Autonomous and Mobile Robotics

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Wheeled Mobile Robots

Motion Control: Regulation

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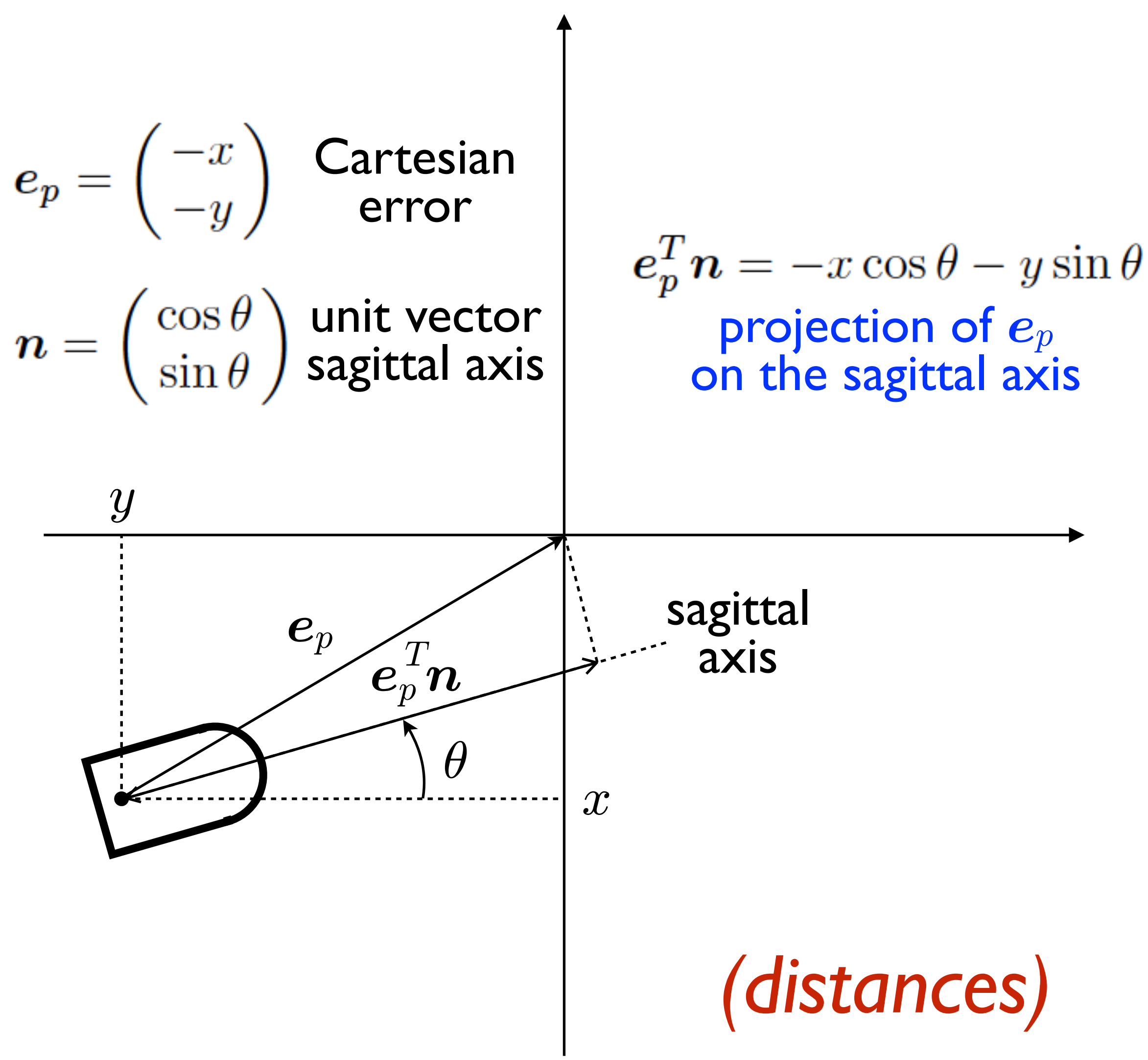
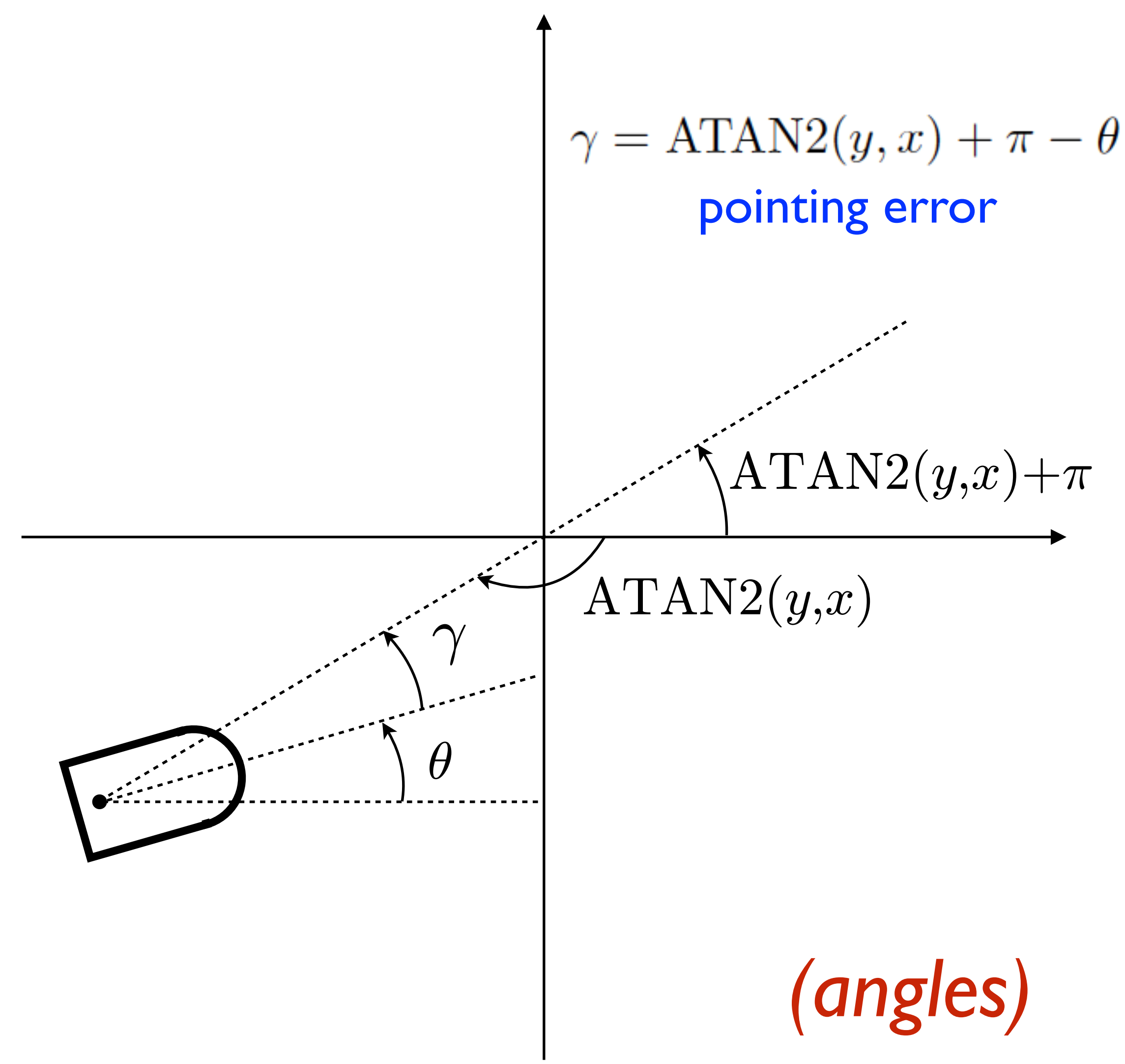
regulation

