y}_1 \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} b_1(\mathbf{x}) \\ b_2(\mathbf{x}) \end{bmatrix} + \begin{bmatrix} m_{11}(\mathbf{x}) & 0 \\ m_{21}(\mathbf{x}) & m_{22}(\mathbf{x}) \end{bmatrix} \mathbf{u} = b(\mathbf{x}) + M(\mathbf{x})\mathbf{u} \tag{7}$$

The expression of the terms $b_i(\mathbf{x})$ and $m_{ij}(\mathbf{x})$ are obtained from (3a) and (6). The matrix $M(\mathbf{x})$ is locally nonsingular, although it inherits the possible singularites introduced by the algorithm.

The functions

$$y_1(\mathbf{x}),\ y_2(\mathbf{x}),\ \dot{y}_1(\mathbf{x}),\ \dot{y}_2(\mathbf{x}),\ \lambda(\mathbf{x})$$

are independent in the region of nonsingularity of $M(\mathbf{x})$; by the implicit function theorem, the knowledge of their values allows to recover the five components of the state **x**. Note that constraining these functions to zero gives $\mathbf{x} \equiv 0$. As a result, the system formed by (1) and (2) turns out to be invertible and with no zero-dynamics.

Following Isidori *et al.* (1986), a dynamic state-feedback can be designed which yields full linearization and decoupling in the closed-loop system. Such a dynamic control law is not unique and may be obtained in various ways. In particular, the following three-step procedure can be used:

1. Apply the static state-feedback

$$\mathbf{u} = M^{-1}(\mathbf{x})\,[\,\mathbf{w} - b(\mathbf{x})\,]$$

which is derived from (7).

2. Extend the system by adding one integrator (with state ξ) on the new input w_1:

$$w_1 = \xi, \qquad \dot{\xi} = \bar{w}_1, \qquad w_2 = \bar{w}_2$$

More in general, μ_i integrators should be added to each ordered input w_i; this number is equal to the difference between the second-highest and the lowest order of derivation of the output y_i appearing in the left-hand side of (7) (Singh, 1981).

3. Use decoupling static feedback from the *extended* state $(\mathbf{x},\xi)$

$$\bar{\mathbf{w}} = \bar{\alpha}(\mathbf{x},\xi) + \bar{\beta}(\mathbf{x})\,\mathbf{v}$$

to complete the design of the full linearizing compensator.

Note that the decoupling matrix $D(\mathbf{x})$ of the extended system depends only on $\mathbf{x}$ and has a simple triangular form:

$$D(\mathbf{x}) = \bar{\beta}^{-1}(\mathbf{x}) = \begin{bmatrix} 1 & 0 \\ \gamma(x_5) & 1 \end{bmatrix}$$

while

$$\bar{\alpha}(\mathbf{x},\xi) = -\bar{\beta}(\mathbf{x})\, L_{\bar{f}}^3 h(\mathbf{x},\xi)$$

where the bar over f denotes the drift vector field obtained after the dynamic extension of step 2 in the above procedure.

Collecting together these terms provides a dynamic state-feedback compensator in the form (see Figure 2):

$$\dot{\xi} = P(\mathbf{x},\xi) + Q(\mathbf{x},\xi)\,\mathbf{v} \qquad \mathbf{u} = R(\mathbf{x},\xi) + S(\mathbf{x},\xi)\,\mathbf{v}$$

In the proper coordinates, the equivalent structure of the closed-loop system is given by two strings, each of three integrators, from $\mathbf{v}$ to $\mathbf{y}$. Since there is no resulting unobservable dynamics in the closed-loop, the stabilization of this linear system is achieved simply by pole-placement design of $\mathbf{v}$.

Input-output linearization via dynamic feedback

The previous choice of outputs gives a closed-loop system without unobservable dynamics. The exact value of the rotor flux representative vector can be freely imposed through a third-order linear dynamics. However, the two components of the rotor flux do not represent suitable quantities to be controlled when the induction motor is used as an actuator in a typical drive. In fact, it is a difficult task to ensure a desired torque by defining only reference values for these outputs. With this respect, the following two alternative sets of outputs are more convenient:

(a) torque and squared norm of the stator flux

$$y_1 = T_m = x_2x_3 - x_1x_4 \tag{8a}$$

$$y_2 = \frac{1}{2}\Phi_s^2 = \frac{1}{2}(x_3^2 + x_4^2) \tag{8b}$$

(b) torque and squared norm of the rotor flux

$$y_1 = T_m = x_2x_3 - x_1x_4 \tag{9a}$$

$$y_2 = \frac{1}{2}\Phi_r^2 = \frac{1}{2}[(x_3 - \sigma\Lambda_s x_1)^2 + (x_4 - \sigma\Lambda_s x_2)^2] \tag{9b}$$

All the above functions are intrinsically coordinate-free, i.e. independent from the absolute position of the vectors representing the state. In particular, these outputs assume constant values at sinusoidal steady-state.

