

Verification of Deployed Artifact Systems via Data Abstraction

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Overview

- ① Motivation: Artifact Systems
- ② Verification of infinite-state data-aware systems
- ③ Key contribution: decidability under boundedness assumption
- ④ Application of the result
- ⑤ Conclusion and future directions

Artifact and Artifact Systems

Recent paradigm for Business Process modeling and development [CH09]

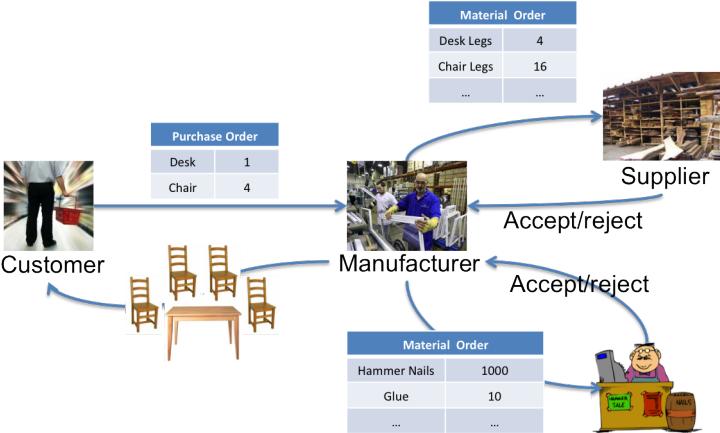
- *Artifact*: information model + lifecycle
 - ▶ (Nested) records equipped with actions
- *Artifact System*: set of interacting artifacts

Features:

- Data and processes are given same emphasis
 - ▶ data affect the actions to execute
 - ▶ actions affect data (content and structure)
- Modularized approach (sort of Object-Orientation)
 - ▶ focus on one artifact at a time

Artifact Systems

Motivating Scenario



Artifact Systems

Motivating Scenario (cont.)

CPO

| <i>id</i> | <i>customer_id</i> | <i>product_code</i> | <i>status</i> |
|-----------|--------------------|---------------------|---------------|
|-----------|--------------------|---------------------|---------------|

- *createPO(id, cid, code)*
- *deletePO(id)*
- *addItemPO(id, itm, qty)*
- ...

WO

| <i>id</i> | <i>cpo</i> | <i>line_itms</i> | <i>status</i> |
|-----------|------------|------------------|---------------|
|-----------|------------|------------------|---------------|

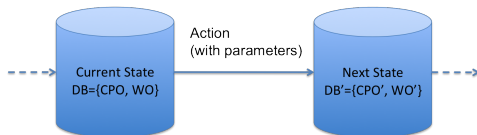
- *createWO(id, cpo)*
- *deleteWO(id)*
- *addLineItemWO(id, mat, qty)*
- ...

Artifact Systems

- As the process goes on, artifact actions are executed
 - ▶ e.g., the Customer Purchase Order is sent to the Manufacturer.
- Actions add/remove artifacts or change artifact attributes
 - ▶ e.g., the CPO status changes from *created* to *submitted*

The whole system can be seen as a *data-aware* dynamic system

- At every step, an action yields a change in the current state



Framework

Preliminaries

Preliminary (standard) notions and notation

- A database schema is a set $\mathcal{D} = \{P_1/a_1, \dots, P_n/a_n\}$ of relation symbols P_i , each with its *arity* a_i
- A \mathcal{D} -interpretation (or *instance*) over (possibly infinite) U is a mapping associating each P_i with a finite a_i -ary relation $D(P_i) \subseteq U^{a_i}$
- Active domain: $adom(D) \subseteq U$ is the (finite) set of all distinct elements occurring in D
- First-Order formulas/sentences are syntactically defined as usual but evaluated under active-domain semantics:
 - ▶ *quantified variables range over the active domain*

Framework

Artifact Systems: Syntax

How do we describe an Artifact System?

