Arbitration: A Commutative Operator for Belief Revision

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Abstract

The ability of Database Systems to cope with changing situations and information coming from different sources is crucial for their applicability in real-world scenarios. Recent work in the field of belief revision and update has provided us techniques to handle change. In this paper we introduce a different form of revision aiming at capturing the process of "merging" possibly inconsistent pieces of information. We call this process arbitration. Along the lines of Gärdenfors' work, we propose a set of postulates for this operator and prove a representation theorem.

Keywords: knowledge representation, belief revision, non-monotonic reasoning.

1 Introduction

One of the main challenges of today's software and databases systems is their ability to manage a large amount of information coming from different sources and at different moments in time. Advanced databases systems must cope with a changing world and not completely reliable sources of information by adopting a "principled" strategy. In the context of logic-based representations of information, this problem has been analyzed in the fields of databases, artificial intelligence and belief revision.

The main focus of this analysis has been providing principles (postulates) that guide the process of accommodating new information into an existing logical theory. In the field of belief revision, Alchourron, Gärdenfors and Makinson in [1, 6] proposed a set of postulates (the AGM postulates from now on) that any operator revising a knowledge should satisfy. Katsuno and Mendelzon in [7, 8] show that AGM postulates correctly capture the process of acquiring new information about a given scenario, but are inadequate to model the process of acquiring new information about a changing scenario. For this purpose, they introduce a new set of postulates (KM postulates) to model the process of updating a knowledge base.

While the AGM and KM postulates define general properties, actual belief revision and update operators have been proposed by several authors [2, 3, 5, 9, 11, 12].

Both update and revision deal with the problem of accommodating new, and completely reliable, information into an existing body of knowledge. A basic assumption of update and revision is that the new piece of information must be in the revised (or updated) knowledge base.

Suppose that we are in a completely different scenario. There are two different sources of information and each one has a different view of the situation, the two views contradicting each other. if we do not have any reason to consider any of the sources completely unreliable, the best we can do is to "merge" the two views in a new and consistent one, trying to preserve as much information as possible. We call this merging process *arbitration*.

A formal study of the properties of arbitration was first done by Revesz in [10], where he discusses the properties that an arbitration operator should satisfy. In this paper we critically review Revesz's work and propose a different characterization of arbitration. In the style of the work on revision and update, we propose a set of postulates for arbitration and then prove a representation theorem for arbitration operators.

2 Notations and Definitions

All along this paper, we use a propositional language. Lower case letters (a, $b, \ldots$) denote propositional formulas, while the corresponding capital ones (A, $B \ldots$) denote the set of models of the formula. Where this convention cannot be applied, we use $mod(a)$ to denote the set of models of a and $form(A)$ to denote one formula whose models are A, furthermore Cn denotes the closure under the consequence operator of propositional logic (i. e. $Cn(a) = \{b | a \vdash b\}$). Notice that these notational conventions differ from those used by Gärdenfors et. a. in [1, 6], where the authors always consider inputs as set of propositional formulas closed under the consequence relation.

In addition, we denote with $\top$ the set of all interpretations and with $\mathcal{P}(\top)$ its power set and, given a set of models M, $\overline{M}$ denotes the set of models not in M.

Belief revision is mainly concerned with the operations of revision and contraction. Let k be a set of propositional formulas and $Cn(k)$ be its closure under the consequence relation. The revision of $Cn(k)$ with respect to a new formula a, written $Cn(k) * a$, is similar to the expansion $Cn(k) + a$, but the new knowledge base ($Cn(k) * a$) must be consistent if a is consistent. The contraction of a theory $Cn(k)$ with respect to a formula $a \in Cn(k)$, written $Cn(k) - a$, is a subset of $Cn(k)$ that does not contain a.

A new operation, recently introduced by Revesz in [10] is arbitration. The arbitration of two theory A and B is a new theory that "if possible contains" the starting sets A and B.

