Autonomous and Mobile Robotics

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Humanoid Robots 3:

Balance

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recap

- the Newton-Euler equations can be used to derive a relation between **CoM** and **ZMP**

- the ZMP represents the point of application of the **resultant ground reaction force**

- sufficient condition for balance: ZMP inside **support polygon**

- approximate model: **Linear Inverted Pendulum (LIP)**

$$\ddot{\boldsymbol{c}}^{\boldsymbol{x},\boldsymbol{y}} = \frac{g^z}{c^z}\left(\boldsymbol{c}^{x,y} - \boldsymbol{z}^{x,y}\right)$$

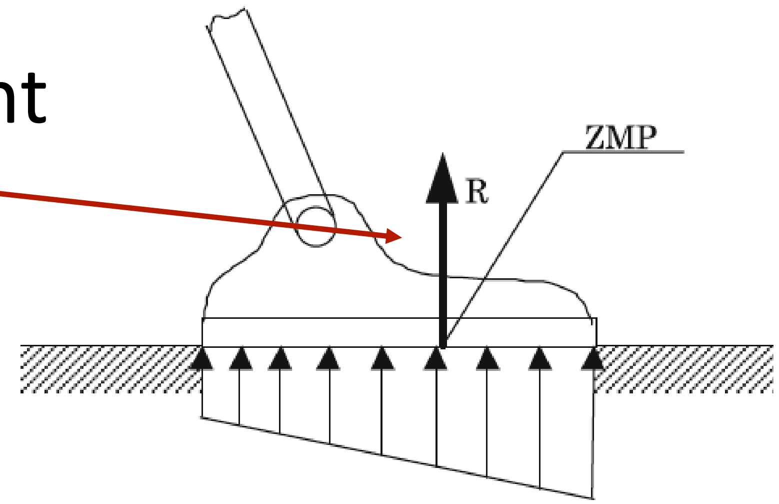
linear inverted pendulum: basic scope

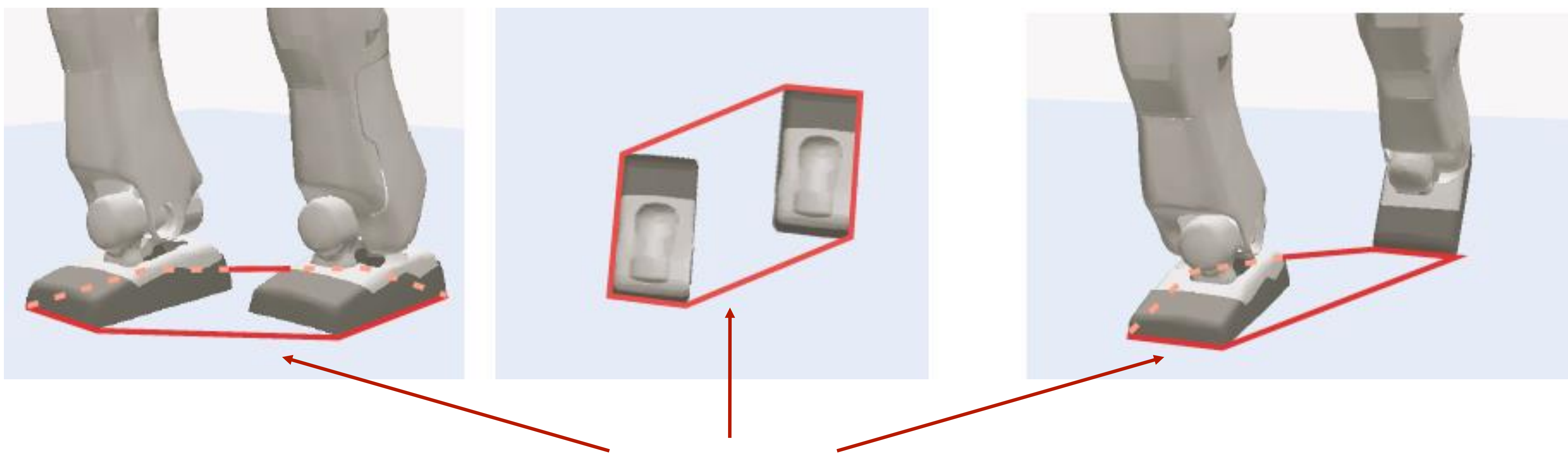
$$\ddot{\boldsymbol{c}}^{\boldsymbol{x},\boldsymbol{y}} = \frac{g^z}{c^z}\left(\boldsymbol{c}^{x,y} - \boldsymbol{z}^{x,y}\right)$$

- although extremely simplified, the LIP equation describes in first approximation the time evolution of the CoM trajectory
- it defines a **differential relationship** between the CoM trajectory and the ZMP time evolution
- a suitable ZMP trajectory can be chosen such to satisfy **dynamic balance** by avoiding tilting
- the associated CoM trajectory can then be **tracked** with a complete robot model

ZMP

vertical component
of the GRF



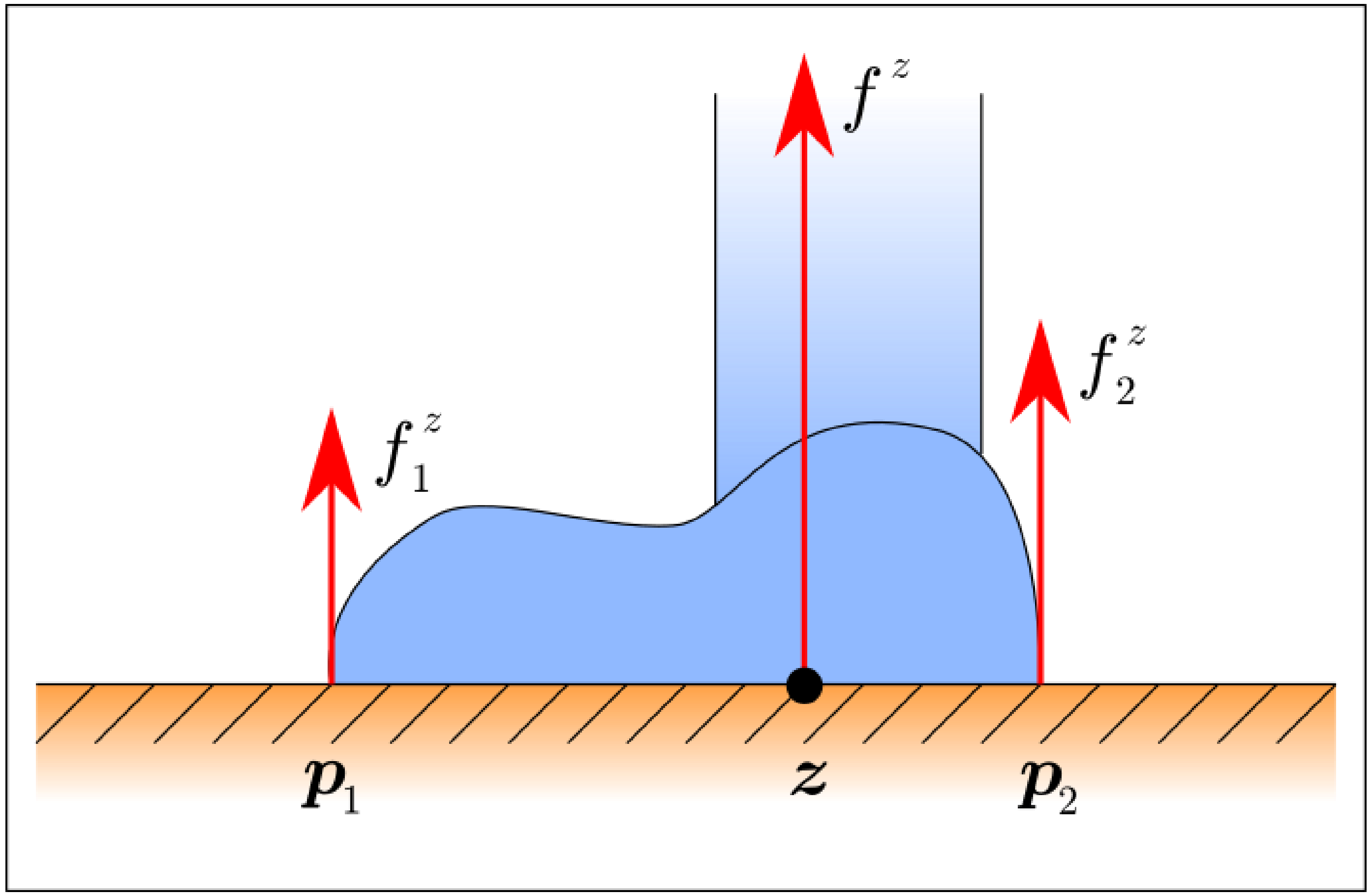


Support Polygons during
Double Support

ZMP – simplified case

- let us consider a simplified example: two vertical ground reaction forces in the x-z plane, applied at $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$