Definition (Artifact System)

An *Artifact System* is specified as a tuple $\mathcal{S} = \langle \mathcal{D}, U, D_0, \Phi \rangle$, where:

- $\mathcal{D} = \{P_1/a_1, \dots, P_n/a_n\}$ is a *database schema*
- U is a possibly infinite *interpretation domain*
- D_0 is an *initial* \mathcal{D} -instance over U
- Φ is a finite set of *parametric actions* of the form $\alpha(\vec{x}) = \langle \pi(\vec{y}), \psi(\vec{z}) \rangle$, where:
 - ▶ $\alpha(\vec{x})$ is the *action signature* and \vec{x} the set of its *formal parameters*
 - ▶ $\vec{x} = \vec{y} \cup \vec{z}$
 - ▶ $\pi(\vec{y})$ is a FO-formula over \mathcal{D} called the *action precondition*
 - ▶ $\psi(\vec{z})$ is a FO-formula over $\mathcal{D} \cup \mathcal{D}'$ called the *action postcondition*, where $\mathcal{D}' \doteq \{P'_1/a_1, \dots, P'_n/a_n\}$

Framework

Artifact Systems: Semantics

Definition (Model of an Artifact System)

Given an *Artifact System* $\mathcal{S} = \langle \mathcal{D}, U, D_0, \Phi \rangle$, its *model* is the Kripke structure $\mathcal{K} = \langle \Sigma, D_0, \tau \rangle$, where:

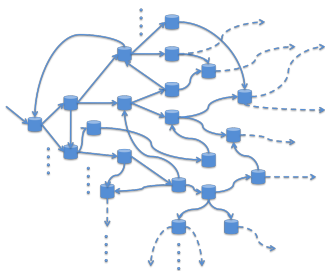
- $\Sigma \subseteq \mathcal{I}_{\mathcal{D}}(U)$ is the set of *states* ($\mathcal{I}_{\mathcal{D}}(U)$: all instances of \mathcal{D} over U)
- $D_0 \in \Sigma$ is the *initial state*
- $\tau : \Sigma \rightarrow \Sigma$ is the *transition relation* s.t. $\tau(D, D')$ iff for some α there exists an execution $\alpha(\vec{u}) = \langle \pi(\vec{v}), \psi(\vec{w}) \rangle$ such that:
 - ▶ $\text{adom}(D') \subseteq \text{adom}(D) \cup \{w_1, \dots, w_\ell\} \cup \text{const}(\psi)$
 - ▶ $D \models \pi(\vec{v})$, i.e., the action is *enabled*
 - ▶ $D \oplus D' \models \psi(\vec{w})$, where $D \oplus D'$ interprets unprimed symbols as in D and primed ones as in D' .

NOTE: First-Order formulas evaluated under *active-domain* semantics.

Framework

Intuition

- Each state is a \mathcal{D} -instance
- As actions are executed, new states are reached
- Action parameters can introduce new values
- Infinite U yields potentially infinitely many distinct states
- In general, infinite branching and infinite run-length



The Problem

Intuition

Check whether all possible system evolutions satisfy a desired property

- Does the system satisfy a (branching-time) *temporal* specification?
E.g.:
 - ▶ It is always the case that every artifact can be deleted
 - ▶ There exists a way to create a certain number of artifacts
 - ▶ A product can be shipped to the customer only after assemblage
- Flavor of Model Checking, but:

Relational states + infinite interpretation domain = infinite state space!

Verification Formalism: FO-CTL

Syntax

How to specify system properties?

Definition (Syntax of FO-CTL over \mathcal{S})

$$\varphi ::= \phi \mid \varphi \wedge \varphi \mid \neg \varphi \mid AX\varphi \mid A\varphi U\varphi \mid E\varphi U\varphi,$$

where ϕ is a FO-sentence over \mathcal{D} and U .

(Other operators derived as usual)

Essentially, CTL with propositional formulas replaced by FO *sentences*

E.g.:

- $\varphi_{ship} = AG \forall c (shippedCPO(c) \rightarrow \forall m (related(c, m) \rightarrow shippedMPO(m)))$
- $\varphi_{t+} = EF \exists x_1, \dots, x_{t+1} \bigwedge_{i \neq j} x_i \neq x_j$
- $\varphi_{empty} = AG EF (emptyCPO \wedge emptyWO \wedge emptyMPO)$

Verification Formalism: FO-CTL

Semantics

(A run r is a sequence of successor states. $r(i)$ selects the i -th r -state.)