This operation is different from classical revision, because the revision is not commutative: the second argument (a in the revision $Cn(k) * a$) is more important than the first one. Consider the so-called full meet revision [1]:

$$Cn(k) * a = \begin{cases} Cn(k) + a & \text{if consistent} \\ Cn(a) & \text{otherwise} \end{cases}$$

This is a valid revision operator (i. e. it satisfies all AGM postulates). The theory $Cn(k) * a$ always contains a. But $Cn(k) * a$ must be consistent, so if $Cn(k) + a$ is inconsistent, it doesn't contain all of $Cn(k)$.

In the arbitration, the two operands are considered to be of equal importance, hence the postulate of commutativity will hold: $A \triangle B = B \triangle A$. We can say that arbitration adds two sets of (possibly incompatible) formulas where no one is preferred with respect to the other one.

Example. During a trial, two witnesses give a different version of the facts. In our process of reconstructing the facts we have two choices:

1. We completely trust the first witness, and some of the facts of second witness. If we denote with T_1 the story of the first witness and T_2 the story of the second one, We come to believe the revision $T_2 * T_1$.
2. We believe that any witness may be partially correct. As a consequence, we consider the stories T_1 and T_2 as equally trustworthy. Therefore, we come to believe the arbitration $T_1 \triangle T_2$.

While the idea of arbitration was first introduced by Revesz in [10], the definition he gives is not, in our opinion, completely adequate. Our major objection concerns Revesz definition. In fact, he defines arbitration as a derived notion of *model-fitting*. in our opinion, this is a *particular* definition, not a general theory of arbitration. The other form in which Revesz express his arbitration (based on models) is more clear, but is still a possible and not general operation. A more detailed analysis of Revesz's proposal is reported in Section 6.

3 Postulates

We first introduce the general postulates that any arbitration operator should satisfy and afterwards we discuss in more detail the properties enforced by these postulates.

3.1 Basic Postulates

In the sequel we assume that sets of formulas are represented by the corresponding set of models. This assumption does not change the generality of the work and simplifies the presentation. Therefore, we suppose that arguments A and B of an arbitration are two sets of models, and the result $A \triangle B$ is a set of models.

1. $A \triangle B$ is a set of models
2. $A \triangle B = B \triangle A$
3. $A \cap B \subseteq A \triangle B$
4. if $A \cap B \neq \emptyset$ then $A \triangle B \subseteq A \cap B$
5. $A \triangle B = \emptyset$ only if $A = \emptyset$ and $B = \emptyset$

These are the more "intuitive" and basic properties of arbitration. In fact, postulate 2 warrants that arguments have the same importance, 3 and 4 together warrant that, when the corresponding formulas a and b are not incompatible, then the arbitration gives the intersection of their models (the logical "and"). Notice the strong similarities with AGM postulates 1-6, which provide basic properties of revision. While these postulates enforce reasonable constraints, they are far too weak to characterize it precisely. For this reason, in the next section we introduce more specific postulates.

3.2 More Specific Postulates

More specific properties are:

6 $A \triangle (B \cup C) = \begin{cases} A \triangle B & \text{or} \\ A \triangle C & \text{or} \\ (A \triangle B) \cup (A \triangle C) & \end{cases}$

7 $A \triangle B \subseteq A \cup B$

8 If $A \neq \emptyset$ then $A \cap (A \triangle B) \neq \emptyset$

Postulate 6 guarantees that arbitration can be obtained via the composition of the arbitration of smaller formulas. It is also the natural extension to arbitration of AGM postulates 7 and 8. A very natural requirement for arbitration is that it is more defined than just logically disjoining the formulas. Postulate 7 warrants that the arbitration of a and b logically implies $a \vee b$. As a parallel, AGM postulate 2 of revision, $a \in K * a$, forces the models of $K * a$ to be subsets of the models of a. In arbitration, postulate 2 is replaced by commutativity, so that the result of $A \triangle B$ if free to vary in the set of all models. Without postulate 7, an arbitration allowing an interpretation which is not in A or B, can be a valid arbitration.

Since arbitration is a merging process of two pieces of information, it is a very reasonable constraint imposing that both pieces contribute to the final result. This is obtained via postulate 8, which ensures that at least one model of any of the two (non-empty) sets will be in the arbitration.