For the first set of outputs, a nonlinear controller has been already designed in De Luca and Ulivi (1988). In this case the system can be input-output linearized and decoupled if a dynamic compensator of order one is used. In particular, an integrator should be added on the voltage modulus input in order to get a nonsingular decoupling matrix. The resulting closed-loop system has a two-dimensional unobservable part whose stability was shown by simulation only.

When the outputs (9) are used, i.e. replacing the stator with the rotor flux norm, it is possible to show that a similar result can be obtained. Namely, a dynamic state-feedback of order one can be designed such that the system becomes input-output linearized and decoupled. Again, the dynamic extension can be realized by adding one integrator to the voltage modulus input. In this case the closed-loop unobservable part becomes of dimension one and its stability has to be investigated.

In the following, explicit expressions will be derived for the unobservable dynamics in both cases. In order to achieve this goal, a proper choice of coordinates is required which helps in getting simpler equations. The full linearization results shown in the previous section suggest to use the components of the rotor flux vector as coordinates for the unobservable parts. The appealing feature of the rotor functions (2) stands in their maximum relative degree (Isidori, 1985). Besides, the choice of other coordinate functions (e.g. the two components of the stator current as in De Luca and Ulivi (1988)) may lead to very complicate expressions which are difficult to handle even using symbolic manipulation languages.

Before analyzing the above cases (a) and (b) it is worth to rewrite the dynamic equations (1) in a compact form after the introduction of the input integrator:

$$\mathbf{x} = \begin{bmatrix} i_{s\alpha} & i_{s\beta} & \varphi_{s\alpha} & \varphi_{s\beta} & \vartheta & V \end{bmatrix}^T = \begin{bmatrix} \bar{\mathbf{x}}^T & x_5 & x_6 \end{bmatrix}^T$$

$$\mathbf{u} = \begin{bmatrix} \dot{V} & \omega_a \end{bmatrix}^T$$

$$\dot{\mathbf{x}} = f(\mathbf{x}) + G\,\mathbf{u} = \begin{bmatrix} \bar{A}\,\bar{\mathbf{x}} + x_6\, g_{11}(x_5) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \mathbf{u} \tag{10}$$

This preliminary addition of one integrator implies that the controllers which will be derived next are always of the dynamic state-feedback type.

Outputs (a): Torque and squared norm of the stator flux

It is easy to see that the outputs $\mathbf{y} = h(\mathbf{x})$ defined by (8) have relative degrees $r_1 = r_2 = 2$, with reference to the dynamics (10). The decoupling matrix $D(\mathbf{x}) = [L_G L_f h(\mathbf{x})]$ is nonsingular for

$$\det D(\mathbf{x}) = x_6 [(x_1x_3 + x_2x_4) - \frac{1}{\sigma\Lambda_s}(x_3^2 + x_4^2)] =: -\frac{V}{\sigma\Lambda_s} \vec{\Phi}_r \cdot \vec{\Phi}_s \neq 0$$

i.e. whenever the two vectors of stator and rotor flux are not parallel and voltage is applied at the motor input. These conditions are always satisfied during standard operation. Since $n = \dim(\mathbf{x}) = 6$, an unobservable sink of order $n-(r_1+r_2) = 2$ arises in the closed-loop system (see Figure 3a). In order to derive dynamic equations for this part, a state transformation is performed:

$$\mathbf{z} = T(\mathbf{x}) = \begin{bmatrix} h_1(\mathbf{x}) & L_f h_1(\mathbf{x}) & h_2(\mathbf{x}) & L_f h_2(\mathbf{x}) & T_5(\mathbf{x}) & T_6(\mathbf{x}) \end{bmatrix}^T =$$

$$= \begin{bmatrix} T_m & \dot{T}_m & \frac{1}{2}\Phi_s^2 & \vec{\Phi}_s \cdot \dot{\vec{\Phi}}_s & \varphi_{r\alpha} & \varphi_{r\beta} \end{bmatrix}^T$$