Definition (Semantics of FO-CTL over \mathcal{S})

Let \mathcal{K} be the model of \mathcal{S} and $D \in \Sigma$ a \mathcal{K} -state.

$(\mathcal{K}, D) \models \varphi$ iff $D \models \varphi$, if φ is an FO-sentence;

$(\mathcal{K}, D) \models \neg\varphi$ iff $(\mathcal{K}, D) \not\models \varphi$;

$(\mathcal{K}, D) \models \varphi \rightarrow \psi$ iff $(\mathcal{K}, D) \not\models \varphi$ or $(\mathcal{K}, D) \models \psi$;

$(\mathcal{K}, D) \models AX\varphi$ iff for all \mathcal{K} -runs r s.t. $r(0) = D$, $(\mathcal{K}, r(1)) \models \varphi$;

$(\mathcal{K}, D) \models A\varphi U\psi$ iff for all \mathcal{K} -runs r s.t. $r(0) = D$, $\exists k \geq 0$ s.t. $(\mathcal{K}, r(k)) \models \psi$
and $\forall j$ s.t. $0 \leq j < k$, $(\mathcal{K}, r(j)) \models \varphi$;

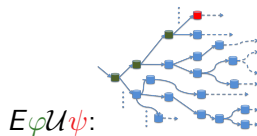
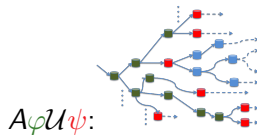
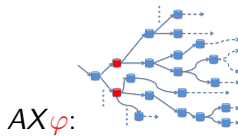
$(\mathcal{K}, D) \models E\varphi U\psi$ iff for some \mathcal{K} -run r , $r(0) = D$, $\exists k \geq 0$ s.t. $(\mathcal{K}, r(k)) \models \psi$,
and $\forall j$ s.t. $0 \leq j < k$, $(\mathcal{K}, r(j)) \models \varphi$.

A formula φ is *true* in \mathcal{K} , written $\mathcal{K} \models \varphi$, if $(\mathcal{K}, D_0) \models \varphi$.

\mathcal{S} satisfies φ , written $\mathcal{S} \models \varphi$, if $\mathcal{K} \models \varphi$.

FO-CTL Semantics

Intuition



Verification of Artifact Systems

General Formulation

- *Model Checking problem for Artifact Systems:*

Given \mathcal{S} and φ , does $\mathcal{S} \models \varphi$ hold?

- ▶ Similar to Model Checking but technically more challenging
 - ★ Relational states
 - ★ Infinite state-space

Theorem

The MC problem for Artifact Systems is undecidable.

- ▶ BUT decidable over finite interpretation domains:
 - ★ by reduction to standard propositional case (*propositionalise* FO facts).

Verification of Bounded Artifact Systems

- Here we devise a notable case of decidability
- If all the \mathcal{D} -instances (states) of the system are **bounded**, then, though **infinite-state**, model-checking the system is decidable.

Bounded Artifact System

Definition (b -Bounded (Artifact) System)

Consider a system $\mathcal{S} = \langle \mathcal{D}, U, D_0, \Phi \rangle$, and a bound $b \in \mathbb{N}$ such that $b \geq |D_0|$. \mathcal{S} is b -bounded if its model $\mathcal{K}_b = \langle \Sigma_b, D_0, \tau_b \rangle$ is such that

- for every $D \in \Sigma_b$, $|D| \leq b$

Verification of Bounded Artifact Systems

We consider the following problem:

- *Model Checking of Bounded Artifact Systems:*

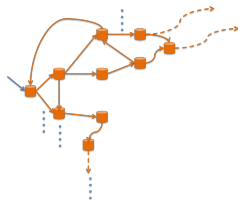
Given a b -bounded artifact system S and a property φ , does $\mathcal{K}_b \models \varphi$?