Let $a \perp b$ be the set of maximal consistent subset of $a + b$ containing either a or b (each element is $Cn(a)$ with a maximal consistent subset of $Cn(b)$, or $Cn(b)$ with a maximal consistent subset of $Cn(a)$). Postulate 7 has two equivalent representations:

Theorem 1 *If arbitration $\triangle$ satisfies 1-5, then the following statements are equivalent:*

1. *$\triangle$ satisfies 7*
2. *if both $a \vdash c$ and $b \vdash c$ then $A \triangle B \models c$ (if $Cn(k_1)$ and $Cn(k_2)$ are two closed set of formulas, and $Cn(k_3)$ the corresponding arbitration, then $Cn(k_1) \cap Cn(k_2) \subseteq Cn(k_3)$)*
3. *Let $Cn(c)$ denote the closed set of formulas corresponding to $A \triangle B$. Thus, $Cn(c)$ can be expressed as the intersection of elements of $a \perp b$.*

If $\triangle$ satisfies 7-8, then this property holds:

$$\{I\}\triangle\{J\} = \{I, J\} \tag{1}$$

where I and J are complete interpretations. We can also prove that (1) implies both 7 and 8, if arbitration satisfies 1-6.

Theorem 2 *Every arbitration satisfying 1-6 and (1), also satisfies 7 and 8*

Proof: We prove first that such an arbitration satisfies 7.

Let $A = \{A_1, \ldots, A_n\}$ and $B = \{B_1, \ldots, B_m\}$, from 6 follows

$$A\triangle B = \{A_1, \ldots, A_n\}\triangle\{B_1, \ldots, B_m\} = \begin{cases} \{A_1\}\triangle\{B_1\} \\ \vdots \\ \{A_n\}\triangle\{B_m\} \\ \text{unions} \end{cases}$$

Where "unions" denotes any possible union of the previous arbitrations. But $\triangle$ satisfies (1), then $\{A_i\}\triangle\{B_j\} = \{A_i, B_j\}$, for every i, j. As a consequence, for every possible choice we have that $\{A_i\}\triangle\{B_j\} \subseteq A \cup B$.

In order to prove 8, we note that $\{A_i\}\triangle\{B_j\} = \{A_i, B_j\}$, hence the arbitration $A\triangle B$ must contains at least one model of A. $\Box$

3.3 Complexity

An unfortunate, but inevitable, consequence of our postulates is the following result:

Theorem 3 *If $\triangle$ satisfies 1-5, then the problem of deciding whether $A\triangle B \models c$, is both NP and CoNP hard.*

Proof: The formula a is a theorem iff $\top\triangle\top \models a$, where $\top$ is the set of all models. The formula a is contradictory if and only if $\top\triangle A \models a$ $\Box$

For a detailed analysis of the computational complexity of belief revision and update, we refer to the paper [4] by Eiter and Gottlob and [9] by Nebel.

4 Arbitration as Commutative Revision

In this section, we study the properties of arbitration satisfying 1-8. Since these postulates are natural extensions of the AGM ones, we also refer to this form of arbitration as commutative revision. All arbitrations satisfying 1-8 can be expressed as:

$$A\triangle B = min(A, \leq_B) \cup min(B, \leq_A) \tag{2}$$

where $\leq_K$ is a relation upon the set of the sets of models (it compares two set of models). Note that we only need to know $\leq_A$ over singletons (set with only one model).

Theorem 4 *The formula (2) is a valid arbitration if and only if $\leq$ satisfies:*

1. *transitivity: if $A \leq B$ and $B \leq C$ then $A \leq C$*
2. *if $A \subseteq B$ then $B \leq A$*
3. *$A \leq A \cup B$ or $B \leq A \cup B$*
4. *$B \leq_A C$ for every C iff $A \cap C \neq \emptyset$*
5. $A \leq_{C \cup D} B \Leftrightarrow \begin{cases} C \leq_{A \cup B} D \quad \text{and} \quad A \leq_C B & \text{or} \\ D \leq_{A \cup B} C \quad \text{and} \quad A \leq_D B \end{cases}$

and any valid arbitration can be represented by a $\leq$ satisfying 1-5.

