Control Systems

Control basics I

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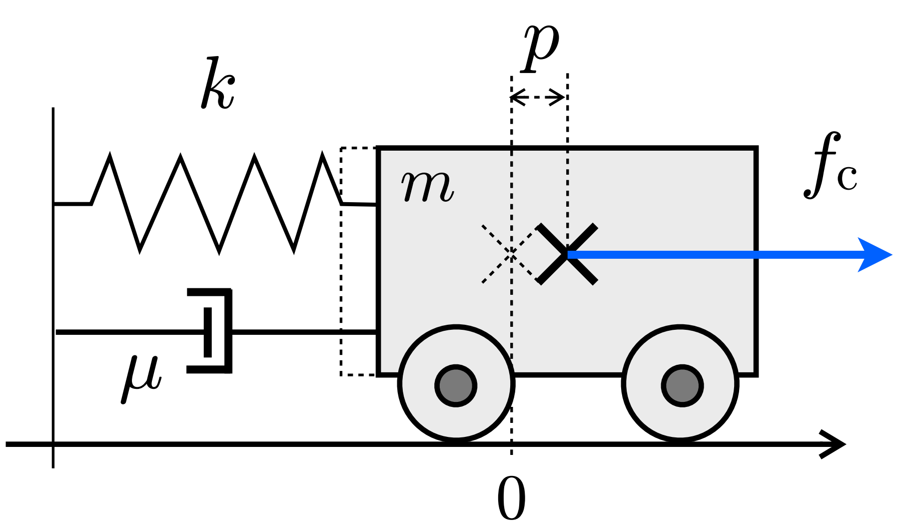
Outline

- two introductory examples
- differences between the open and closed-loop approaches
- general feedback control scheme

Preliminary example I



f_c control input (force)

p mass position (0 at rest position with no external forces applied)

$$\frac{p(s)}{f_c(s)} = \frac{1}{ms^2 + \mu s + k}$$

Goal

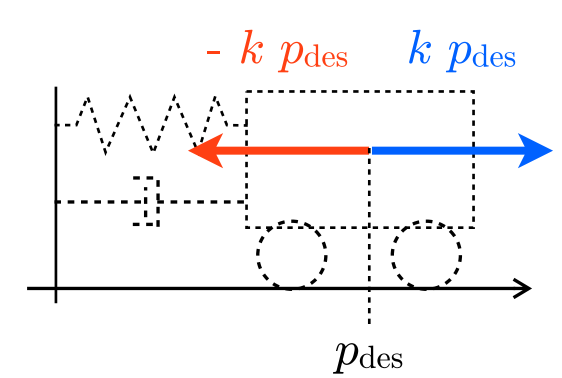
choose the proper value of the force f_c such that the mass will stop in p_{des} constant

Solution

in static conditions (forces balance at equilibrium) the mass will be in the p_{des} position

iff f_c exactly counterbalances the elastic force - $k.p_{des}$

$$f_c = k\ p_{des}$$

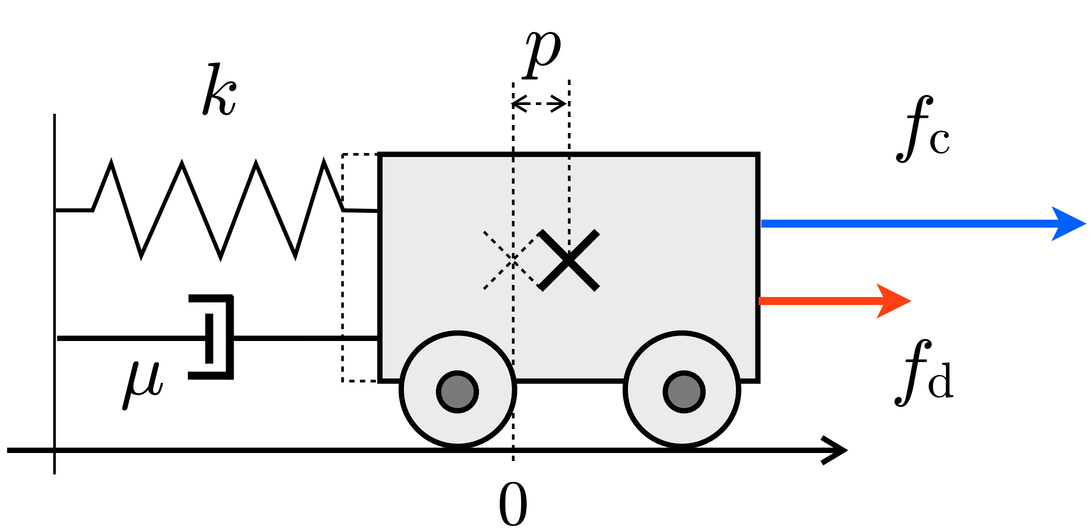


(in other words we want p_{des} to be the new equilibrium point with zero velocity) check

