Autonomous and Mobile Robotics
Final Class Test, 2013/2014

Problem 1

Consider the following nonlinear system that arises in quantum mechanics

$$\begin{aligned} \dot{x}_1 &= u_1 \\ \dot{x}_2 &= u_2 \\ \dot{x}_3 &= x_1 u_2 - x_2 u_1 \end{aligned}$$

This is called a *nonholonomic integrator* or *Heisenberg control system*.

1. Write the differential constraint underlying the above model.
2. Prove that such constraint is nonholonomic.
3. Prove that the system is controllable.
4. Compute the final state displacement after a Lie Bracket control maneuver.

Problem 2

Discuss the nature and the dimension of the configuration space for the following mechanical systems.

1. A team of two mobile robots that can translate and rotate in the plane.
2. A team of two mobile robots that can translate and rotate in the plane and are connected by a rope.
3. A team of two mobile robots that can translate and rotate in the plane and are connected by a rigid bar.
4. A spacecraft with a 6R robot arm.
5. The last link of 6R robot arm mounted on a spacecraft.

Problem 3

Consider a fixed-wing Unmanned Aerial Vehicle (UAV) flying at a constant altitude $z = \bar{z}$ with zero pitch angle. In this particular condition, the UAV configuration can be described as $(x \quad y \quad \psi \quad \phi)^T$, where (x, y) are the Cartesian coordinates of its center of gravity, ψ is the yaw angle, and ϕ is the roll angle. The UAV dynamic model is

$$\begin{aligned} \dot{x} &= v \cos\psi \\ \dot{y} &= v \sin\psi \\ \dot{\psi} &= -\frac{g}{v} \tan\phi \\ \dot{\phi} &= u_\phi \end{aligned}$$

where g is the gravity acceleration. The UAV speed v and roll rate u_ϕ are the available control inputs. The UAV is equipped with a sensor, located at the center of gravity, that can measure the distance between itself and two radio beacons, located on the ground ($z = 0$) at unknown positions. Under the assumption that the sensor can distinguish between the two beacons, build an Extended Kalman Filter for estimating simultaneously the configuration of the UAV and the position of the beacons.

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