Proof (sketch): If $\triangle$ satisfies 1-8, then it is possible to show that $A \cap (A \triangle B)$ is a valid revision (satisfies all AGM postulates). Therefore, we can express:

$$A \cap (A \triangle B) = min(B, \leq_A)$$

and, from 7:

$$A \triangle B = [A \cap (A \triangle B)] \cup [B \cap (A \triangle B)] = min(A, \leq_B) \cup min(B, \leq_A)$$

If $\leq$ satisfies 5, then the corresponding $\triangle$ satisfies 6, and viceversa, if $\triangle$ satisfies 6 then $\leq$ satisfies 5. Postulates 7 and 8 are self-evident. $\Box$

Notice that, given an arbitration operator, the corresponding ordering $\leq_A$ over models is defined as: $I \leq_A J$ iff $I \in A \triangle \{I, J\}$. A simple representation of any commutative revision follows from the next theorem.

Theorem 5 *If we know $X \triangle Y$, where X and Y are sets with at most two interpretations, then one can calculate $A \triangle B$ for any pair of sets of models A and B (if we assume that $\triangle$ satisfies 1-8).*

Proof: Note, first of all, that $\{I\} \triangle \{L, M\}$ define $\leq_{\{I\}}$, and $\{I, J\} \triangle \{L, M\}$ define $\leq_{\{I,J\}}$ as well as $\leq_{\{L,M\}}$.

If $A = \emptyset$ (or $B = \emptyset$), then $A \triangle B = min(B, \leq_\emptyset)$. If A is a singleton (set with only one element), $A = \{I\}$, then $A \triangle B = \{I\} \cup min(B, \leq_{\{I\}})$

Now we can consider A and B a generic couple of sets. From the previous theorem, if we know $\leq_A$ for every set of models A, then we can calculate $A \triangle B$ for any A and B. The relation $\leq_A$ is defined by

$$I \leq_A J \qquad \text{iff} \qquad I \in A \triangle \{I, J\}$$

Thus, we need to know whether $I \in A \triangle \{I, J\}$. From $\{I, J\} \triangle \{L, M\}$ we can derive $\leq_{\{I,J\}}$, and then

$$A \cap (A \triangle \{I, J\}) = min(A, \leq_{\{I,J\}}) = \{A_1, \ldots, A_k\}$$

Now, from 6 we obtain:

$$A \triangle \{I, J\} = \begin{cases} \{A_1\} \triangle \{I, J\} \\ \vdots \\ \{A_n\} \triangle \{I, J\} \\ \text{unions} \end{cases}$$

But $A_i \in \{A_i\} \triangle \{I, J\}$ and the only models of A which are in $A \triangle \{I, J\}$ are $\{A_1, \ldots, A_k\}$. Hence we have

$$A \triangle \{I, J\} = (A_1 \triangle \{I, J\}) \cup \cdots \cup (A_k \triangle \{I, J\})$$

We can now conclude that $I \in A \triangle \{I, J\}$ if and only if $I \in A_i \triangle \{I, J\}$ for some $A_i \in min(A, \leq_{\{I,J\}})$. $\square$

4.1 An arbitration operator

A possible commutative revision is

$$A \triangle_{\leq} B = \bigcup \{\{I\} \triangle \{J\} | \langle I, J \rangle \in min(A \times B, \leq)\}$$

where $\leq$ is a reflexive, transitive and total relation over non ordered pair of interpretations.

Theorem 6 *If $\leq$ is reflexive, transitive, total and satisfies the condition $\langle J, J \rangle = \langle I, I \rangle < \langle I, J \rangle$ for every I, J (if $I \neq J$) then $\triangle_{\leq}$ satisfies 1-6.*

We can obtain 7 imposing $\{I\} \triangle \{J\} \subseteq \{I, J\}$. This revision satisfies also 8 if $\{I\} \triangle \{J\}$ is replaced by $\{I, J\}$. This is a very simple definition, but, unfortunately, there are arbitrations satisfying 1-8 but such that no relation $\leq$ can represent them.