- system is asymptotically stable ($m,\ \mu$ and k all strictly positive)
- steady-state exists and since the input is constant, the output will tend to the constant value given by (step response)

$$p_{\rm ss} = \text{(static gain) x (input magnitude)} = 1/k\ k\ p_{\rm des} = p_{\rm des}$$

let's add the effect of a disturbance constant force $f_{\rm d}$

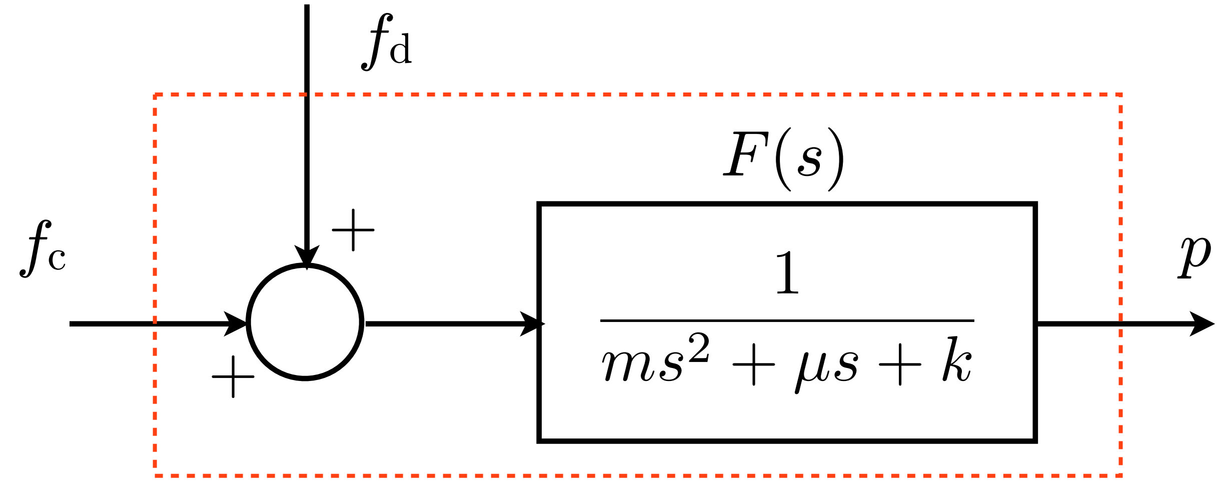


- same goal ($p_{\rm des}$) but in the presence of $f_{\rm d}$
- same solution principle: at steady-state the force needs to balance both the elastic force -$k\ p_{\rm des}$ and the disturbance $f_{\rm d}$

$$f_{\rm c} = k\ p_{\rm des} - f_{\rm d}$$

check

system can be represented as



- system has not changed (and therefore still asymptotically stable)
- using the superposition principle (or just considering $f_c + f_d$ as a unique input) we have

$$p(s) = F(s)(f_c + f_d)$$

and therefore at steady-state we have

$$p_{ss} = 1/k\ (f_c + f_d) = 1/k\ (k\, p_{des} - f_d + f_d) = p_{des}$$

but implicit hypothesis are

- perfect knowledge of the system's parameters, in particular k
- perfect knowledge of the disturbance (constant) magnitude f_d

in real systems these hypothesis are not met: it is equivalent to considering the effect of uncertainties

we need to distinguish the **estimated values** (those values we think the parameters or the disturbance have) from the true values

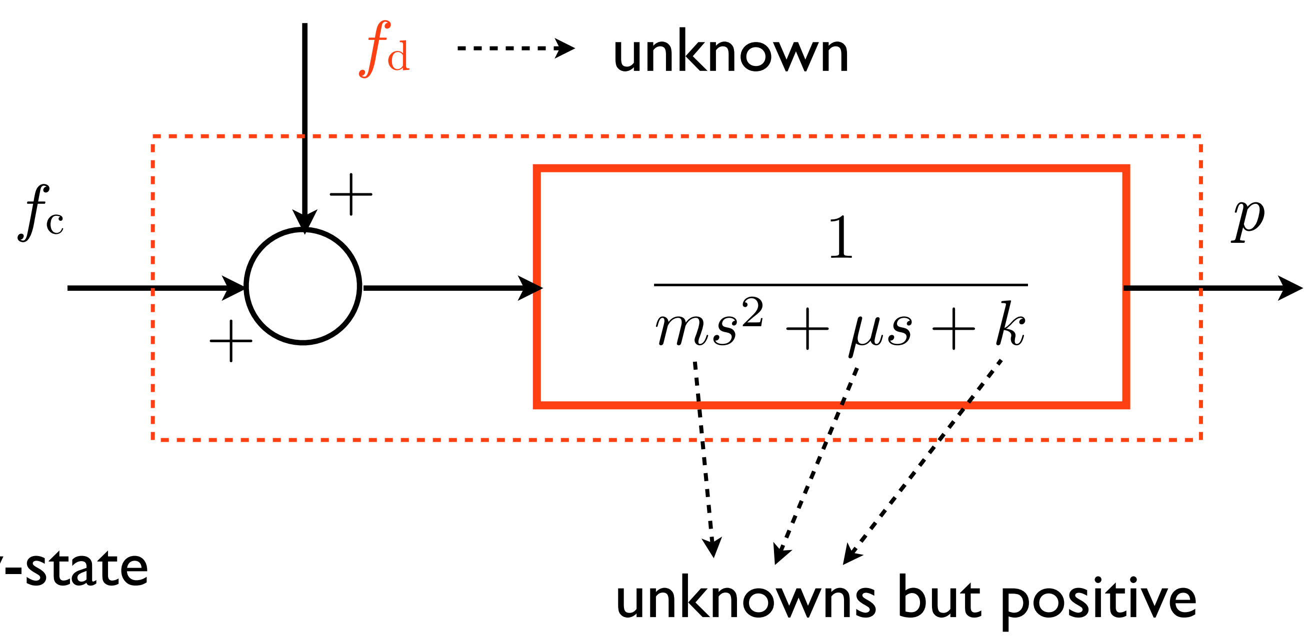
true values $m,\ \mu,\ k$ and $f_{\rm d}$