- drive the unicycle to a desired configuration $\boldsymbol{q}_d$
- the obvious approach (choose a path/trajectory that stops in $\boldsymbol{q}_d$, then track it via feedback) does not work:
  - the controller based on approximate linearization requires persistent trajectories
  - i/o linearization via static feedback would lead point B to the destination rather than the wheel contact point
  - i/o linearization via dynamic feedback requires persistent trajectories
- being nonholonomic, WMRs (unlike manipulators) do not admit universal controllers, i.e., controllers that can stabilize arbitrary trajectories, persistent or not

Cartesian regulation

- drive the unicycle to a given Cartesian position (w.l.o.g., the origin $(0\ 0)$), regardless of orientation
- geometry:





Cartesian regulation

- consider this feedback control law

$$v = -k_1(x\cos\theta + y\sin\theta)$$
$$\omega = k_2(\mathrm{Atan2}(y,x) - \theta + \pi)$$

- geometrical interpretation:
  - v is proportional to the orthogonal projection of the Cartesian error e_p on the sagittal axis
  - ω is proportional to the pointing error (i.e., the difference between the orientation of e_p and that of the unicycle)

- does it work? consider the Lyapunov-like function

$$V = \frac{1}{2}(x^2 + y^2)$$ positive semidefinite (PSD)

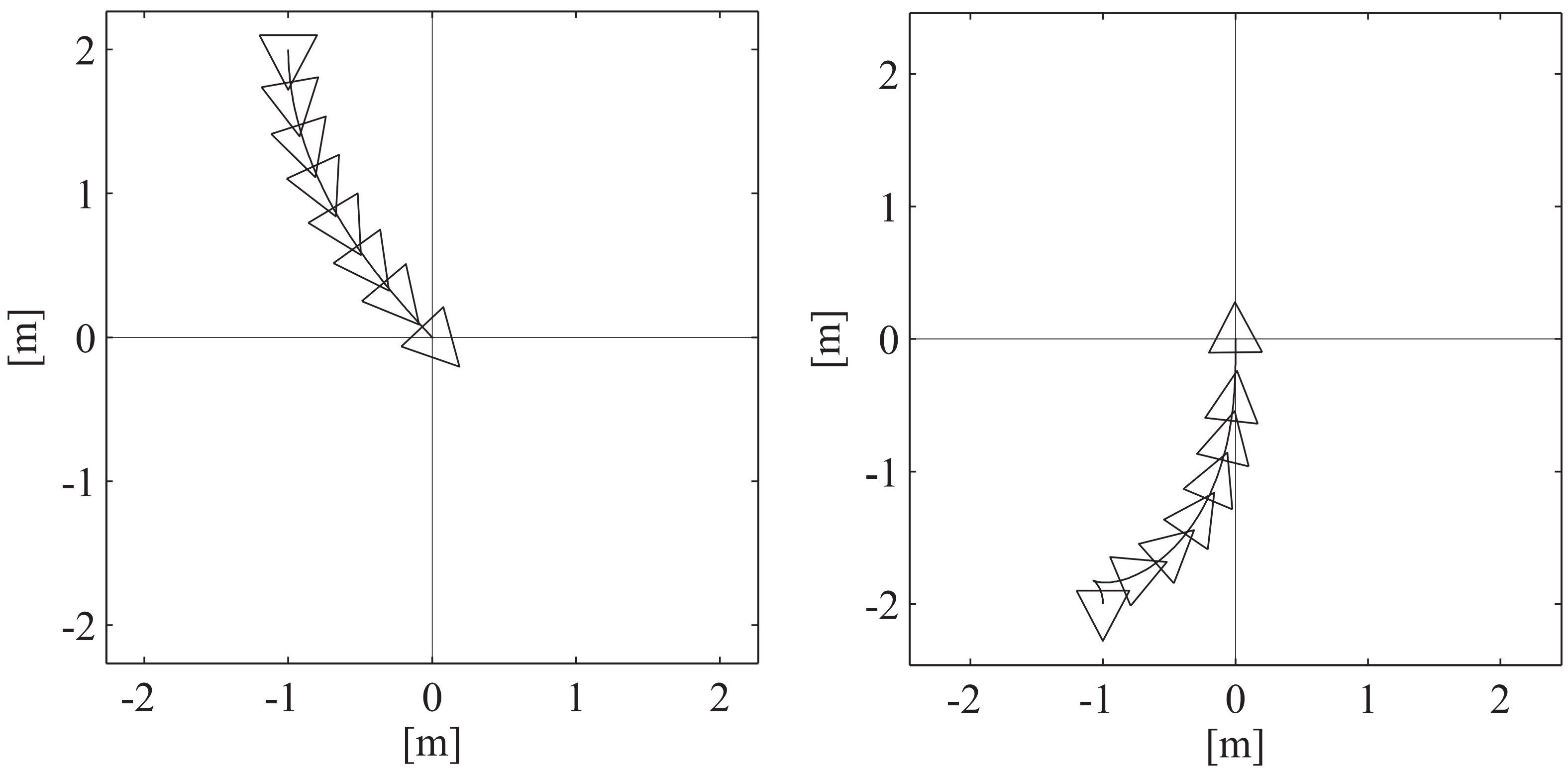
$$\dot{V} = -k_1(x\cos\theta + y\sin\theta)^2$$ negative semidefinite (NSD)