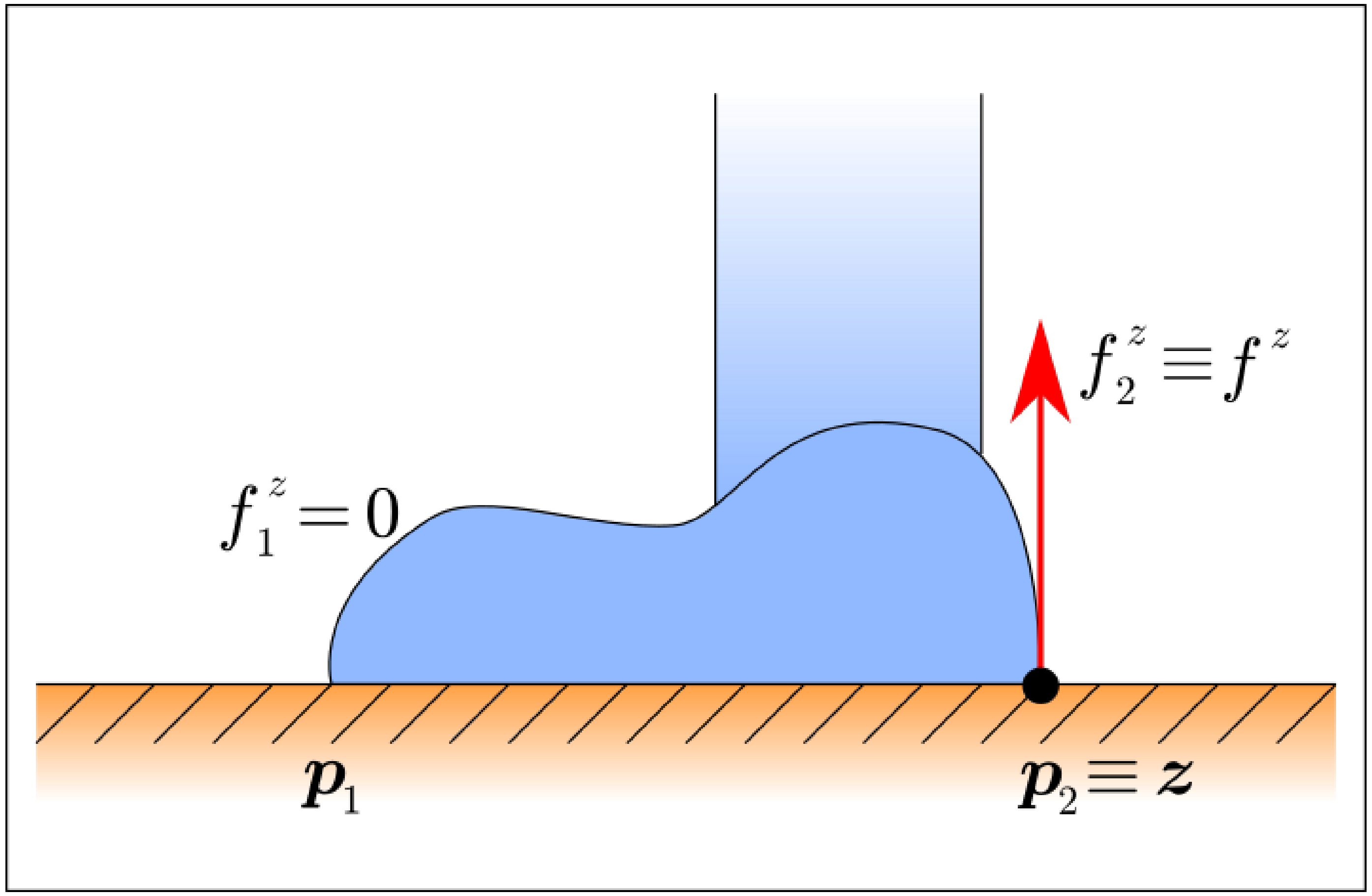
- the ZMP is somewhere in between $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$, shifted towards the larger of the two forces



ZMP – simplified case

- if we reduce the magnitude of f_1^z, the resultant force moves towards $\boldsymbol{p}_2$, up until the point when f_1^z equals zero

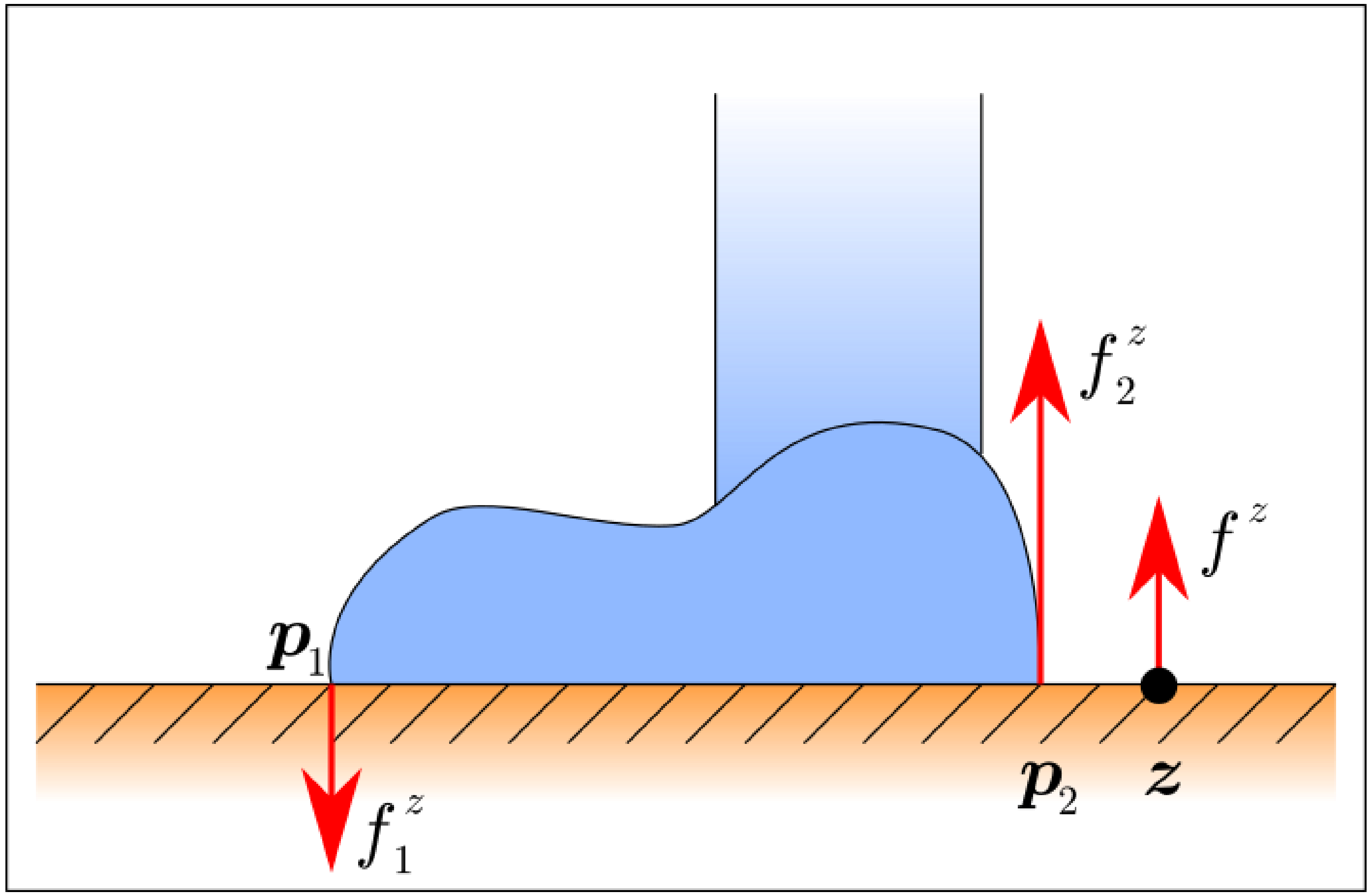
- at this point the ZMP exactly coincides with $\boldsymbol{p}_2$