Incidentally, we note that the revision operators of Dalal [3], Satoh [11], Borgida [2] and Forbus [5] can be represented as

$$mod(Cn(k) * a) = \{J | \exists I. \langle I, J \rangle \in min(K \times A, \leq)\}$$

where $\leq$ is defined as follows:

Dalal	$IJ \leq_{Dalal} LM$	iff $\lvert diff(I, J) \rvert \leq \lvert diff(L, M) \rvert$
Borgida	$IJ \leq_{Borgida} LM$	iff $I = L$ and $diff(I, J) \subseteq diff(L, M)$
Satoh	$IJ \leq_{Satoh} LM$	iff $diff(I, J) \subseteq diff(L, M)$
Forbus	$IJ \leq_{Forbus} LM$	iff $I = L$ and $\lvert diff(I, J) \rvert \leq \lvert diff(L, M) \rvert$

Only Dalal's relation is total. The others are reflexive and transitive but not total.

An example of arbitration can be defined by $\triangle_{\leq}$ using one of these relations. Since only Dalal's relation is total (Borgida's and Forbus's ones are relation over *ordered* pairs of interpretations, Satoh's one is not total), it seems the better choice:

$$A \triangle_{Dalal-like} B = \{I, J | \langle I, J \rangle \in min(A \times B, \leq_{Dalal})\}$$

This is very near to Dalal's revision: one can transform a revision into arbitration and viceversa using the following formulas:

$$K * A = (K \triangle A) \cap A$$

$$A \triangle B = (A * B) \cup (B * A)$$

where the revision $K * A$ operates on sets of models and is expressed by a set of models.

As a consequence of the last properties, we can find out the computational complexity of this method. The computational task that has been most investigated in the literature and that we consider here, is deciding the following relation:

Input:	formulas a, b and c.
Output:	$true$ if $A \triangle B \models c$
	$false$ otherwise

Using the correlation between revision and arbitration, we have that $A * B = (A \triangle B) \wedge B$. Hence, deciding if $A * B \models c$ is equivalent to $A \triangle B \models b \rightarrow c$. Thus, Dalal's revision is at most as difficult as Dalal-like arbitration. Conversely, deciding whether $A \triangle B \vdash c$ holds requires valuation of $(a * b) \vee (b * a) \vdash c$. This control requires two operations of revision: $a * b \vdash c$ and $b * a \vdash c$ that can be made in parallel. Using Eiter and Gottlob's result [4] on revision, the complexity of Dalal-like arbitration is, in the general case, $P^{NP[O(log n)]}$-complete.

As a drawback of this method, we show that Dalal-like arbitration isn't associative. Let us consider the following scenario:

$$\begin{array}{lcl} a & = & (x \wedge y) \vee (x \wedge \neg y) \\ b & = & \neg x \wedge y \\ c & = & \neg x \wedge \neg y \end{array}$$

It's easy to verify that:

$(A \triangle B) \triangle C = \{(\neg x, \neg y), (\neg x, y)\}$

$A \triangle (B \triangle C) = \top$

Hence, Dalal-like arbitration is not associative, and $\triangle_{\leq}$ is, in general, not associative too.

5 Alternative Arbitrations

In the previous sections we introduced a set of postulates for arbitration. Here we investigate on alternative choices for the specific postulates. Two interesting properties for arbitration, not yet considered, are the following ones:

(monot') if $A \subseteq C$ and $B \subseteq D$ then $A \triangle B \subseteq C \triangle D$

(assoc) $A \triangle (B \triangle C) = (A \triangle B) \triangle C$.

In (assoc), we assume that all arbitrations can't be made as an intersection of arguments (if we do not assume that, one may obtain inconsistent arbitrations).

Monotonicity (monot') ensures that arbitration of larger sets obtains as result a larger set, while associativity (assoc) is a natural property to impose on the arbitration of three or more theories (i.e. $\triangle(T_1, \ldots, T_n)$ with $n > 2$): we can express $\triangle(A, B, C) = (A \triangle B) \triangle C = (C \triangle B) \triangle A = (A \triangle C) \triangle B$.