Verification of Bounded Artifact Systems

Cont.

As a result of the infinite interpretation domain, we still have:

- Infinite branching
- Infinite state-space



QUESTIONS:

- Is the problem decidable?
- 🖱️ How can we model-check a bounded system?

Non-trivial! (we cannot *construct* the (infinite) model)

Abstract System

Definition

Given a b -bounded system $\mathcal{S} = \langle \mathcal{D}, U, D_0, \Phi \rangle$ and a property φ , the (b, φ) -bounded Abstract System of \mathcal{S} is the Artifact System

$\hat{\mathcal{S}}_{b,\varphi} = \langle \mathcal{D}, \hat{U}, D_0, \Phi \rangle$, s.t. $\hat{U} = \mathcal{C}_{\mathcal{S},\varphi} \cup \hat{\mathcal{C}}$, with:

- $\mathcal{C}_{\mathcal{S},\varphi} = \text{const}(\varphi) \cup \bigcup_{\phi \in \Phi} \text{const}(\phi)$
- $\hat{\mathcal{C}} \cap \mathcal{C}_{\mathcal{S},\varphi} = \emptyset$
- $|\hat{\mathcal{C}}| = b + v$, with $v = \max_{\phi \in \Phi} \{|\text{vars}(\phi)|\}$

Intuition:

- $\hat{\mathcal{S}}_{b,\varphi}$ analogous to \mathcal{S} except for $U \neq \hat{U}$
- \hat{U} contains:
 - ▶ all constants mentioned in \mathcal{S} and φ
 - ▶ enough distinct abstract symbols to “fill” the bound and have “fresh” actual parameters for action executions
- \hat{U} is finite!

Abstract System Verification

- Obviously, $\hat{S}_{b,\varphi} \models \varphi$ is decidable, as \hat{U} is finite
- But we want to check whether $\mathcal{K}_b \models \varphi$
- So, what is the relationship between \mathcal{K}_b and $\hat{S}_{b,\varphi}$?

Theorem

Consider a b -bounded system S with U infinite, and a FO-CTL specification φ .^a If $\hat{S}_{b,\varphi}$ is the (b, φ) -bounded abstract system of S then

$$\mathcal{K}_b \models \varphi \Leftrightarrow \hat{\mathcal{K}}_{b,\varphi} \models \varphi,$$

where:

- \mathcal{K}_b is the model of S , and
- $\hat{\mathcal{K}}_{b,\varphi}$ is the model of $\hat{S}_{b,\varphi}$.

^aIn fact for the whole FO μ -calc

Complexity

- Upper bound:

$$\mathcal{O}(2^{|\hat{U}|^a + |\hat{U}||\varphi|})$$

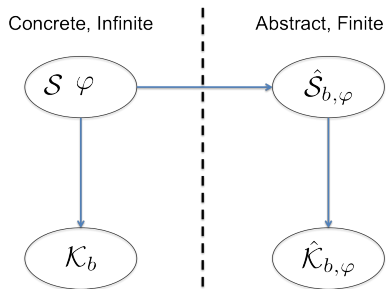
- Technique based on reduction to propositional CTL MC (viable as abstract interpretation domain finite)
- $\hat{\mathcal{K}}_{b,\varphi}$ -states propositionalised (single exponential wrt $|\hat{U}| = b + v + |C_{S,\varphi}|$ but doubly wrt a)
- Quantifiers eliminated from φ (single exponential in $|\varphi|$)

Observations:

- Not far from similar results ([DSV07, DHPV09, BCD⁺11])
- Some performing well in practice ([DSV07])
- Non-optimal technique

Abstract System Verification

Technique



- $\hat{\mathcal{K}}_{b, \varphi} \models \varphi$ can be reduced to standard MC
- We have an actual technique to model-check \mathcal{K}_b !

Data Abstraction

$$\mathcal{K}_b \models \varphi \Leftrightarrow \hat{\mathcal{K}}_{b,\varphi} \models \varphi$$

- What's behind the scene?
- How did we get rid of an infinite number of elements and transitions?