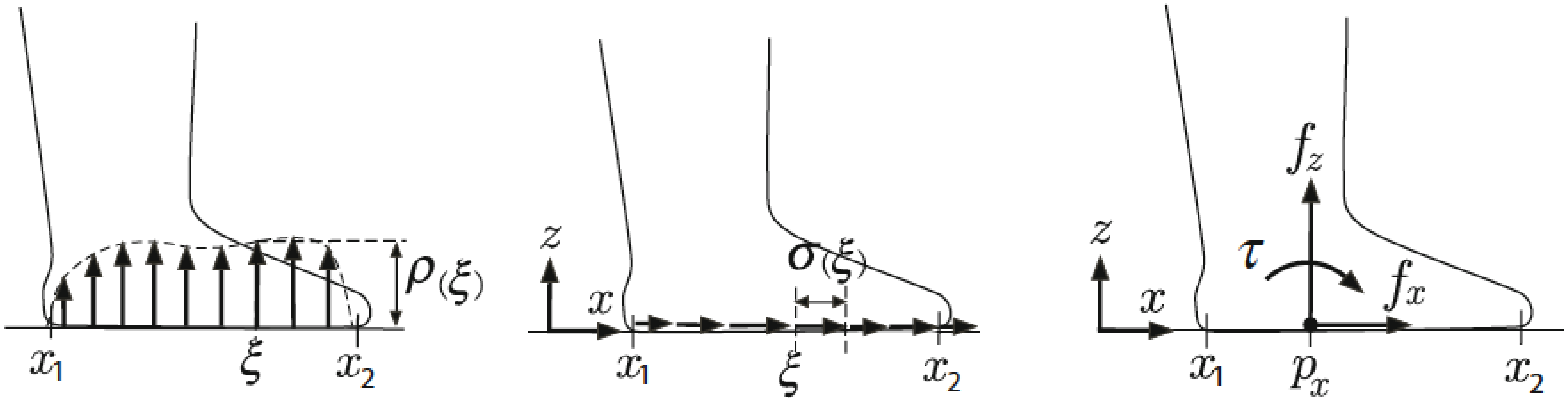
ZMP – simplified case

- imagine we could keep moving the ZMP to the right: it would end up outside the support polygon, requiring a negative f_1^z

- this is impossible because the vertical ground reaction force is **unilateral**: the ground can only push on the robot and not pull



ZMP – 2D case



(a) Vertical force (b) Horizontal force

horizontal force $$f_x = \int_{x_1}^{x_2} \sigma(\xi) d\xi$$

vertical force $$f_z = \int_{x_1}^{x_2} \rho(\xi) d\xi$$

torque $$\tau(p_x) = -\int_{x_1}^{x_2} (\xi - p_x)\rho(\xi) d\xi$$

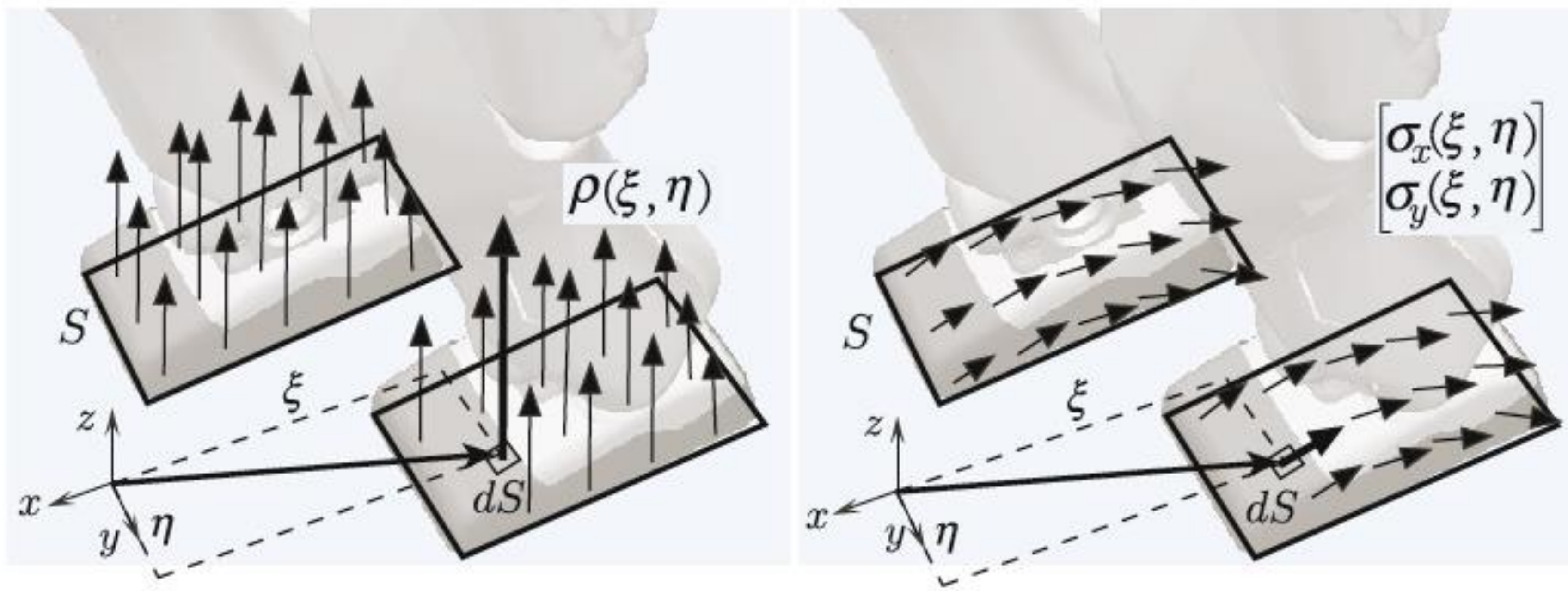
(computed wrt to a generic point p_x)

ZMP definition: $\tau(z_x) = 0$

ZMP

$$z_x = \frac{\int_{x_1}^{x_2} \xi\rho(\xi) d\xi}{f_z}$$

ZMP – 3D case



(a) Vertical reaction forces (b) Horizontal reaction forces

vertical component of the GRF

$$f_z = \int_S \rho(\xi,\eta)dS$$

$$\boldsymbol{\tau}_n(\boldsymbol{p}) \equiv [\tau_{nx}\ \tau_{ny}\ \tau_{nz}]^T$$