The Jacobian matrix $J(\mathbf{x}) = [\partial T/\partial \mathbf{x}](\mathbf{x})$ of this state transformation is nonsingular if and only if the decoupling matrix $D(\mathbf{x})$ is nonsingular. By inspection it is easy to see that $T(\mathbf{x})$ has the following block triangular dependence on $\mathbf{x}$:

$$\begin{bmatrix} \mathbf{z}' \\ \mathbf{z}'' \end{bmatrix} = \begin{bmatrix} T'(\bar{\mathbf{x}}) \\ T''(\bar{\mathbf{x}}, x_5, x_6) \end{bmatrix}, \quad \mathbf{z}' = (z_1, z_3, z_5, z_6), \quad \mathbf{z}'' = (z_2, z_4) \tag{11}$$

Therefore, one can obtain explicit expressions for $\mathbf{x} = \mathbf{x}(\mathbf{z})$ first inverting T' and then using the last two rows of (11) to determine $x_5 = x_5(\mathbf{z})$ and $x_6 = x_6(\mathbf{z})$. The inversion of the mapping T' provides:

$$x_1 = -\frac{z_5}{\sigma\Lambda_s}\left[1 - \frac{\sqrt{2z_3(z_5^2 + z_6^2) - (\sigma\Lambda_s z_1)^2}}{z_5^2 + z_6^2}\right] - \frac{z_1 z_6}{z_5^2 + z_6^2} =: x_1(\mathbf{z})$$

$$x_2 = \frac{z_1 + x_1(\mathbf{z})\, z_6}{z_5} =: x_2(\mathbf{z}), \qquad \text{for } z_5 \neq 0$$
$$x_3 = z_5 + \sigma\Lambda_s x_1(\mathbf{z}) =: x_3(\mathbf{z}) \tag{12}$$
$$x_4 = z_6 + \sigma\Lambda_s x_2(\mathbf{z}) =: x_4(\mathbf{z})$$

The first two expressions in (12) hold for z_5 ($= \varphi_{r\alpha}$) $\neq 0$, while an alternative representation can be obtained for z_6 ($= \varphi_{r\beta}$) $\neq 0$. Since z_5 and z_6 are never both zero in the operating region, no special restrictions are imposed on the validity of the inversion of this mapping. By replacing these four solutions into

$$z_2 = T_2(\mathbf{x}) = \omega(x_1x_3+x_2x_4) - (\alpha+\beta)(x_2x_3-x_1x_4) - \omega/\sigma\Lambda_s\,(x_3^2+x_4^2) + \\ + x_6\,[\,(x_3-\sigma\Lambda_s x_1)\sin x_5 - (x_4-\sigma\Lambda_s x_2)\cos x_5\,]\,/\,\sigma\Lambda_s$$

$$z_4 = T_4(\mathbf{x}) = -\alpha\sigma\Lambda_s(x_1x_3+x_2x_4) + x_6\,[x_3\cos x_5 + x_4\sin x_5]$$

and noting that these equations are linear w.r.t.

$$v_\alpha = x_6\cos x_5\,, \qquad v_\beta = x_6\sin x_5$$

it is possible to solve for $v_\alpha = v_\alpha(\mathbf{z})$ and $v_\beta = v_\beta(\mathbf{z})$ from

$$\begin{bmatrix} -z_6/\sigma\Lambda_s & z_5/\sigma\Lambda_s \\ x_3(\mathbf{z}) & x_4(\mathbf{z}) \end{bmatrix} \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \begin{bmatrix} z_2 - \Psi_2[\bar{\mathbf{x}}(\mathbf{z})] \\ z_4 - \Psi_4[\bar{\mathbf{x}}(\mathbf{z})] \end{bmatrix} \tag{13}$$

The solution of (13) is unique and well defined in the same region where the decoupling controller is assumed to operate. Finally, the last two components of **x** are computed as

$$x_5 = \mathrm{ATAN2}\,\{\,v_\beta(\mathbf{z}),\,v_\alpha(\mathbf{z})\,\} =: x_5(\mathbf{z})$$
$$x_6 = \sqrt{v_\alpha^2(\mathbf{z}) + v_\beta^2(\mathbf{z})} =: x_6(\mathbf{z}) \tag{14}$$

where ATAN2 is the four-quadrant arctangent function.