We applied an *abstraction process* based on two formal notions:

- 1 *Isomorphism* between DB instances
- 2 *Bisimulation* between Kripke structures

Data Abstraction

Isomorphic instances

Definition (C-isomorphic \mathcal{D} -instances)

Two \mathcal{D} -instances D and \hat{D} , respectively over U and \hat{U} , are said C-isomorphic, for $C \subseteq U, \hat{U}$, written $D \sim_C \hat{D}$, iff there exists a bijection $i : \text{atom}(D) \cup C \mapsto \text{atom}(\hat{D}) \cup C$ that is the identity on C , and such that for every $j = 1, \dots, n$, and for every $\vec{u} \in \text{atom}(D)^{a_j}$, $D \models P_j(\vec{u}) \Leftrightarrow \hat{D} \models P_j(i(\vec{u}))$, where $i(\vec{u}) \doteq \langle i(u_1), \dots, i(u_{a_j}) \rangle$.

In words: Instances obtained by uniformly renaming the elements not in C
E.g., for $C = \{1\}$, $i(1) = 1$, $i(2) = a$, $i(3) = b$, $i(4) = c$.

| D | | | \hat{D} | | |
|-----|---|---|-----------|---|---|
| 1 | 2 | 3 | 1 | a | b |
| 2 | 4 | 3 | a | c | b |

Data Abstraction

Isomorphic instances (cont.)

Isomorphic instances have a notable (well-known) property:

Lemma

If $D \sim_C \hat{D}$ then for every FOL φ s.t. $\text{const}(\varphi) \subseteq C$, $D \models \varphi \Leftrightarrow \hat{D} \models \varphi$.

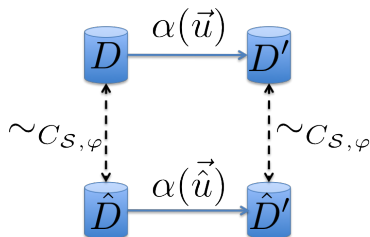
| | | |
|------|-------|--------|
| 1 | blue | orange |
| blue | green | orange |

- The “coloured instance” satisfies φ iff all the instances isomorphic to it do
- The “coloured” instance stands for infinitely many isomorphic instances (*isomorphism type*):
 - ▶ same values iff same colours
- IDEA: No FO (sub-)formula from \mathcal{S} or φ can distinguish two $C_{\mathcal{S},\varphi}$ -isomorphic instances
- Observation: for given b , only finitely many isomorphism types

Crux of the Result

Theorem

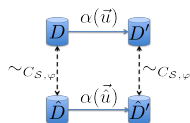
If $D \sim_{C_{S,\varphi}} \hat{D}$, every concrete transition $\langle D, D' \rangle$ has an abstract counterpart $\langle \hat{D}, \hat{D}' \rangle$ s.t. $D' \sim_{C_{S,\varphi}} \hat{D}'$, and viceversa.



- Execution: $\alpha(\vec{u}) = \langle \pi(\vec{v}), \psi(\vec{w}) \rangle$

Crux of the Result

If-Part (Intuition)



Need to prove that there exist $\vec{\hat{v}}, \vec{\hat{w}}, \hat{D}'$ s.t.

(i) $\hat{D} \models \pi(\vec{\hat{v}})$, (ii) $\hat{D} \oplus \hat{D}' \models \psi(\vec{\hat{w}})$, and (iii) $D' \sim_{C_{S,\varphi}} \hat{D}'$

- See \vec{u} as a (1-tuple) relation
- We can prove that there exists \hat{D}' and $\vec{\hat{u}}$, and a $C_{S,\varphi}$ -isomorphism between

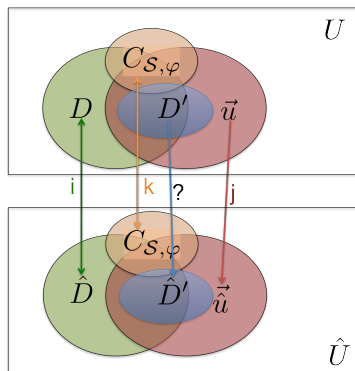
$$\{D, D', \vec{u}\} \text{ and } \{\hat{D}, \hat{D}', \vec{\hat{u}}\}$$

- This is enough, as π and φ are invariant wrt $C_{S,\varphi}$ -isomorphic instances

Crux of the Result

If-Part (Intuition) Cont.