- cannot use LaSalle theorem, but being V PSD, $\dot{V}$ NSD and $\ddot{V}$ bounded (can be shown) we can use Barbalat lemma to infer that $\dot{V}$ tends to zero, i.e.

$$\lim_{t \to \infty} (x\cos\theta + y\sin\theta) = 0$$

- this implies that the Cartesian error goes to zero (the other possibility would be e_p becoming orthogonal to $\boldsymbol{n}$, but this cannot be steady-state since in such configuration it would be $v = 0$ and $\omega = k_2\pi/2$)

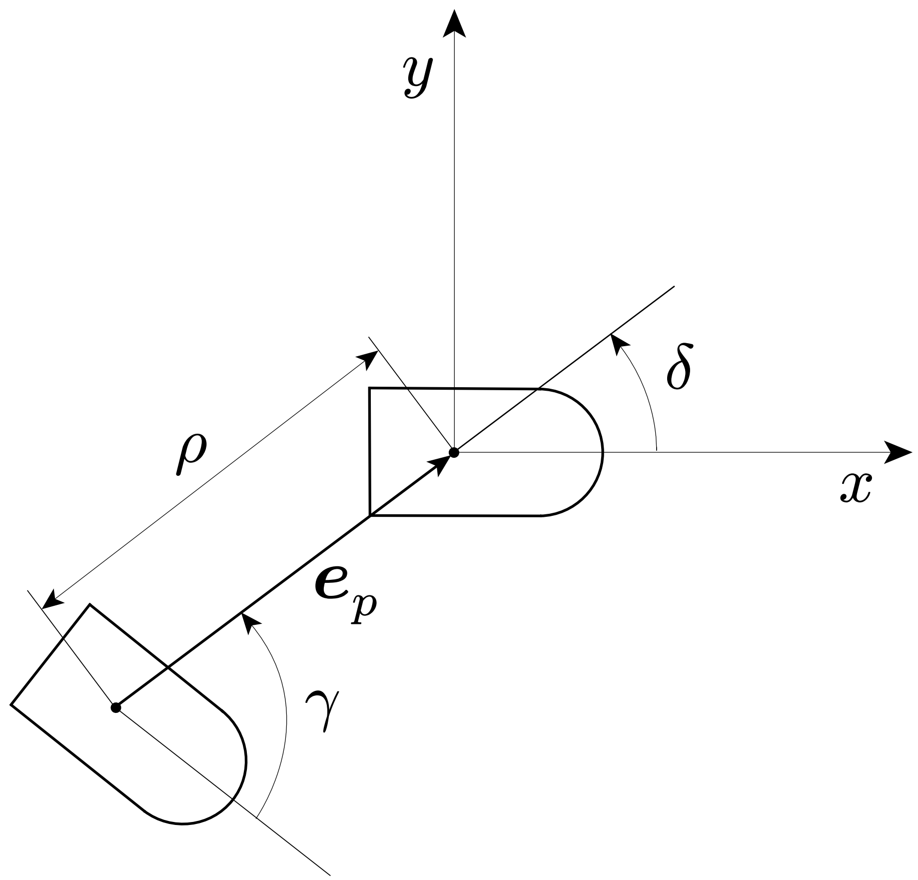
simulation



- final orientation is not controlled
- at most one backup maneuver

posture regulation

- drive the unicycle to a given configuration (w.l.o.g., the origin $(0\ 0\ 0)$)
- convert to polar coordinates



$$\rho = \sqrt{x^2 + y^2}$$
$$\gamma = \text{Atan2}(y, x) - \theta + \pi$$
$$\delta = \gamma + \theta$$

- γ and δ are undefined at the Cartesian origin; however, if ρ, γ and δ converge to zero so do x, y and θ

- kinematic model in polar coordinates

$$\dot{\rho} = -v\cos\gamma$$
$$\dot{\gamma} = \frac{\sin\gamma}{\rho}v - \omega$$
$$\dot{\delta} = \frac{\sin\gamma}{\rho}v$$

note the potential singularity when $\rho = 0$

- consider this control law (compare with previous)

$$v = k_1\rho\cos\gamma$$
$$\omega = k_2\gamma + k_1\frac{\sin\gamma\cos\gamma}{\gamma}(\gamma + \delta)$$