we do not know these values

estimated/perturbed values $m^*,\ \mu^*,\ k^*$ and $f_{\rm d}{}^*$

control input is based on estimated values $f_{\rm c} = k^*\, p_{\rm des} - f_{\rm d}{}^*$

and is applied to the real system



we can compute what happens at steady-state at steady-state

$$p_{ss} = 1/k\ (f_c + f_d) = 1/k\ (k^* p_{des} - f_d^* + f_d)$$

and defining the error at steady-state as $e_{ss} = p_{des} - p_{ss}$ we obtain

$$e_{ss} = \frac{k - k^*}{k} p_{des} - \frac{f_d - f_d^*}{k}$$

Note that

- the greater the uncertainty the greater the error
- no uncertainty (perfect knowledge of the parameters and the disturbance) zero error
- we have just discussed what happens at steady-state but we do not know how long it will take to reach the steady-state and how it will be reached. Moreover we have no control over the transient if we apply the chosen constant control input.

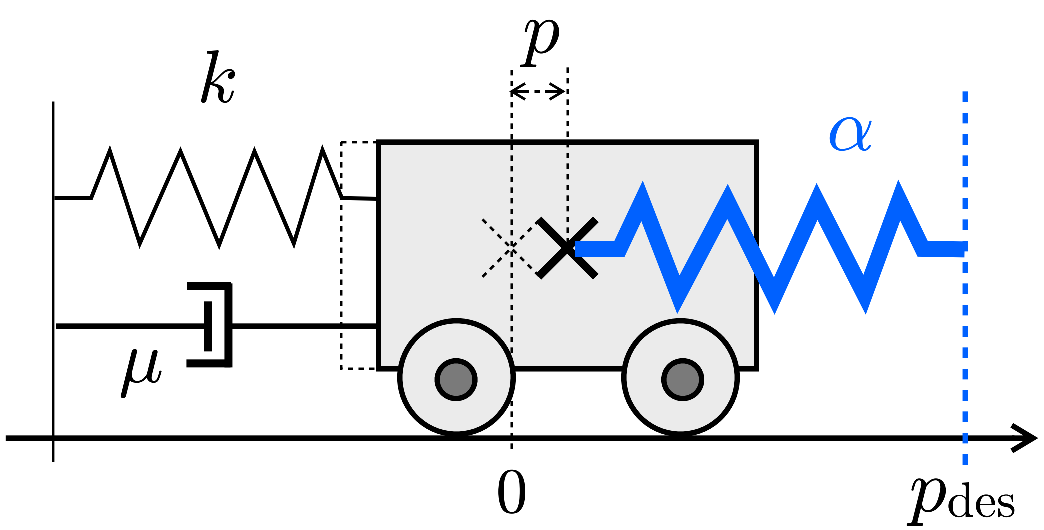
↑ open-loop approach **not robust** w.r.t. uncertainties and/or poor knowledge of the system

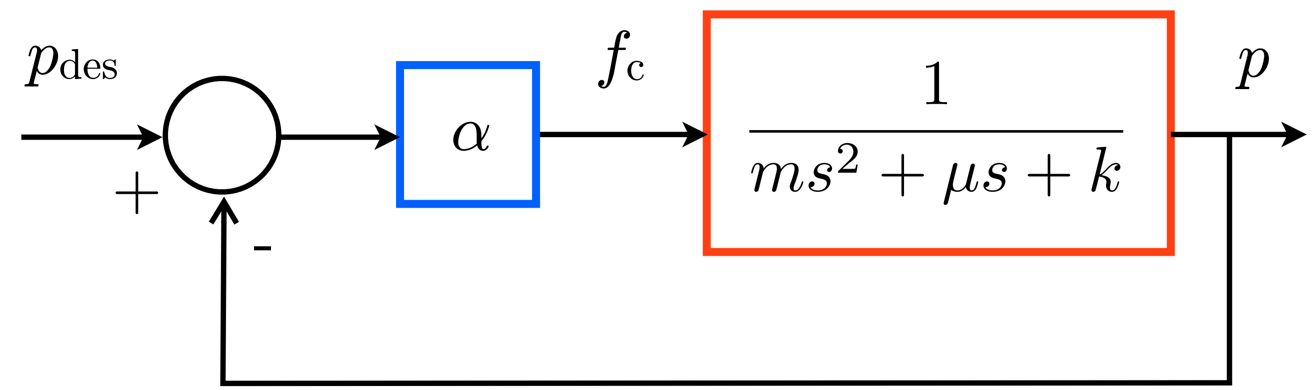
New goal

we want to reach the desired position in spite of the uncertainties and the unknown disturbance

idea

we introduce a virtual spring $\alpha > 0$ with rest position p_{des} which generates a force $\alpha(p_{\text{des}} - p)$