$C_{S,\varphi}$ -isomorphism between $\{D, D', \vec{u}\}$ and $\{\hat{D}, \hat{D}', \vec{\hat{u}}\}$:



- 1 obtain $\vec{\hat{u}}$ by renaming the elements in \vec{u} according to i , k , and preserving (in)equalities – \hat{U} contains enough elements
- 2 obtain \hat{D}' by renaming the elements in D' according to i and j

Data Abstraction

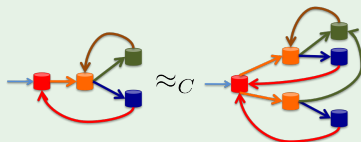
Bisimilar Kripke Structures

Definition (C -bisimilar Kripke structures)

Given $\mathcal{K} = \langle \Sigma, D_0, \tau \rangle$, $\hat{\mathcal{K}} = \langle \hat{\Sigma}, \hat{D}_0, \hat{\tau} \rangle$, and C , \mathcal{K} and $\hat{\mathcal{K}}$ are C -bisimilar ($\mathcal{K} \approx_C \hat{\mathcal{K}}$) iff there exists a relation $R \subseteq \Sigma \times \hat{\Sigma}$, called C -preserving bisimulation, s.t. $\langle D_0, \hat{D}_0 \rangle \in R$, and if $\langle D, \hat{D} \rangle \in R$ then:

- $D \sim_C \hat{D}$;
- for all D' s.t. $\tau(D, D')$ there exists \hat{D}' s.t. $\hat{\tau}(\hat{D}, \hat{D}')$ and $\langle D', \hat{D}' \rangle \in R$;
- for all \hat{D}' s.t. $\hat{\tau}(\hat{D}, \hat{D}')$ there exists D' s.t. $\tau(D, D')$ and $\langle D', \hat{D}' \rangle \in R$.

Example



Data Abstraction

Bisimilar Kripke Structures (cont.)

Lemma

If $\mathcal{K} \approx_C \hat{\mathcal{K}}$, for every FO-CTL (μ -calc) sentence φ such that $\text{const}(\varphi) \subseteq C$,

$$\mathcal{K} \models \varphi \Leftrightarrow \hat{\mathcal{K}} \models \varphi.$$

That is, C -bisimilar Kripke structures cannot be distinguished by FO-CTL formulas using only constants from C . Thus

- If $\hat{\mathcal{K}}$ is finite-state, we are able to check whether $\mathcal{K} \models \varphi$
- (In this case each $\hat{\mathcal{K}}$ transition abstracts infinitely many \mathcal{K} -transitions)

Back to the Abstract System

Lemma

Consider a b -bounded $S = \langle \mathcal{D}, U, D_0, \Phi \rangle$, and a FO-CTL formula φ .

Let:

- $\hat{S}_{b,\varphi} = \langle \mathcal{D}, \hat{U}, D_0, \Phi \rangle$ be the (b, φ) -bounded abstract system of S
- \mathcal{K}_b be the model of S
- $\hat{\mathcal{K}}_{b,\varphi}$ be the model of $\hat{S}_{b,\varphi}$

Then

$$\mathcal{K}_b \approx_{C_{S,\varphi}} \hat{\mathcal{K}}_{b,\varphi}$$

Proof by induction:

- base case: D_0 is $C_{S,\varphi}$ -isomorphic wrt itself
- induction step: crux of the result shown above

Given a b -bounded S and φ ,

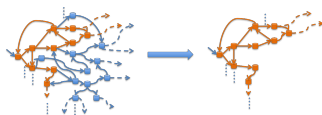
$$\mathcal{K}_b \models \varphi \Leftrightarrow \hat{\mathcal{K}}_{b,\varphi} \models \varphi$$