Using the obtained inverse transformations from **z** to **x** = **x**(**z**), the closed-loop unobservable dynamics can be written as

$$\dot z_5 = [\,\dot x_3 - \sigma\Lambda_s\dot x_1\,]_{\mathbf{x}=\mathbf{x}(\mathbf{z})} =$$
$$= -\beta\left[1 - (1-\sigma)\frac{\sqrt{2z_3\,(z_5^2+z_6^2) - (\sigma\Lambda_s z_1)^2}}{z_5^2+z_6^2}\right] z_5 - \left[\frac{k\,z_1}{z_5^2+z_6^2} + \omega\right] z_6 \tag{15}$$

$$\dot z_6 = [\,\dot x_4 - \sigma\Lambda_s\dot x_2\,]_{\mathbf{x}=\mathbf{x}(\mathbf{z})} =$$
$$= \left[\frac{k\,z_1}{z_5^2+z_6^2} + \omega\right] z_5 - \beta\left[1 - (1-\sigma)\frac{\sqrt{2z_3\,(z_5^2+z_6^2) - (\sigma\Lambda_s z_1)^2}}{z_5^2+z_6^2}\right] z_6$$

where $k := \beta\sigma(1-\sigma)\Lambda_s > 0$. This dynamics has the form of a nonlinear harmonic system. When the chosen motor outputs z_1 and z_3 are forced to their desired constant steady-state values, then the solution of (15) converges to a sinusoidal form characterized by the zeroing of the damping terms. In this condition the squared norm of the rotor flux $\Phi_r^2 = (z_5^2 + z_6^2)$ is constant and its value can be easily derived. Moreover, the angular frequency of the rotor flux vector can be computed from the off diagonal coefficients in (15) and it coincides with that of the motor supply ω_a.

Outputs (b): Torque and squared norm of the rotor flux

When the squared norm of the rotor flux (9b) is used in place of the similar quantity defined for the stator (8b), the relative degree of this second output becomes $r_2 = 3$. It can be shown that in this case the decoupling matrix $D(\mathbf{x})$ associated to the system dynamics (10) is nonsingular for:

$$\det D(\mathbf{x}) = -\frac{\beta(1-\sigma)}{\sigma\Lambda_s}\, x_6\,[(x_3-\sigma\Lambda_s x_1)^2 + (x_4-\sigma\Lambda_s x_2)^2] =$$
$$= -\frac{\beta(1-\sigma)}{\sigma\Lambda_s}\, V\,\Phi_r^2 \neq 0$$

In this case the unobservable sink which arises in the closed-loop system has only dimension one (see Figure 3b). As before, the stability of this part needs to be investigated in order to validate the decoupling and input-output linearizing control approach. The derivation of the scalar equation governing the behavior of this unobservable dynamics follows the same steps seen above. In particular, the use of one rotor flux component for completing the change of coordinates in the state-space proves to be satisfactory. Therefore, by setting

$$\mathbf{z} = T(\mathbf{x}) = \begin{bmatrix} h_1(\mathbf{x}) & L_f h_1(\mathbf{x}) & h_2(\mathbf{x}) & L_f h_2(\mathbf{x}) & L_f^2 h_2(\mathbf{x}) & T_6(\mathbf{x}) \end{bmatrix}^T =$$
$$= \begin{bmatrix} T_m & \dot T_m & \tfrac{1}{2}\Phi_r^2 & \vec\Phi_r\cdot\dot{\vec\Phi}_r & \tfrac{d}{dt}(\vec\Phi_r\cdot\dot{\vec\Phi}_r) & \varphi_{r\alpha} \end{bmatrix}^T$$

The Jacobian matrix of this transformation has determinant

$$\det J(\mathbf{x}) = \det\left[\frac{\partial T(\mathbf{x})}{\partial \mathbf{x}}\right] = -\beta^2(1-\sigma)^2\, V\,\Phi_r^2\,\varphi_{r\beta}$$

The mapping $T(\mathbf{x})$ is singular whenever the decoupling matrix $D(\mathbf{x})$ is singular or when $\varphi_{r\beta}$ is zero: in this second situation, $\varphi_{r\beta}$ should be chosen in place of $\varphi_{r\alpha}$ as $T_6(\mathbf{x})$. Therefore, the dynamic description which follows does not hold globally but only within a domain made of (any) two quadrants.