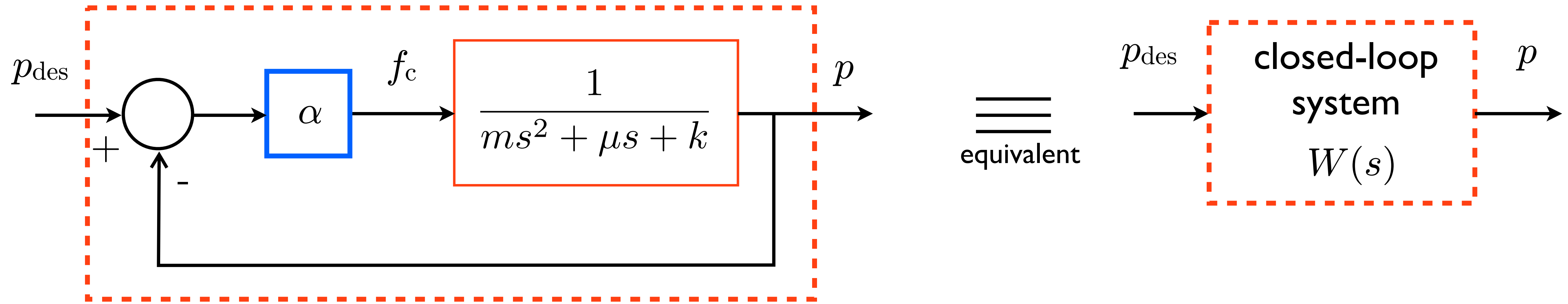
for now we consider $f_d = 0$

implementation of a constant controller in a unit feedback loop control scheme

intuitively

- the final desired position can be approached but not reached
- the true final position will be closer to the desired one for stronger springs

we can analyze the effect of the feedback $f_c = \alpha(p_{\text{des}} - p)$ using well-known results



$$W(s) = \frac{\alpha F(s)}{1 + \alpha F(s)} = \frac{\alpha}{ms^2 + \mu s + k + \alpha}$$

we note

- the new system (closed-loop system) is asymptotically stable (being also $\alpha > 0$)
- the new system has a different static gain which depends upon the design parameter α

$$W(0) = \frac{\alpha}{k + \alpha}$$

to distinguish the different transfer functions, we define $W_{ry}(s) = W(s)$

moreover

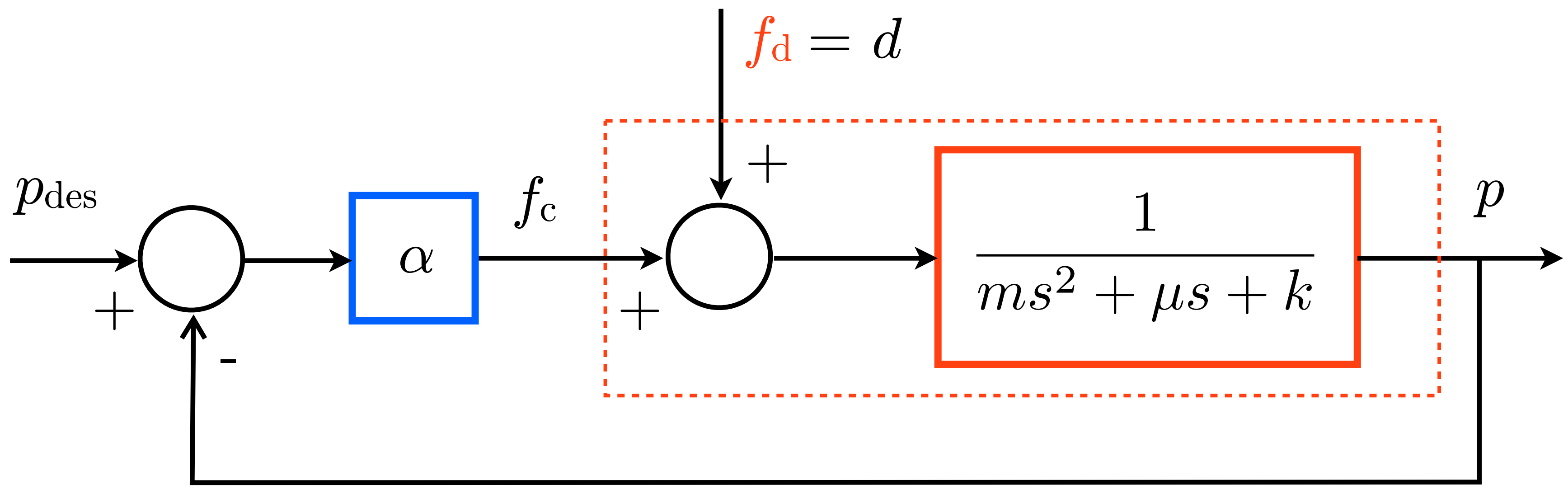
- as α increases the closed-loop system gain $W(0)$ approaches 1 and therefore the steady-state output $p_{\rm ss}$ approaches the constant reference $p_{\rm des}$ independently of the uncertain values of the parameter k

feedback helps reducing the effect of uncertainties

we can evaluate the error at steady-state $e_{\rm ss} = p_{\rm des}$ - $p_{\rm ss}$

$$e_{\rm ss} = p_{\rm des} - W(0)p_{\rm des} = (1 - W(0))p_{\rm des} = \frac{k}{k+\alpha}p_{\rm des}$$

what happens if also the constant (unknown) disturbance force $d = f_{\rm d}$ is present?



same feedback $f_{\rm c} = \alpha(p_{\rm des} - p)$ as before, independent from the value of the disturbance since it reacts to the **effect** of the disturbance on the system

we can use the **superposition principle** to separate the effect of the two inputs on the output

$W_{ry}(s)$ transfer function: effect of the reference $p_{\rm des}$ to the output p (when $f_{\rm d} = 0$)