new term

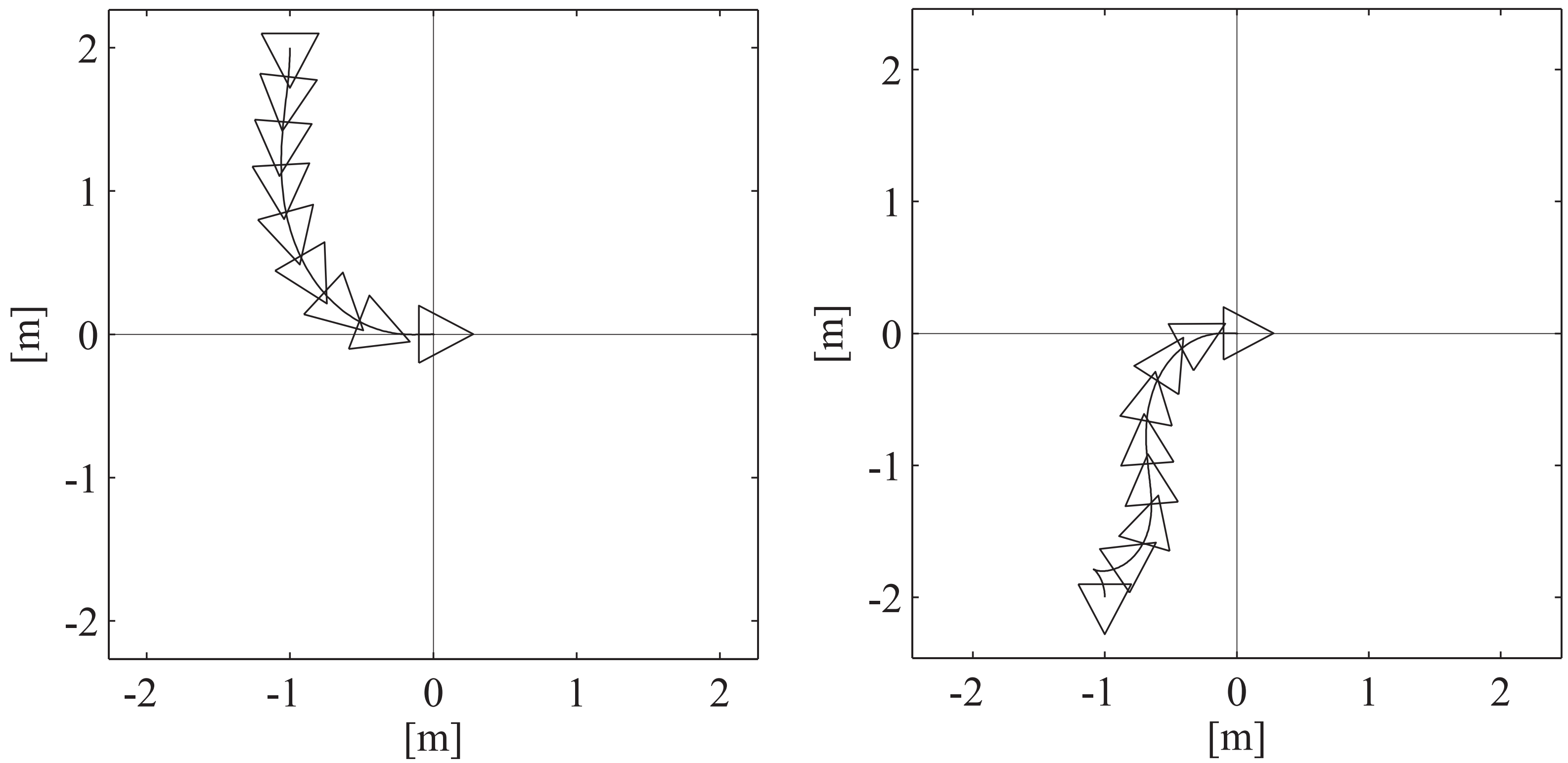
- does it work? consider the Lyapunov candidate

$$V = \frac{1}{2}\left(\rho^2 + \gamma^2 + \delta^2\right) \qquad \text{positive definite}$$

$$\dot{V} = -k_1 \cos^2\gamma\, \rho^2 - k_2\, \gamma^2 \qquad \text{negative semidefinite}$$

- Barbalat lemma implies that $\dot{V}$ goes to zero, i.e., both ρ and γ go to zero; in turn, this can be shown to imply that also δ goes to zero
- the above control law, once mapped back to the original coordinates, is discontinuous at the origin
- it can be shown that, due to the nonholonomy, all posture stabilizers must be discontinuous w.r.t. the state or time-varying (Brockett theorem)

simulation



- final orientation is zeroed as well
- at most one backup maneuver