Following the same reasoning which led to (12), the inverse expressions $\mathbf{x} = \mathbf{x}(\mathbf{z})$ are computed as

$$x_2 = \frac{\beta\sigma(1-\sigma)\Lambda_s z_1 z_6 + (2z_3\beta\sigma + z_4)\sqrt{2z_3 - z_6^2}}{2z_3\,\beta\sigma(1-\sigma)\Lambda_s} =: x_2(\mathbf{z})$$
$$x_1 = \frac{x_2(\mathbf{z})\,z_6 - z_1}{\sqrt{2z_3 - z_6^2}} =: x_1(\mathbf{z})$$
$$x_3 = z_6 + \sigma\Lambda_s\,x_1(\mathbf{z}) =: x_3(\mathbf{z}) \tag{16}$$
$$x_4 = \sqrt{2z_3 - z_6^2} + \sigma\Lambda_s x_2(\mathbf{z}) =: x_4(\mathbf{z})$$

while $x_5(\mathbf{z})$ and $x_6(\mathbf{z})$ are obtained as in (14), i.e. solving a linear system of equations similar to (13). Their explicit functional forms are not reported here; in fact, they are not needed for the derivation of the unobservable dynamics. Substituting (16) into the closed-loop equation of z_6 gives finally:

$$\dot z_6 = -\frac{1}{2z_3}\,\{\,z_4 z_6 - [\beta\sigma(1-\sigma)\Lambda_s z_1 + 2\omega z_3]\sqrt{2z_3 - z_6^2}\,\} \tag{17}$$

When the outputs are forced to their desired constant values

$$z_1 = T_{m,des}\,,\quad z_2 = 0\,,\quad z_3 = \frac{1}{2}\Phi_{r,des}^2\,,\quad z_4 = z_5 = 0$$

then (17) becomes the zero-dynamics of the system

$$\dot z_6 = -\frac{1}{\Phi_{r,des}^2}\,[\,kT_{m,des} + \omega\Phi_{r,des}^2\,]\sqrt{\Phi_{r,des}^2 - z_6^2}$$

where $k > 0$ is the same one defined in (15). This equation is still nonlinear but its solution in time is unique for a given initial condition at time $t = 0$ and has the simple form

$$z_6(t) = |\vec\Phi_{r,des}|\cos(\omega_a t + \delta)\,, \qquad \omega_a = \omega + \frac{kT_{m,des}}{\Phi_{r,des}^2}$$

Therefore, the internal unobservable dynamics is proven to be stable also in the case of rotor flux output.

Conclusions

Three feasible inversion-based control schemes have been proposed for voltage-frequency controlled induction motors. These schemes differ in the definition of the controlled outputs: the two components of the rotor flux, the torque and the norm of the stator flux, and the torque and the norm of the rotor flux. All of them require dynamic state-feedback to achieve input-output linearization and decoupling. Using the components of the rotor flux yields also full state linearization in the closed-loop, but this choice is the less attractive from a practical point of view.

The above analysis shows that the length of the obtained input-output chains of integrators is a rough but valid measure of the intrinsic difficulty in controlling torque (two integrators), stator flux (two) and rotor flux (three) in a VFC induction motor.

Another general result is related to the existence of zero-dynamics for this system. Whenever the inputs and the outputs of an induction motor are coordinate-independent, an internal dynamics arises in the closed-loop due to the intrinsic rotating nature of the vectors representing the motor physical quantities. In fact, once the inversion controller is applied, constant nonzero values for the inputs and outputs are perfectly admissible: an harmonic system should then be present which, at the steady state, internally drives the motor so to provide the proper output torque and flux values. Thus, the (marginal) stability of the zero-dynamics should not be surprising. The same reasoning also explains why the only set of fully linearizing outputs, the two components of the rotor flux, is a time-varying one in steady-state conditions.

It is interesting to remark that if a third independent input is added, as in De Luca and Ulivi (1987), still not enough degrees of freedom are provided so to control the torque, the (stator or rotor) flux norm and the current norm in a decoupled way. In fact, the inherent coupling of these quantities can be made explicit also in the case of VFC induction motors.