$W_{dy}(s)$ transfer function: effect of the disturbance $f_{\rm d}$ to the output p (when $p_{\rm des} = 0$)

$$p(s) = W_{ry}(s) p_{\rm des}(s) + W_{dy}(s) f_{\rm d}(s)$$

we need to compute $W_{dy}(s)$

$$W_{dy}(s) = \frac{F(s)}{1+\alpha F(s)} = \frac{1}{ms^2 + \mu s + k + \alpha}$$

which has static gain

$$W_{dy}(0) = \frac{1}{k+\alpha}$$

at steady-state the position (output) is

$$p_{\text{ss}} = W_{ry}(0) p_{\text{des}} + W_{dy}(0) f_{\text{d}} = \boxed{\frac{\alpha}{k+\alpha}} p_{\text{des}} + \boxed{\frac{1}{k+\alpha}} f_{\text{d}}$$

as $\alpha \nearrow$ $\qquad \nearrow 1 \qquad \searrow 0$

note that we only know the disturbance is constant but not its value

Intuition was correct, it is true even with a constant unknown disturbance that

- the final desired position can be approached but not reached
- the true final position will be closer to the desired one for stronger springs

remarks

- we have only seen what happens for the simplest feedback choice (proportional to the error)
- we have only studied what happens at steady-state and not how this steady-state will be reached

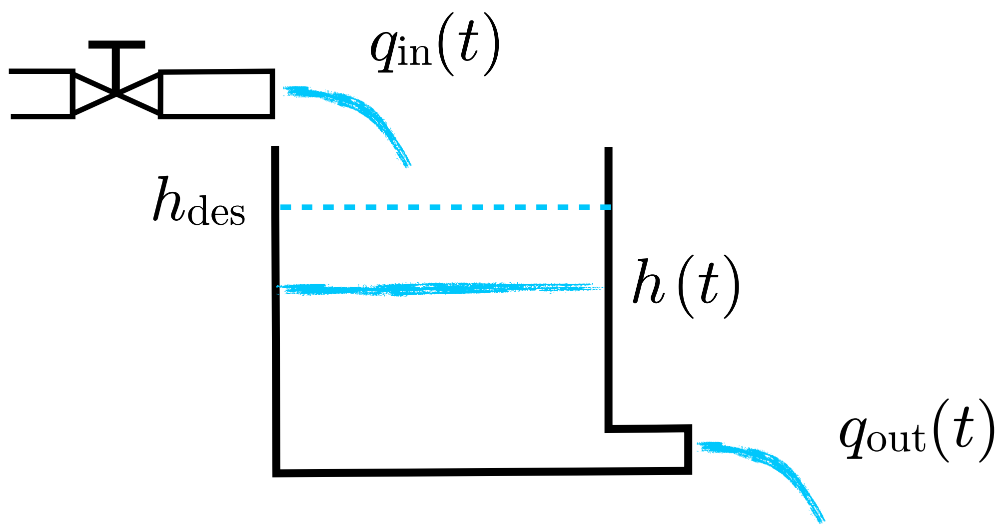
we have observed that

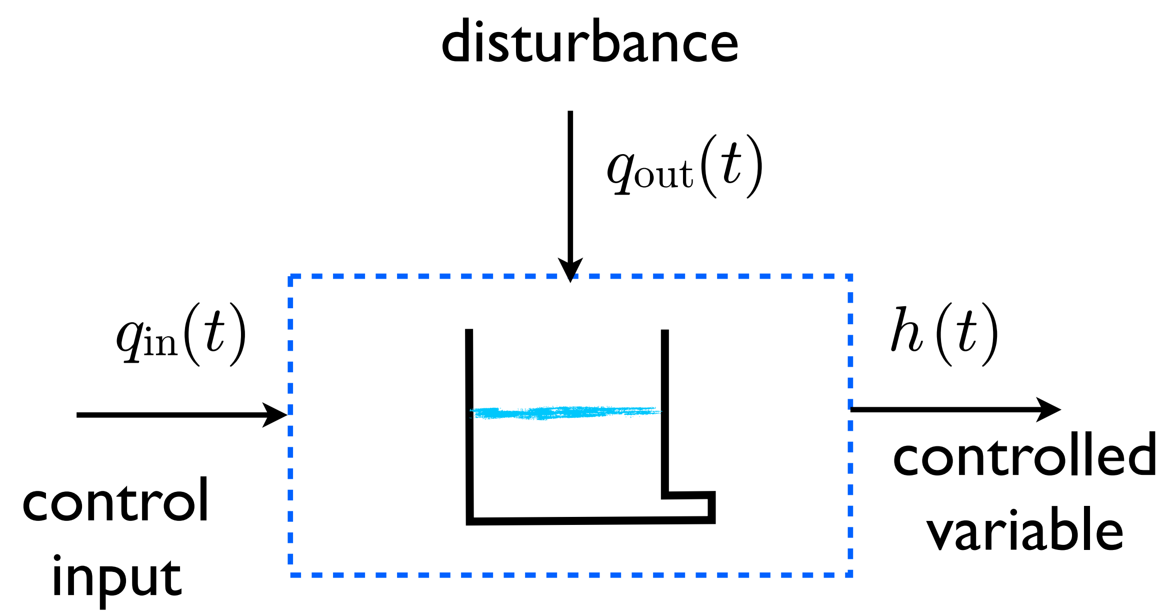
- feedback helps reducing the effect of uncertainties
- feedback helps reducing the effect of disturbances

In general we can design a feedback control system with a dynamical controller $C(s)$ instead of a static one (α) and satisfy requirements on