$$\tau_{nx} = \int_S (\eta - p_y)\rho(\xi,\eta)dS$$

$$\tau_{ny} = -\int_S (\xi - p_x)\rho(\xi,\eta)dS$$

$$\tau_{nz} = 0.$$

$$\tau_{nx} = 0$$

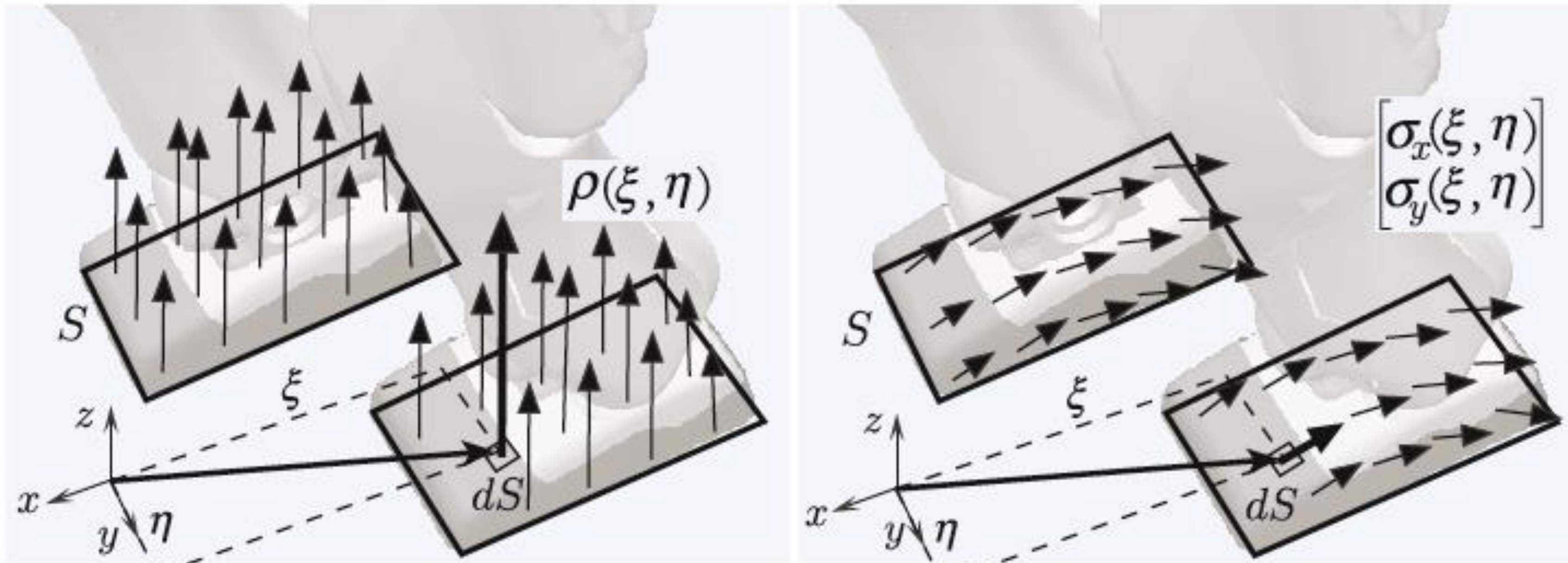
$$\tau_{ny} = 0$$

ZMP

$$p_x = \frac{\int_S \xi\rho(\xi,\eta)dS}{\int_S \rho(\xi,\eta)dS}$$

$$p_y = \frac{\int_S \eta\rho(\xi,\eta)dS}{\int_S \rho(\xi,\eta)dS}.$$

ZMP – 3D case



(a) Vertical reaction forces (b) Horizontal reaction forces

horizontal component of the GRF

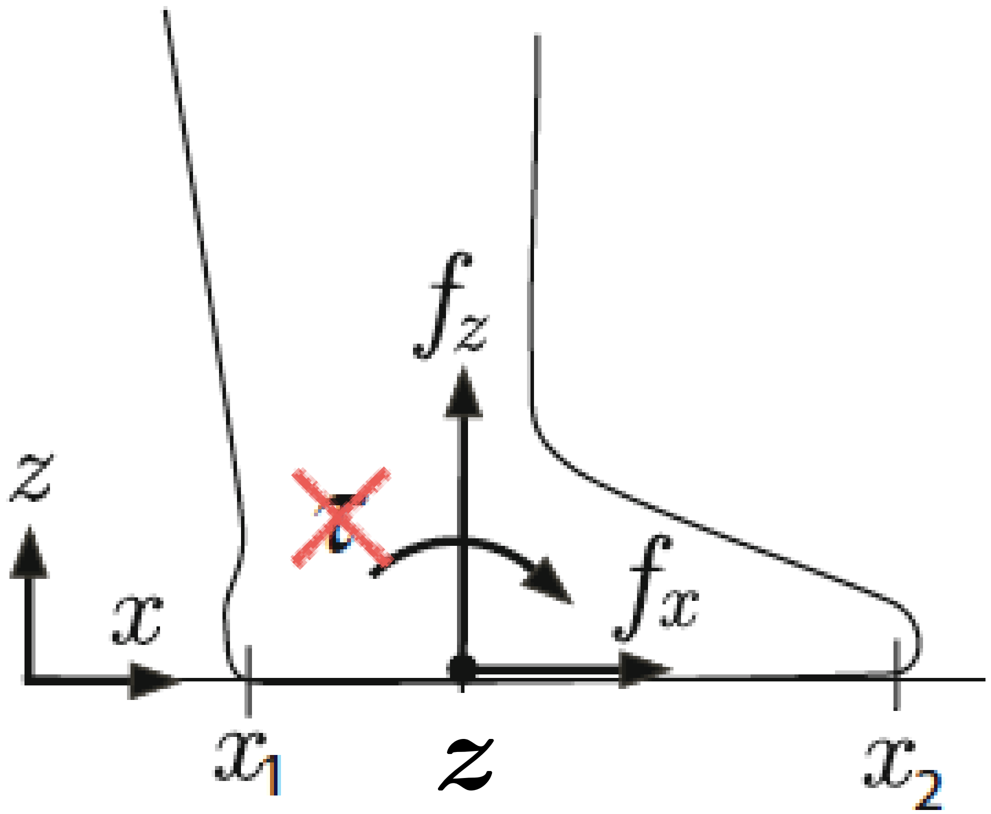
$$f_x = \int_S \sigma_x(\xi,\eta)dS$$
$$f_y = \int_S \sigma_y(\xi,\eta)dS.$$

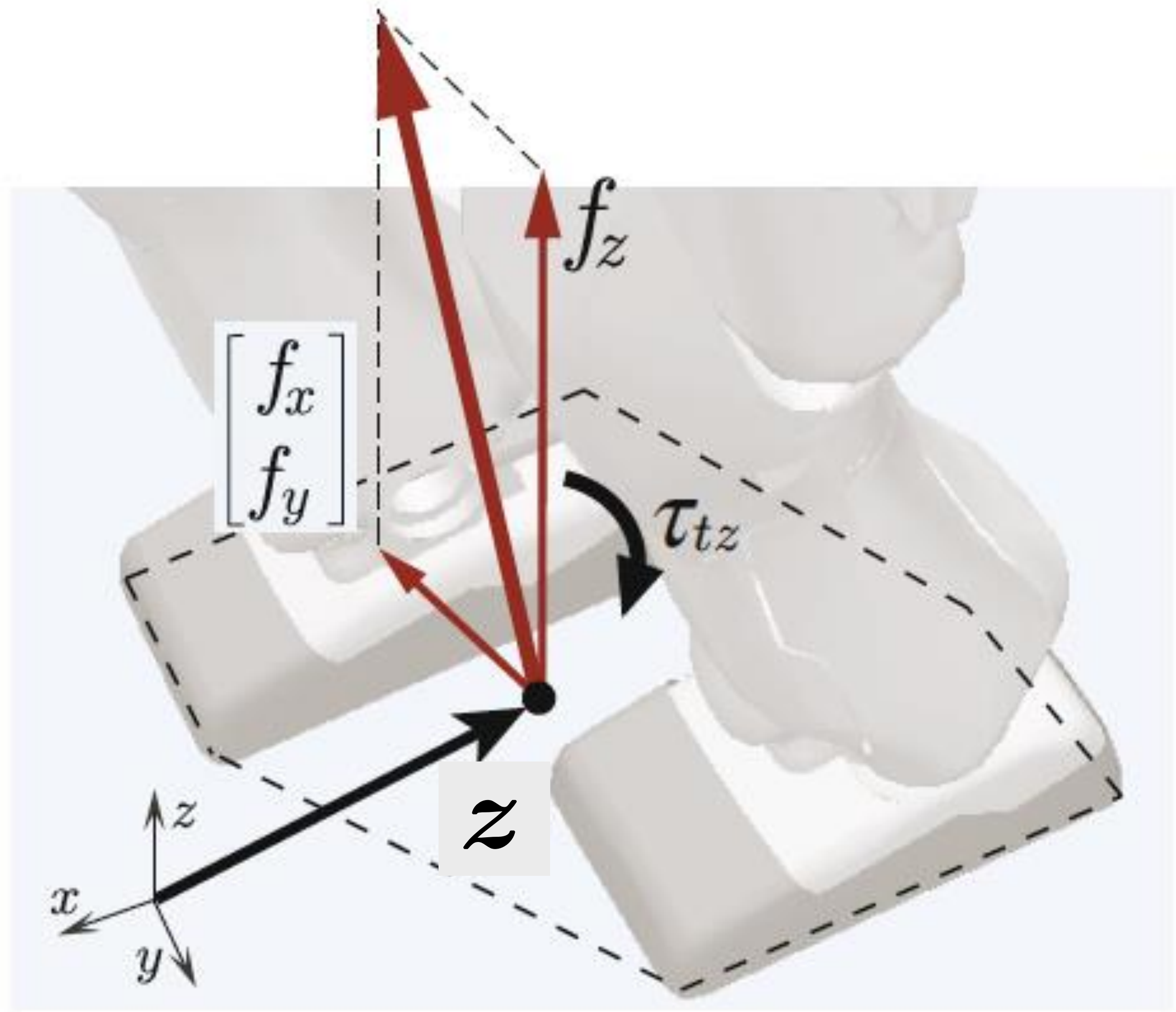
$$\boldsymbol{\tau}_t(\boldsymbol{p}) \equiv [\tau_{tx}\ \tau_{ty}\ \tau_{tz}]^T$$
$$\tau_{tx} = 0$$
$$\tau_{ty} = 0$$
$$\tau_{tz} = \int_S \{(\xi - p_x)\sigma_y(\xi,\eta) - (\eta - p_y)\sigma_x(\xi,\eta)\}dS$$

$$\boldsymbol{\tau}_p = \boldsymbol{\tau}_n(\boldsymbol{p}) + \boldsymbol{\tau}_t(\boldsymbol{p}) = [0\ 0\ \tau_{tz}]^T,$$