Acknowledgements. The Authors are indebted to Professor Alberto Isidori for proposing the choice (2) as linearizing outputs.

References

Byrnes, C.I., Isidori, A. (1988). The Analysis and Design of Nonlinear Feedback Systems. Part I: Zero Dynamics and Global Normal Forms. Nonlinear Control and Robotics Preprints, Università di Roma "La Sapienza", Dipartimento di Informatica e Sistemistica.

De Luca, A., Ulivi, G. (1987). Full Linearization of Induction Motors via Nonlinear State-Feedback, *26th IEEE Conf. on Decision and Control*, Los Angeles, pp.1765-1770.

De Luca, A., Ulivi, G. (1988). Dynamic Decoupling of Voltage Frequency Controlled Induction Motors, *8th Int. Conf. on Analysis and Optimization of Systems, INRIA,* Antibes, pp.127-137.

Isidori, A. (1985). *Nonlinear Control Systems: An Introduction*, Lecture Notes in Control and Information Sciences, **72**, Springer Verlag.

Isidori, A., Moog, C.H., De Luca, A. (1986). A Sufficient Condition for Full Linearization Via Dynamic State Feedback, *25th IEEE Conf. on Decision and Control,* Athens, pp.203-208.

Leonhard, W. (1985). *Control of Electrical Drives*, Springer Verlag.

Singh, S.N. (1981). A Modified Algorithm for Invertibility of Nonlinear Systems. *IEEE Trans. Automatic Control*, **AC-26**, pp.595-598.

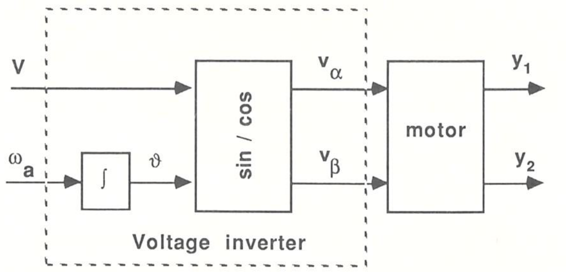

Figure 1. Block diagram of a VFC induction motor

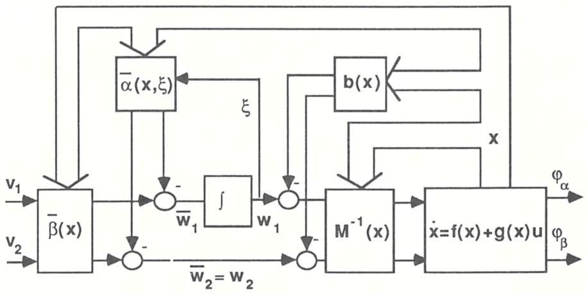

Figure 2. Full linearizing dynamic control scheme

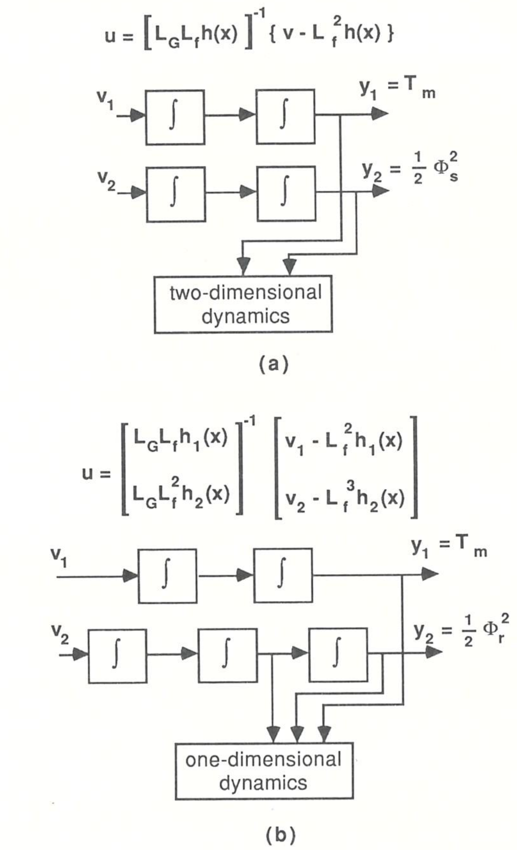

Figure 3. Closed-loop equivalent input-output linearized systems and associated control laws for the two sets of outputs.

This work was partially supported by MPI 40%.