- closed-loop system stability
- steady-state behavior w.r.t. reference and disturbances for general classes of signals
- transient behavior
- guarantee robustness for stability and performance

Preliminary example II





two inputs

- the inlet flow rate $q_{\rm in}(t)$ is our **control input**
- we cannot manipulate the outlet flow rate $q_{\rm out}(t)$ which acts as a **disturbance** (consumer supply rate)

controlled variable

- liquid level $h(t)$ in the tank

reference

- desired liquid level $h_{\rm des}$

Goal

we want to refill the tank and keep the level of the liquid at a desired value h_{des}

Strategy 1

if we know (measure) the disturbance $q_{\text{out}}(t)$ we can compensate exactly the outlet flow

- if $h(0) = h_{\text{des}}$ we choose

$$q_{\text{in}}(t) = q_{\text{out}}(t)$$

direct compensation of the disturbance

- if $h(0) < h_{\text{des}}$ we know the missing volume to reach h_{des} provided we know the system's parameters (for example the tank's section surface) we can choose to integrate with an additional flow $q_{\text{int}}(t)$ what is missing. We choose

$$q_{\text{in}}(t) = q_{\text{out}}(t) + q_{\text{int}}(t)$$

direct compensation of the disturbance + open-loop control

with

$$\int q_{\text{int}}(t)dt = V_{\text{missing}}$$

model

variation of volume is due to the difference between inlet and outlet flow rates

$$\frac{dV(t)}{dt} = \frac{dAh(t)}{dt} = A\frac{dh(t)}{dt} = q_{\text{in}}(t) - q_{\text{out}}(t)$$

A constant area of the section

the state can only be $h(t)$

$$\dot{h} = \frac{1}{A}(q_{\text{in}} - q_{\text{out}})$$

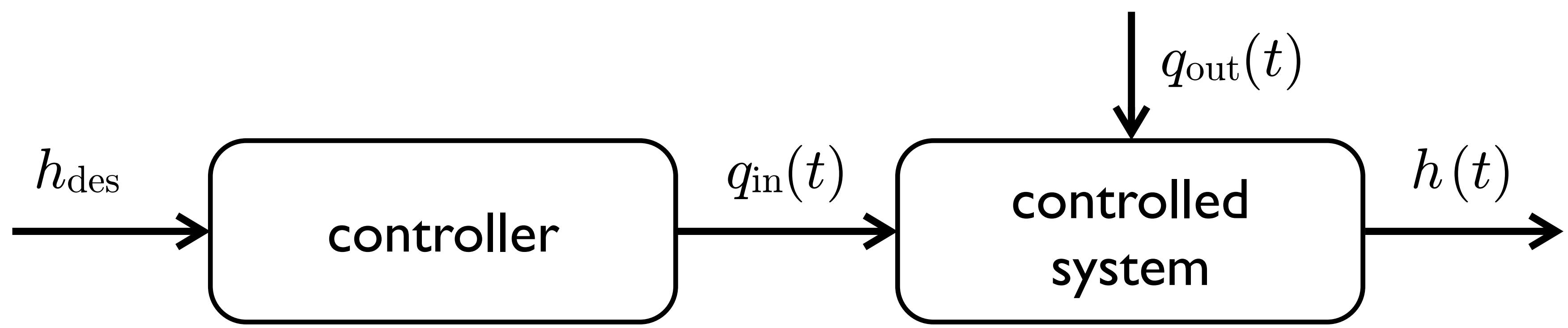
therefore the system is not asymptotically stable but just marginally stable due to the eigenvalue in $\lambda = 0$

> note the difference w.r.t. the previous example: now we have a system which is **not asymptotically stable** (at open-loop) so the control scheme needs also to cope with this situation

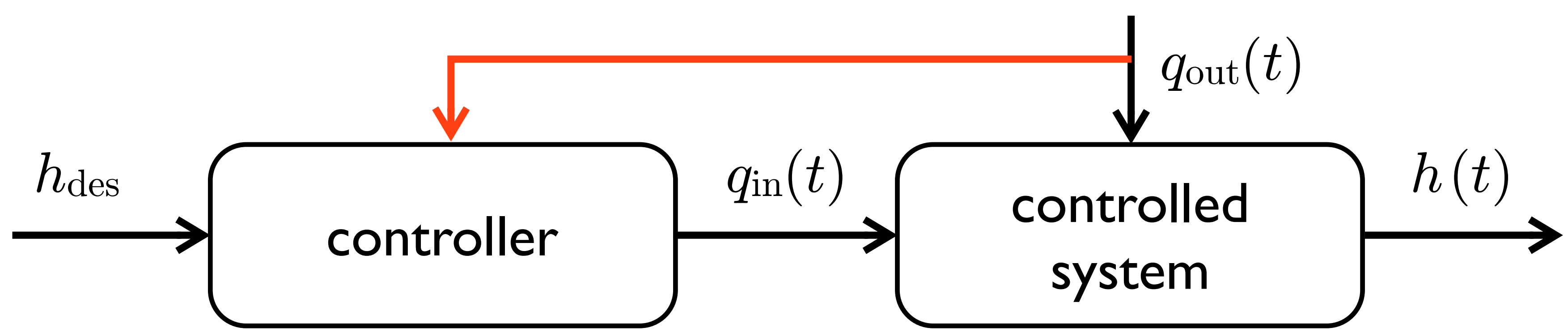
potential problems

- we need to know exactly the system's parameters (no robustness w.r.t. uncertainty in the system's parameters)
- any failure or malfunction in the disturbance measurement device is not detected so an error accumulates and we are not aware of it
- there may be other unconsidered disturbances which can influence the controlled output $h(t)$ (tank leakage, evaporation, ...) thus producing an undetectable error