if robot moves, the z component will be different from 0

ZMP

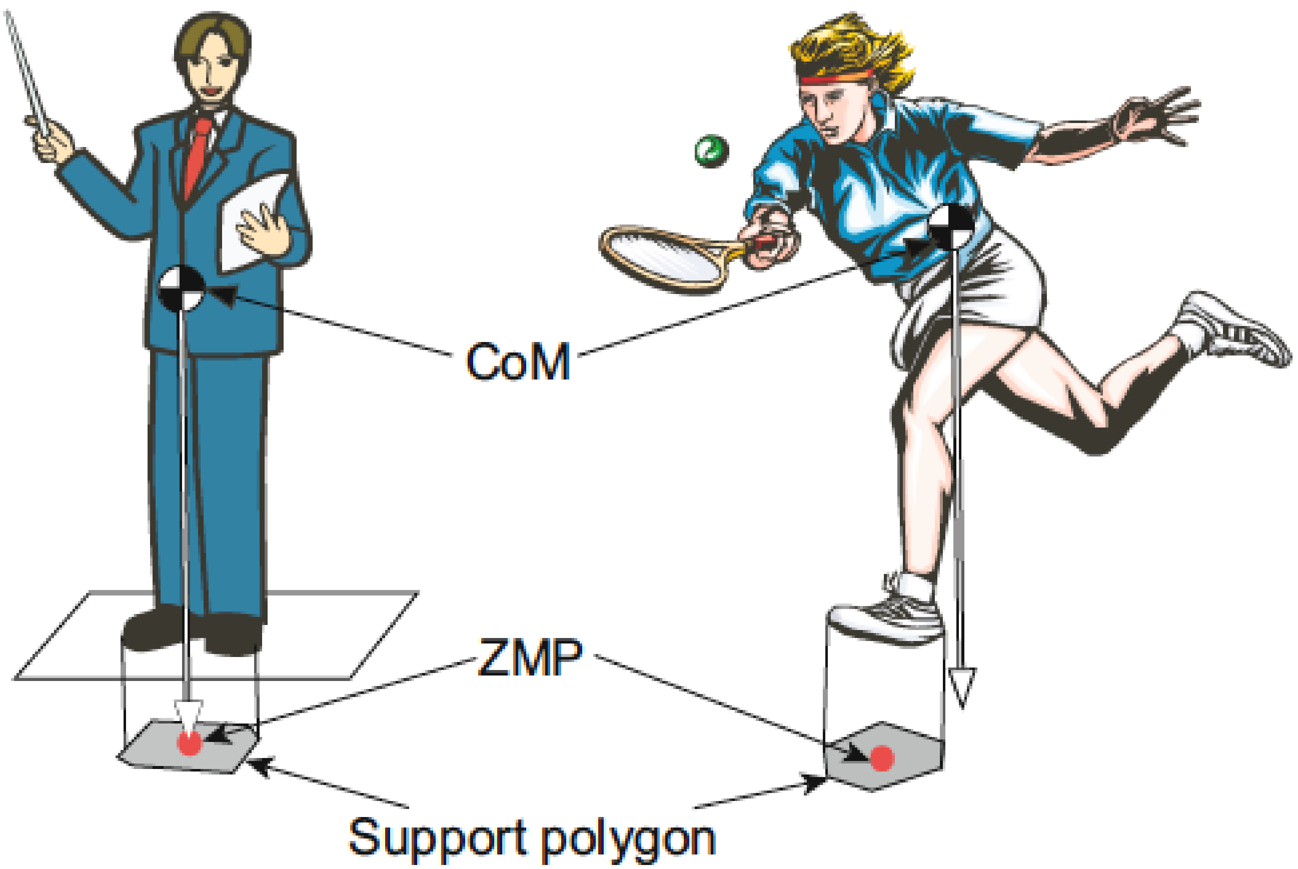




as long as the ZMP is in the Support Polygon,

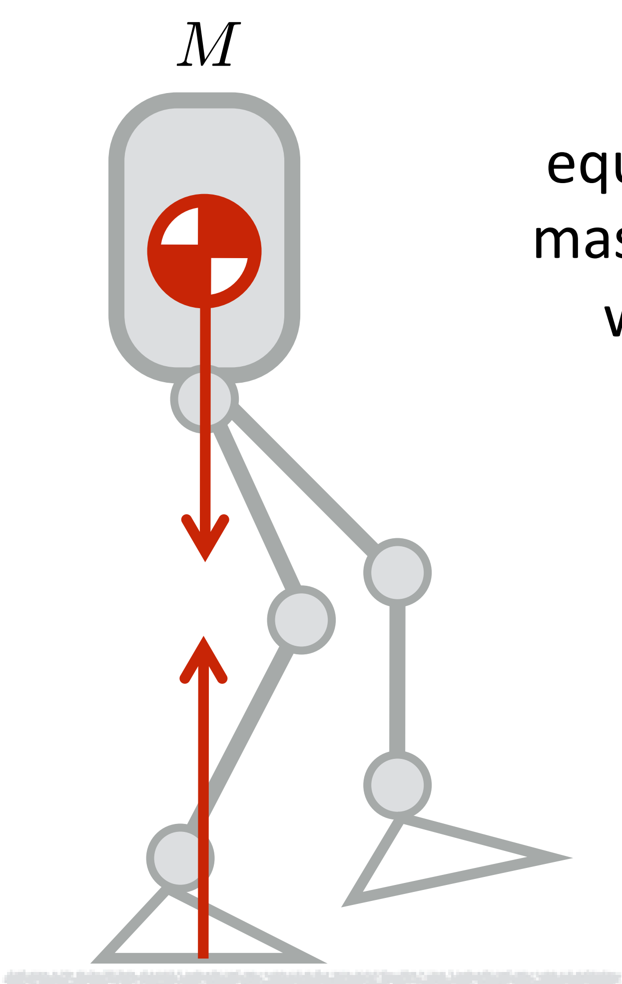
the support foot will not rotate

ZMP



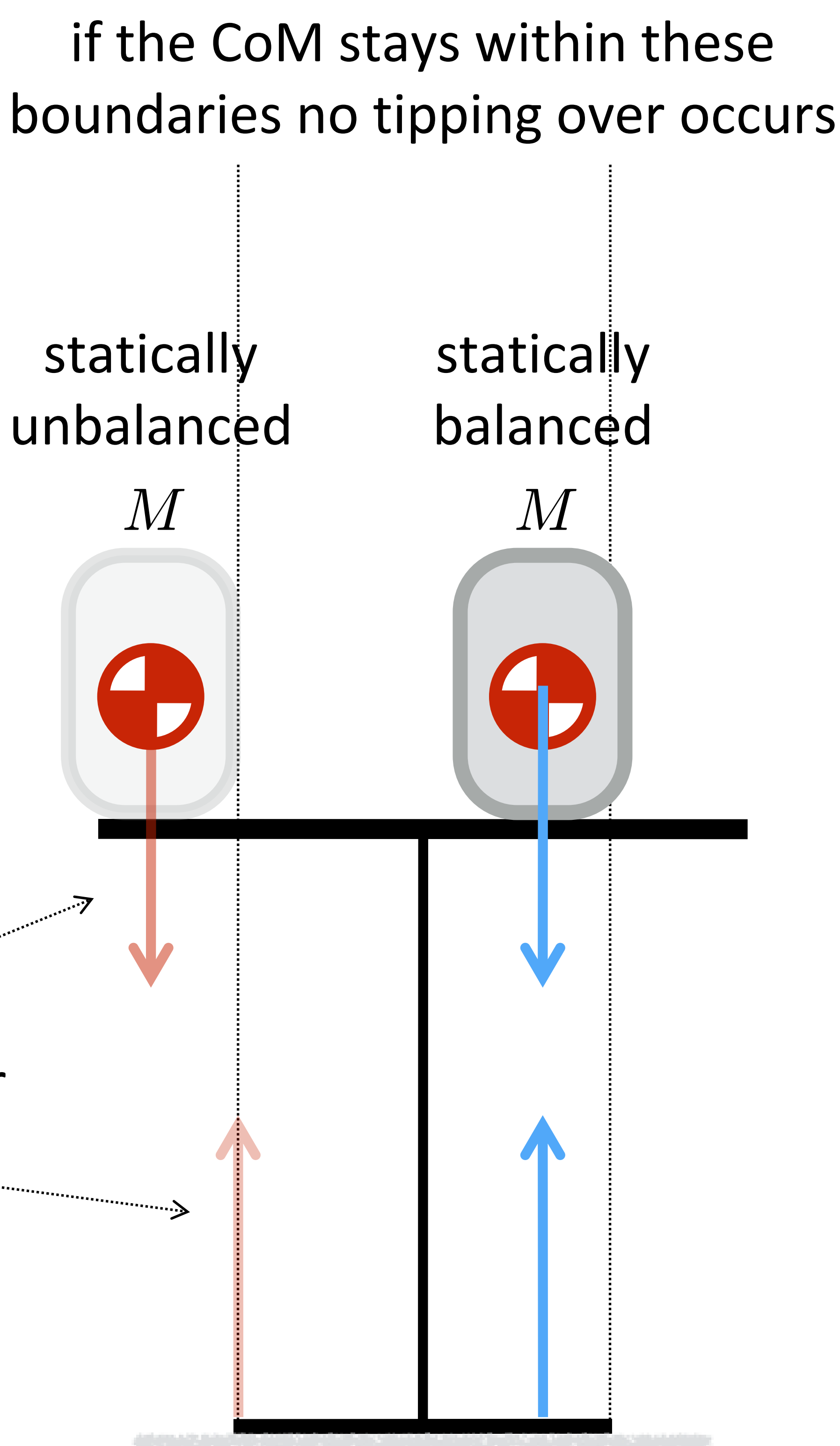
static balance

humanoid motionless:
statically balanced robots keep the center of mass within the polygon of support in order to maintain postural stability (sufficient when the robot moves slow enough so all the inertial forces are negligible)