Application to the General Case

Preservation Theorem

- What if \mathcal{S} is unbounded? (Apart from undecidability)

Observation: for fixed b , the (b, φ) -bounded abstract system $\mathcal{S}_{b,\varphi}$ corresponds to an (infinite) fragment of \mathcal{S}



Preservation theorem for the *existential fragment* FO-ECTL.

$$\varphi ::= \phi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid EX\varphi \mid E\varphi U\varphi$$

Theorem

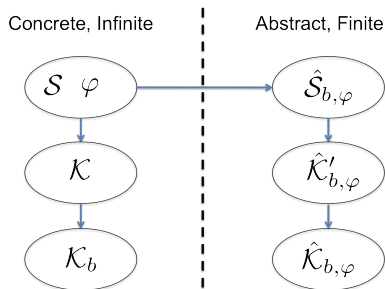
Given \mathcal{S} , $b \geq |D_0|$, and a FO-ECTL formula φ , if $\hat{\mathcal{K}}_{b,\varphi} \models \varphi$ then $\mathcal{S} \models \varphi$.

Observe we can iterate on b

Application to Deployed Systems

What if \mathcal{S} is unbounded?

- Actual machines are memory-bounded
- Executed artifact systems cannot exceed the memory bound
- We can verify the artifact system up to a given bound



Technically requires an additional step, but conceptually same approach as for bounded systems

Conclusion

- Problem originating in the context of Business Processes
- Related to verification of database-driven systems (cf. ICDT 09)
- Contribution to scarcely investigated field (verification of processes in presence of data)
- Abstraction-based approach to bounded verification
 - ▶ Decidability
 - ▶ Actual technique, complete wrt bounded version
 - ▶ Practically relevant: any system runs on an actual, memory-bounded machine
- Partial solution to general case:
 - ▶ satisfied FO-ECTL properties preserved from abstract bounded to concrete unbounded system
- High complexity, but:
 - ▶ comparable to similar work (sometime good practical performance)
 - ▶ current technique non-optimal, space for improvements
 - ★ e.g., CEGAR [CGL94] applied to the abstract system?

Future Directions

- 1 Quantification across modal operators (bounded case)
 - ▶ $AG\ EF\ \forall x\exists y.P(x, y)$ ✓
 - ▶ $AG\ \forall x\ EF\exists y.P(x, y)$? Ongoing
 - ★ Decidable? We conjecture so! (FO-CTL with active-domain quantification)
 - ★ Complexity? (at least) double exponential
- 2 Extension to MAS, in the context of Quantified Interpreted Systems [BL09, BLP11]
 - ▶ Agents capture the actors that execute the actions
 - ▶ Epistemic operators: K (Ongoing), C , D
- 3 Transfer results to settings with similar (low-level) semantics:
 - ▶ E.g., Situation Calculus Ongoing.
- 4 Unbounded systems: what for formulas practically relevant?

Questions?

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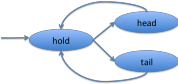


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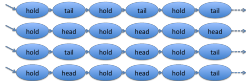
Model Checking

In one slide

Problem: check whether a finite-state *transition-system* satisfies a *temporal specification*[CGP00]

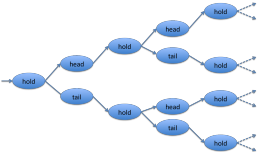


Linear-time: the system defines (infinite-length) runs



E.g., LTL: $\Box \Diamond hold, \neg \Box \Diamond tail$

Branching-time: the system defines an (infinite-depth) tree



E.g., CTL: $AG(hold \rightarrow EX(head) \wedge EX(tail))$

Model Checking

(Well... two!)

Model Checking for *finite systems* is very well understood

The main challenge is *efficiency*, not decidability.

- CTL:
 - ▶ Check whether the property holds over the generated tree
 - ▶ PTIME-complete
- LTL:
 - ▶ Check whether the property holds over the generated runs
 - ▶ PSPACE-complete
- CTL*:
 - ▶ Mixes the above
 - ▶ PSPACE-complete

