Minimum Cycle Bases in Graphs Algorithms and Applications

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A cycle basis of a graph is a family of cycles which spans all cycles of the graph. In an undirected graph, a cycle is simply a set of edges with respect to which every vertex has even degree. We view cycles as vectors indexed by edges. The entry for an edge is one if the edge belongs to the cycle and is zero otherwise. Addition of cycles corresponds to vector addition modulo 2 (symmetric difference of the underlying edge sets). In this way, the cycles of a graph form a vector space and a cycle basis is simply a basis of this vector space. The notion for directed graphs is slightly more involved.

The weight of a cycle is either the number of edges in the cycle (in unweighted graphs) or the sum of the weights of the edges in the cycle (in weighted graphs). A minimum cycle basis is basis of total minimum weight.

The analysis of the cycle space has applications in various fields, e.g., electrical engineering [Kir47], structural analysis [CHR76], biology and chemistry [Gle01], surface reconstruction [GKM$^+$], and periodic timetabling [Lie06]. Some of these applications require bases with special properties [LR07]. In the first part of the talk, I will discuss applications of cycle basis.

In the second part, I turn to construction algorithms. The first polynomial time algorithms for constructing minimum cycle bases in undirected graphs are due to Horton [Hor87] and de Pina [dP95]. Faster realizations of the latter approach are discussed in the papers [BGdV04,KMMP04,MM]. Both approaches can be generalized to directed graphs [LR05,KM05,HKM06,Kav05]. Approximation algorithms are discussed in [KMM07].

Integral cycle basis are required for the application to periodic timetabling. The complexity status of finding minimal integral cycle basis is open. Construction and approximation algorithms are described in [Lie03,Lie06,Kav,ELR07].

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