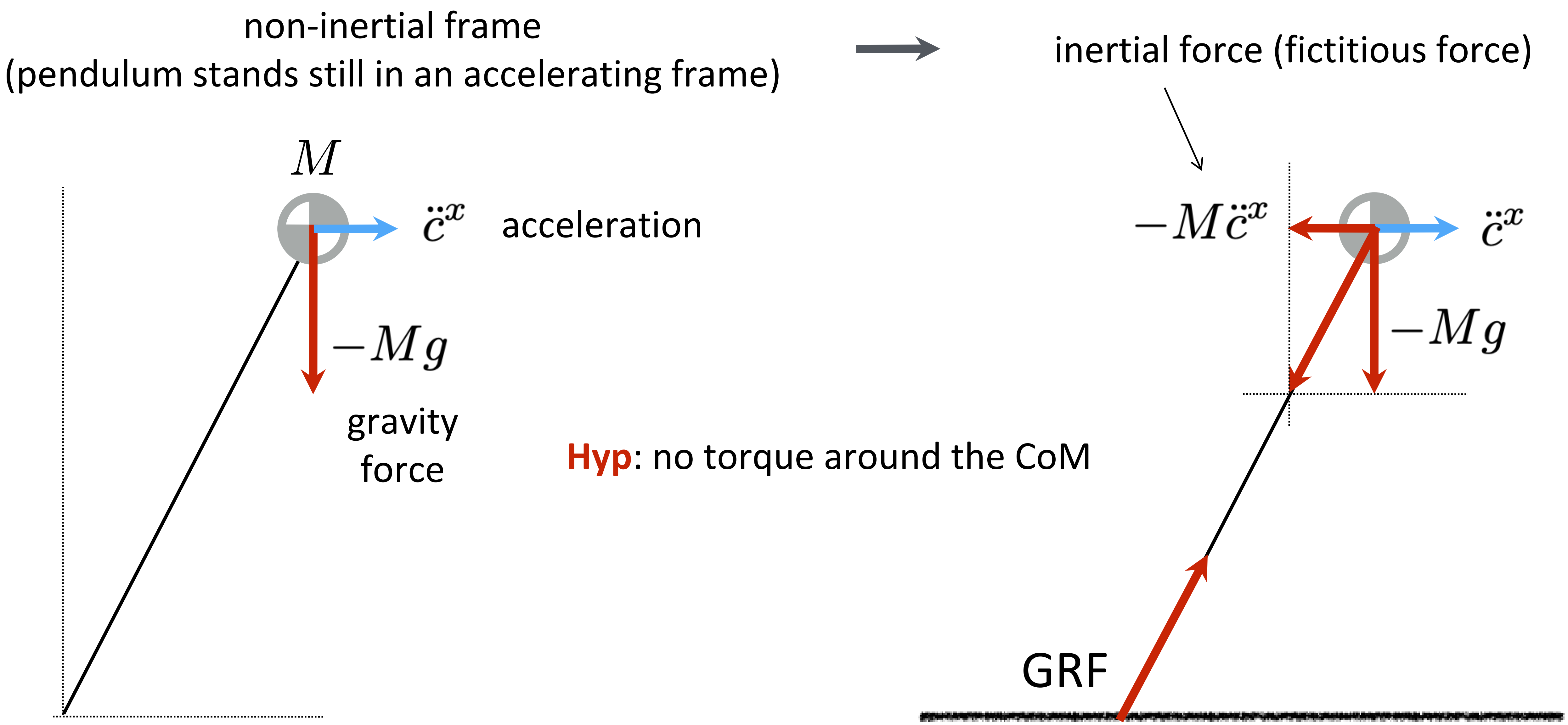
equivalent representation:
mass M on a massless table
with finite length base

the table starts tipping over



dynamic balance

- we can analyze dynamic balance in the same way by adding an **inertial force** (fictitious force in an accelerating frame)

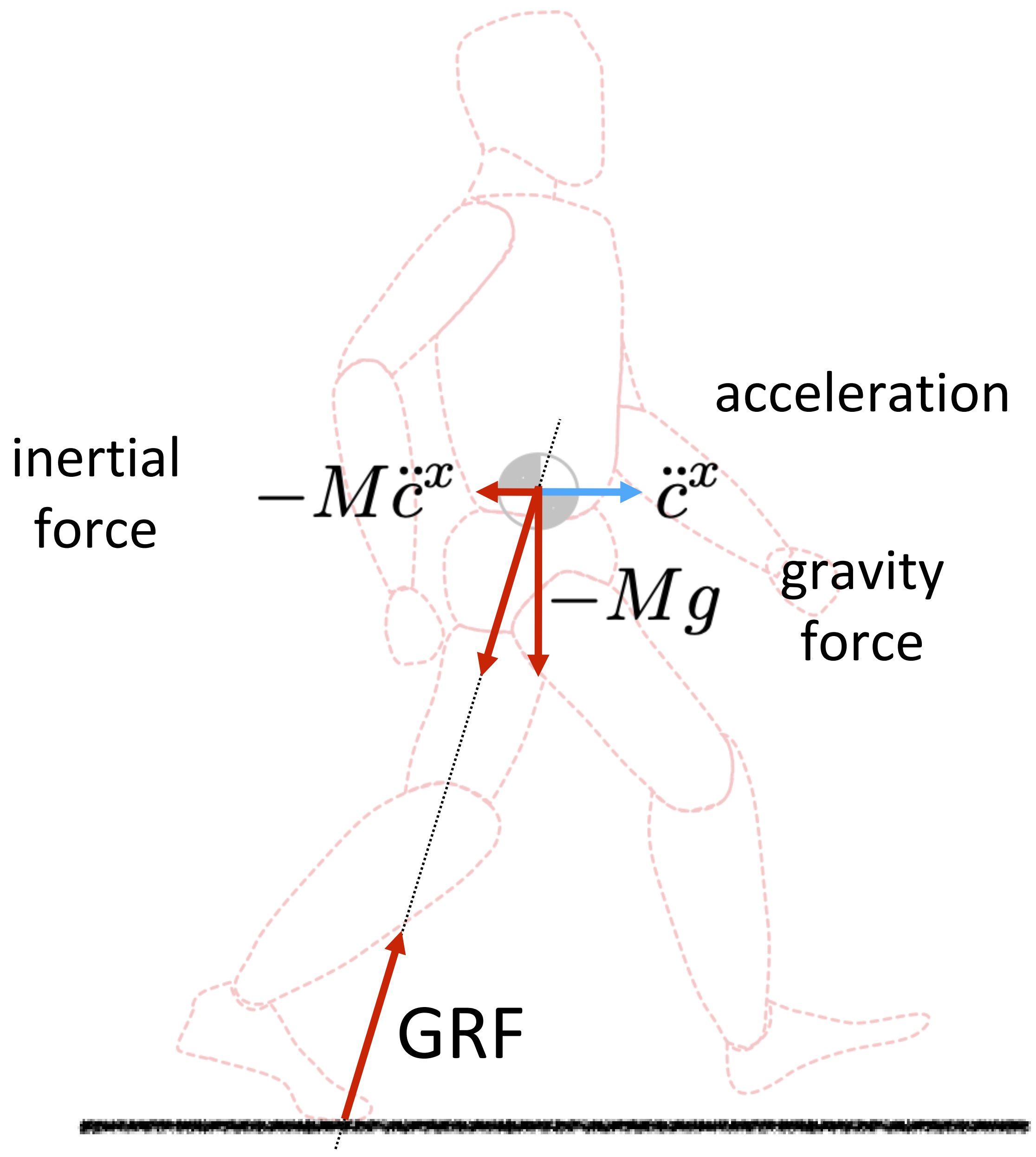


dynamic balance

humanoid walking:

the GRF will also have a component parallel to the ground;