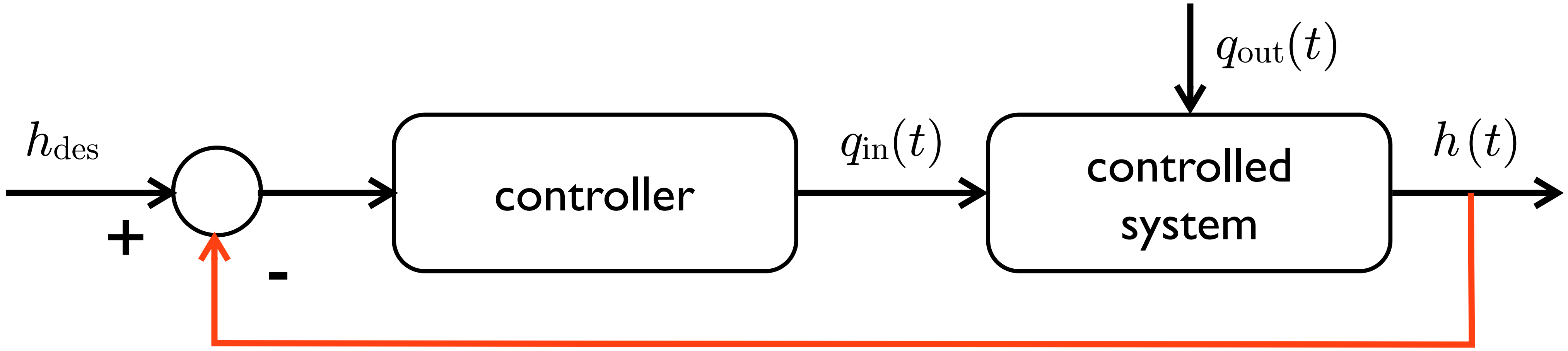
open-loop control scheme



open-loop control scheme with **direct disturbance compensation**



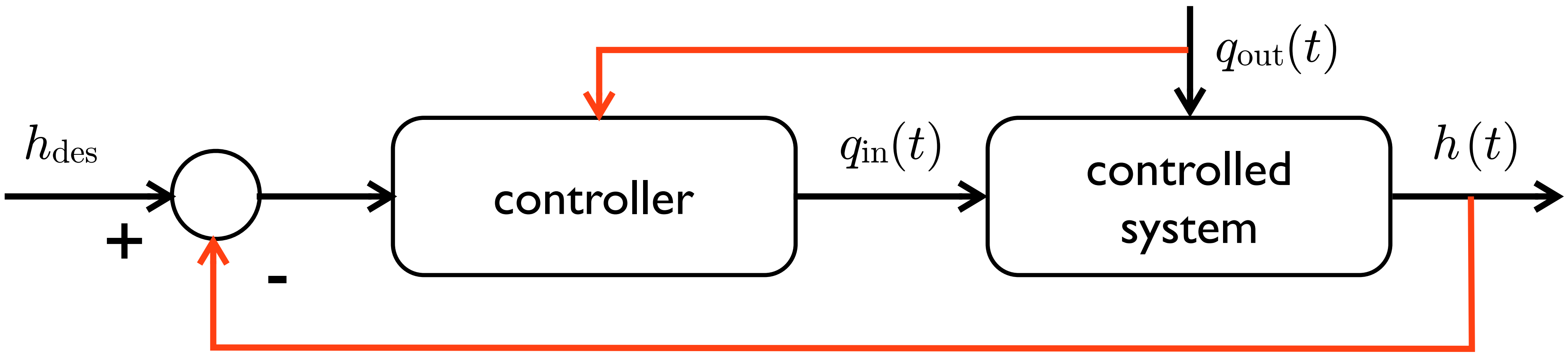
feedback



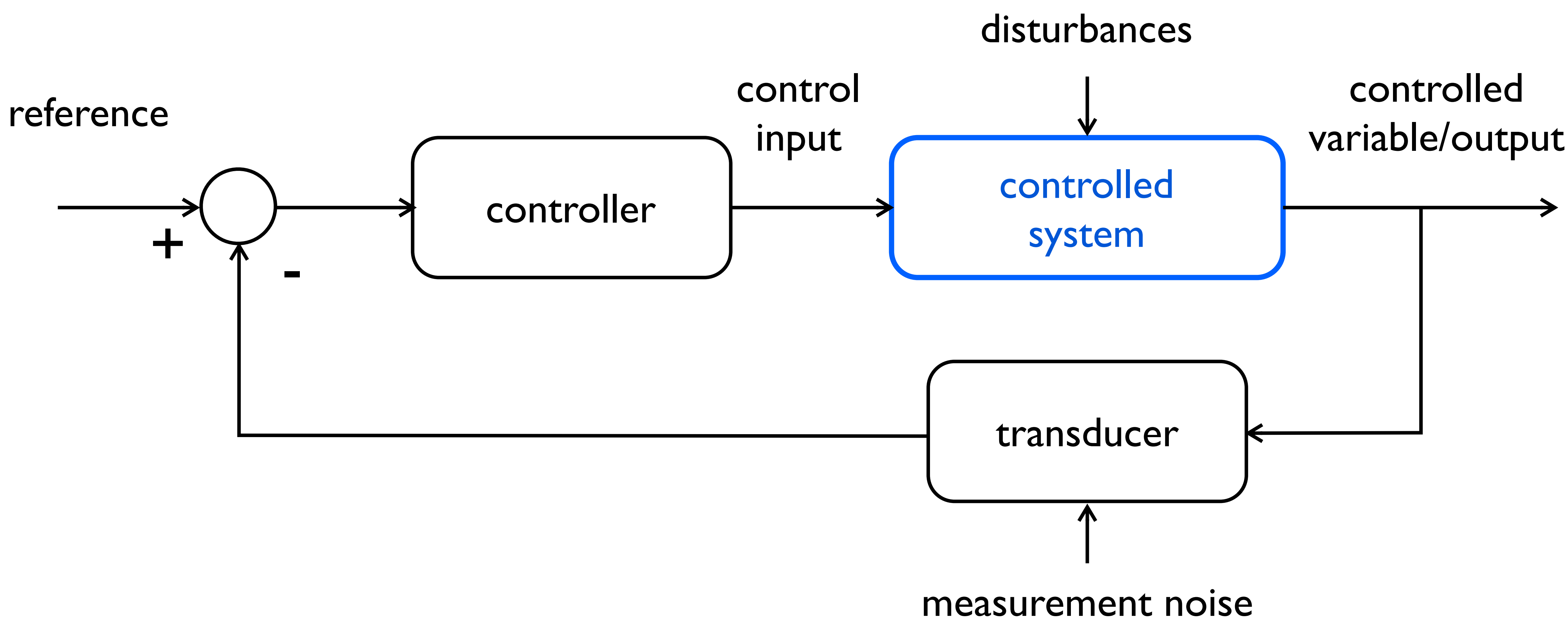
- effects of the disturbances on the controlled variable $h(t)$ are detected (any disturbance even tank leakage or evaporation, not only $q_{\text{out}}(t)$)
- some performance can be guaranteed even in the presence of disturbances
- in spite of the presence of uncertainty in the system's parameters we can guarantee (to a certain extent) the closed-loop behavior (for example in terms of stability, thus obtaining **robust stability**)

feedback + disturbance compensation

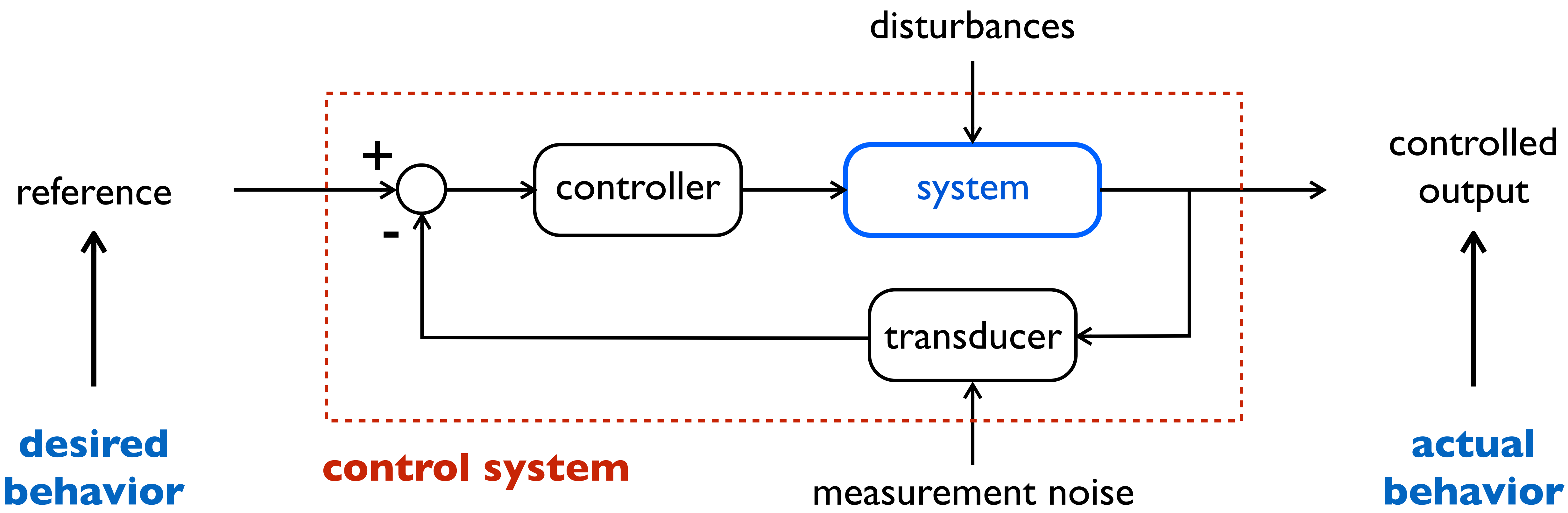
of course if we have a measure available of some disturbance acting on the system we can take advantage of this knowledge and combine the two approaches



note that even if the disturbance measurement device has some malfunctioning the feedback will take care of attenuating the effect of the disturbance general **feedback control scheme**



controlled system	→	models (state-space, transfer function)
controller	→	dynamical system with its representation
transducer	→	dynamical system with its representation



Goal

we need to design a controller in our feedback control scheme such that the controlled output follows as closely as possible a desired behavior represented by the reference even in the presence of disturbances, measurement noise and model uncertainty