the motion requires the exchange of horizontal frictional force with the ground



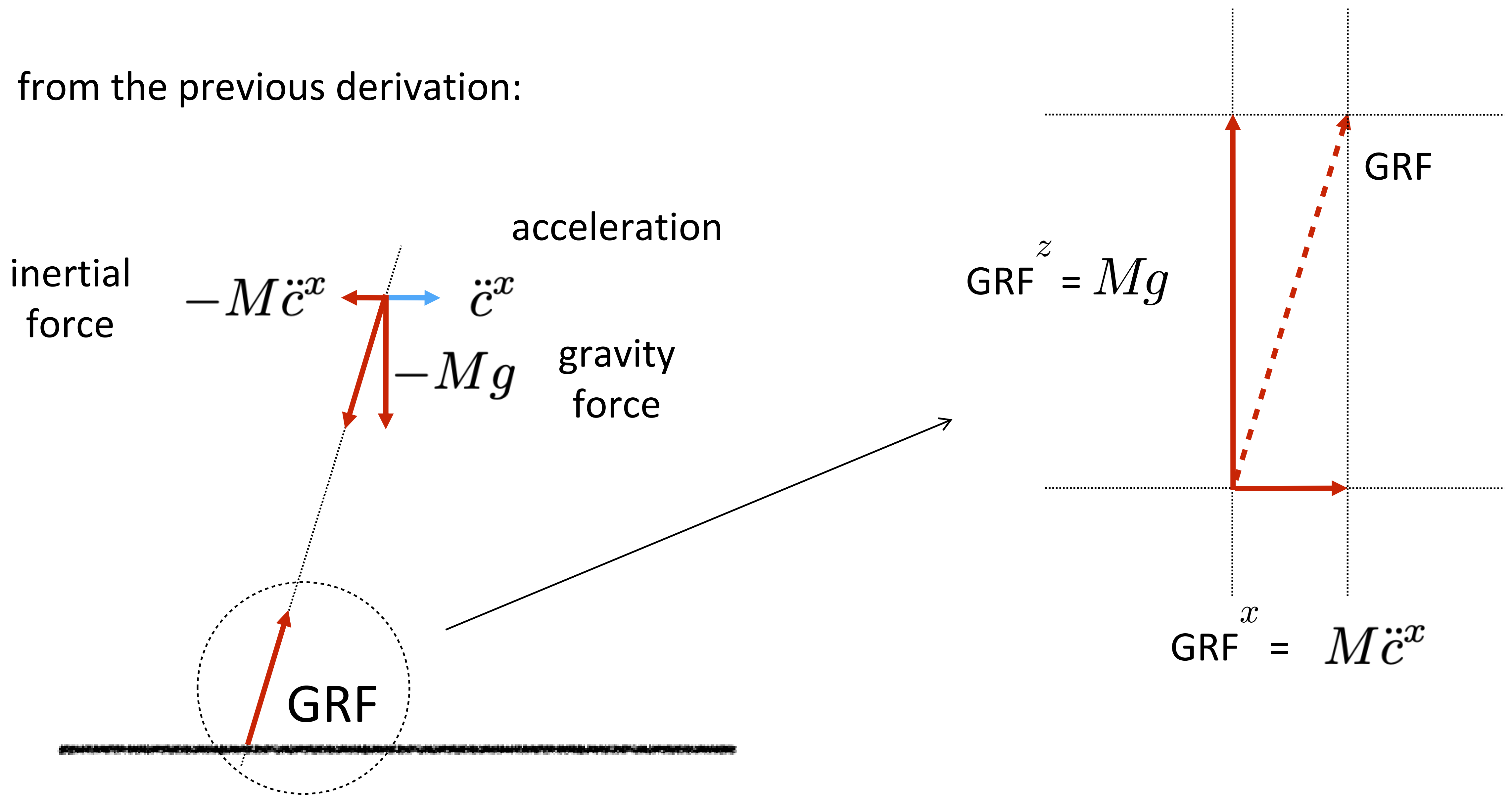
hyp: no torque around the CoM

dynamic balance

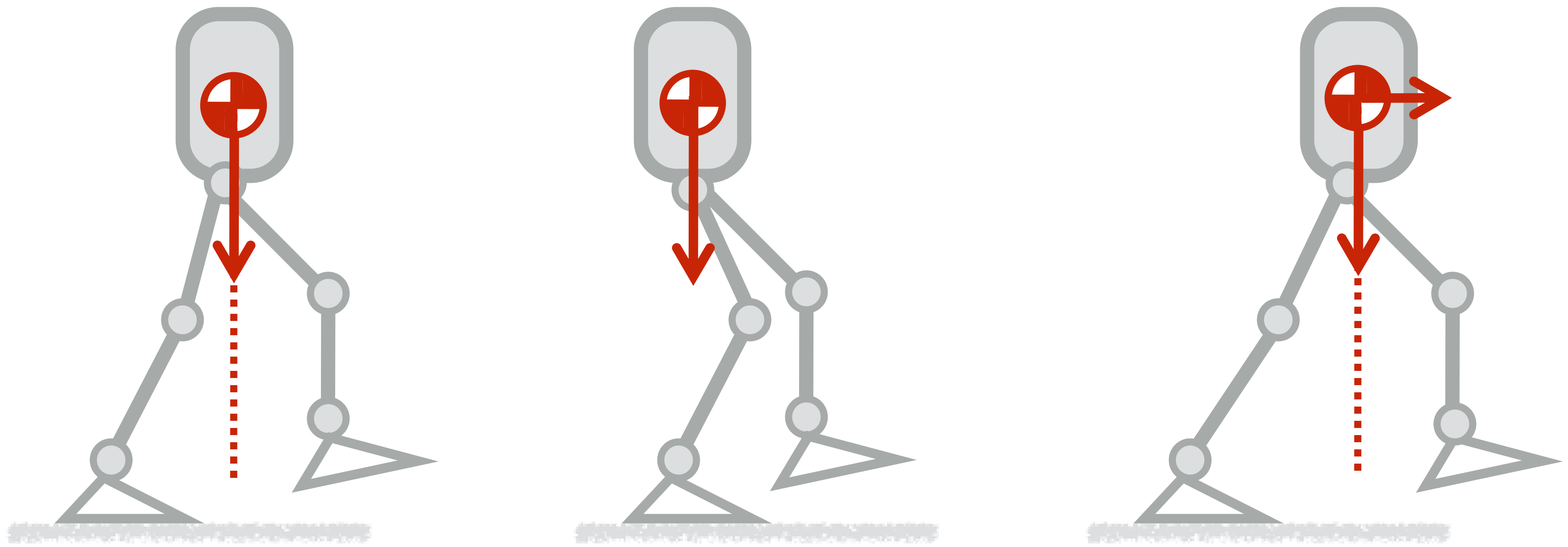
Ground Reaction Force (GRF): 2D components (x,z)

hyp: no torque around the CoM

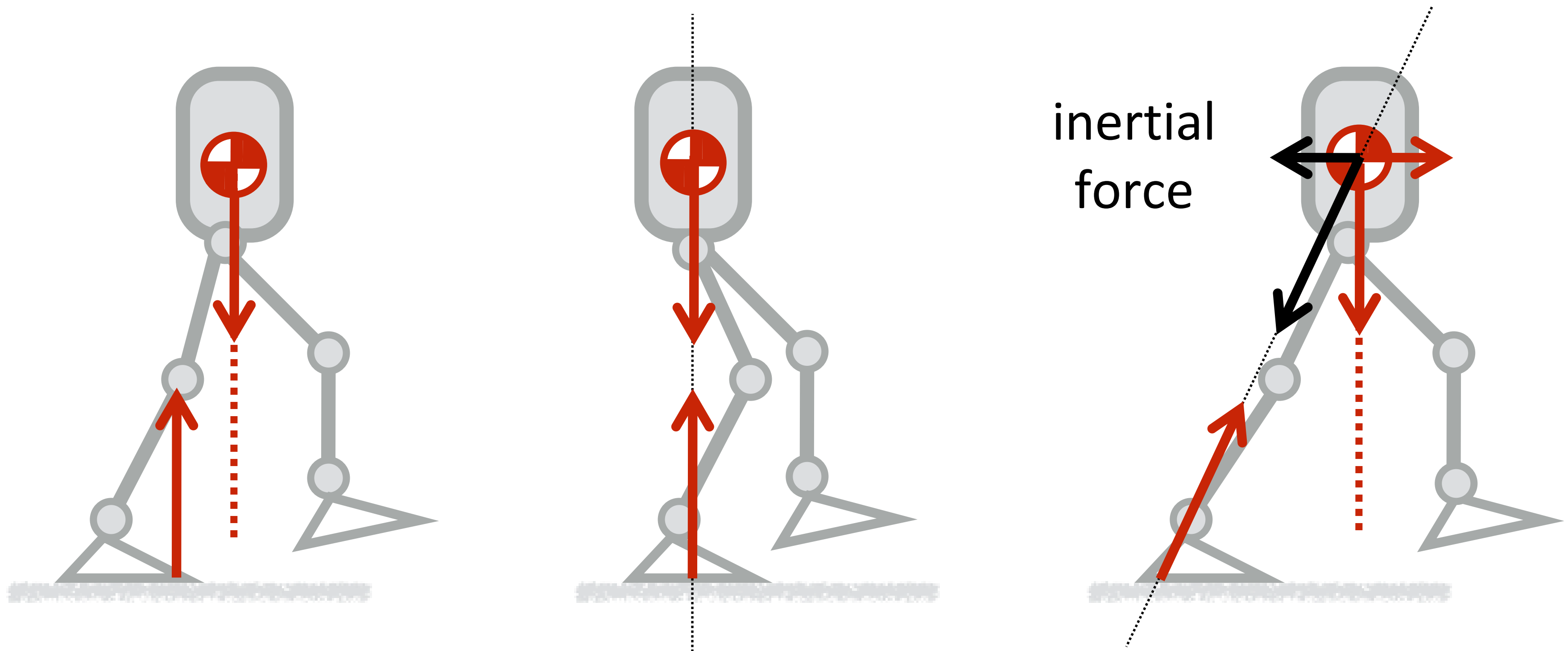
from the previous derivation:



which robot falls down?



which robot falls down?



falls

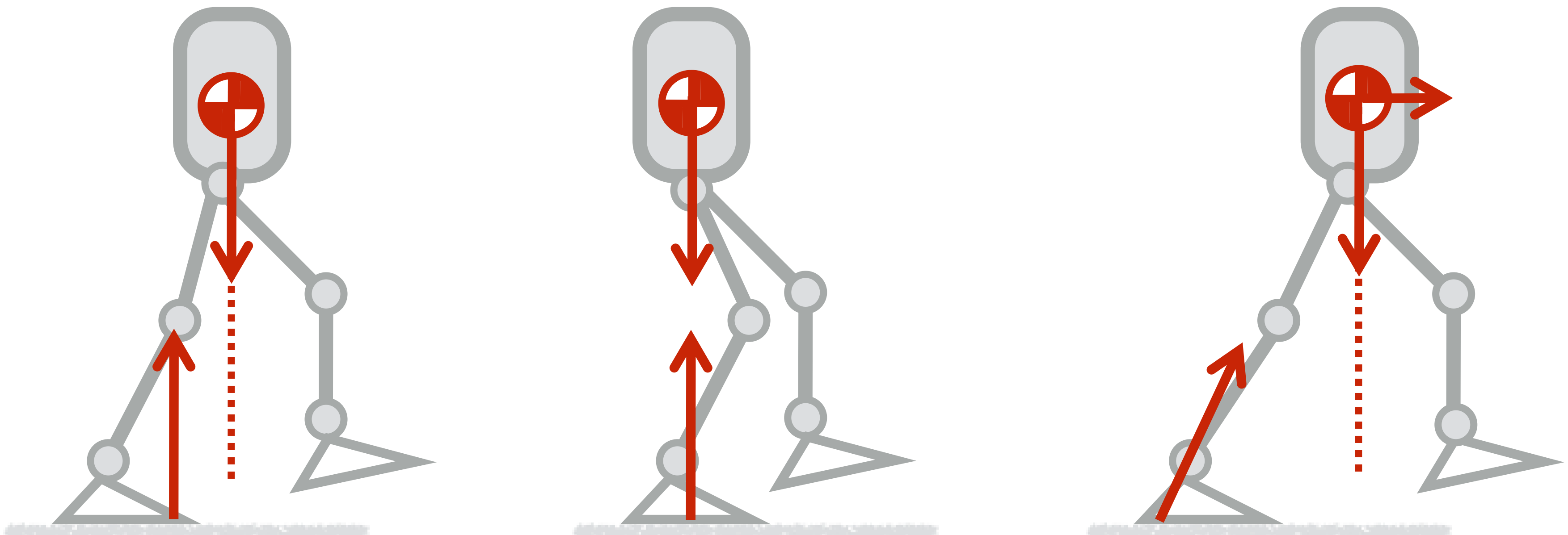
statically balanced

dynamically balanced

where is the ZMP?

z^x (ZMP): point on the ground where the GRF is applied

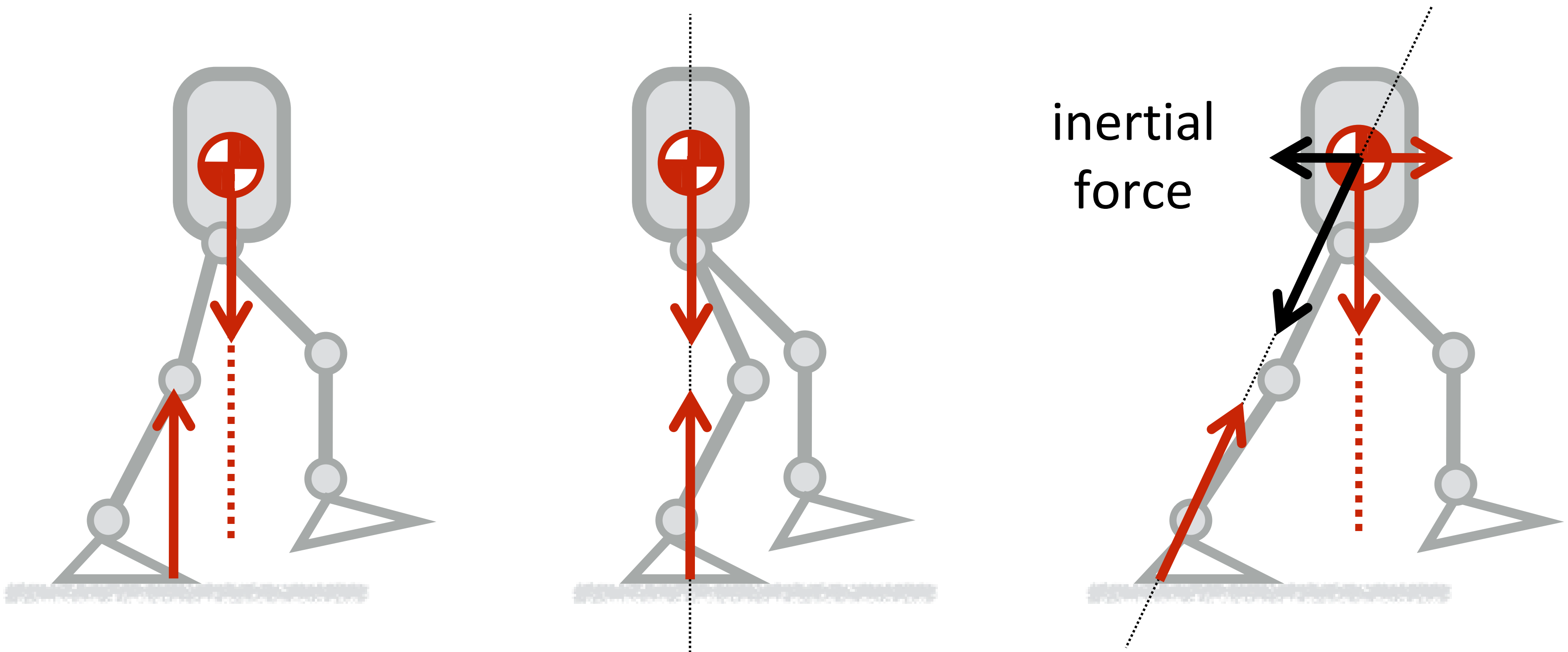
use the dynamics equation on **horizontal** flat ground and neglect $\dot{\boldsymbol{L}}^{x,y}$



$$\frac{c^z}{\ddot{c}^z + g^z}(\ddot{\boldsymbol{c}}^{x,y} + \boldsymbol{g}^{x,y}) = (\boldsymbol{c}^{x,y} - \boldsymbol{z}^{x,y}) + \frac{\boldsymbol{S}\dot{\boldsymbol{L}}^{x,y}}{M(\ddot{c}^z + g^z)}$$

where is the ZMP?

hyp CoM at **constant height** $c^z = \text{constant}$ → LIP equation in the sagittal plane



falls	statically balanced	dynamically balanced
	$z^x = c^x$	$z^x = c^x - \frac{c^z}{g^z}\ddot{c}^x$

dynamically balanced locomotion

generate a gait for walking while maintaining balance

maintaining balance assumed to be equivalent to **not tipping over the support foot**

↓

ZMP-based criterion

←

ZMP needs to stay inside the Support Polygon