5.1 Monotonicity

While (monot') seems a very reasonable constraint, it nevertheless has a dramatic impact, as shown by the following negative result:

Theorem 7 *There is no arbitration that satisfies (monot') and 4.*

Proof: Let A and B two sets of models such that $A \cap B = \emptyset$. We prove that if one of these has at least two models and arbitration satisfies (monot'), then $A \triangle B = \emptyset$, so that arbitration violates postulate 5.

Suppose $A \cap B = \emptyset$. Let $I, J \in A$ be two models of A. If arbitration satisfies 3 and 4, $A \triangle (B \cup \{I\}) = \{I\}$, and $A \triangle (B \cup \{J\}) = \{J\}$. By (monot') it follows that: $A \triangle B \subseteq A \triangle (B \cup \{I\}) = \{I\}$ and $A \triangle B \subseteq A \triangle (B \cup \{J\}) = \{J\}$. Hence $A \triangle B = \emptyset$. □

As a consequence, we can assume (monot') but not 4, or assume (monot') only when $A \cap B = \emptyset$. This leads to a revised form for it:

(monot) if $A \cap B = \emptyset$, $A \subseteq C$ and $B \subseteq D$ then $A \triangle B \subseteq C \triangle D$

While for commutative revision there exists an elegant representation theorem, when we assume postulates 1-5 and (monot) we can only find two approximating formulas.

Let

$$A \triangle_h B = \begin{cases} A \cap B & \text{if } A \cap B \neq \emptyset \\ \bigcup_{I \in A, J \in B} h(I, J) & \text{otherwise} \end{cases}$$

where $h : \top \times \top \rightarrow \mathcal{P}(\top)$: is a function with two interpretation as operands, and a set of interpretations as result. This arbitration is monotone (and satisfies 6).

Theorem 8 *If $\triangle$ is monotone, then there exist two functions, f and g, such that $A \triangle_f B \subseteq A \triangle B \subseteq A \triangle_g B$ for any $A, B \in \mathcal{P}(\top)$, where:*

$$f(I, J) = \{I\} \triangle \{J\}$$

$$g(I, J) = \bigcup_{\{R | I \in R, J \notin R\}} R \triangle \neg R$$

Another monotonic arbitration is

$$A \triangle^h B = \begin{cases} A \cap B & \text{if } A \cap B \neq \emptyset \\ \bigcap_{I \notin A, J \notin B} h(I, J) & \textit{otherwise} \end{cases}$$

A property of this arbitration is given by the following theorem

Theorem 9 *For any monotonic $\triangle$, there exist a function g s. t.*

$$A \triangle B \subseteq A \triangle^g B$$

for any $A, B \in \mathcal{P}(\top)$ where g is defined as:

$$g(I, J) = \begin{cases} \top & \textit{if } I = J \\ \bigcup_{\{M | I \notin M, J \in M\}} M \triangle \neg M & \textit{otherwise} \end{cases}$$

5.2 Associativity

In this section, we consider arbitrations satisfying the associativity postulate. We show that the only associative arbitration satisfying condition 7-8 is $A \triangle B = A \cup B$.

Theorem 10 *If $\triangle$ satisfies (assoc), 1-5, 7 and 8, then $\triangle = \cup$.*

Proof: First of all, we show that:

$$\{I_1\} \triangle \cdots \triangle \{I_n\} = \{I_1, \ldots, I_n\}$$

This can be rephrased as $\{I_1\} \triangle \cdots \triangle \{I_n\} = \{I_i\} \triangle (\textit{the rest})$, hence it is also $I_i \in \{I_1\} \triangle \cdots \triangle \{I_n\}$ for every I_i.

We show that $\{I_1\} \triangle \cdots \triangle \{I_n\} \subseteq \{I_1, \ldots, I_n\}$ by induction on n: $\{I_1\} \triangle \{I_2\} = \{I_1, I_2\}$ is the basic step. For $n > 2$ interpretations we have that

$$\{I_1\} \triangle (\{I_2\} \triangle \cdots \triangle \{I_n\}) \subseteq \{I_1\} \cup (\{I_2\} \triangle \cdots \triangle \{I_n\})$$

But $\{I_2\} \triangle \cdots \triangle \{I_n\} \subseteq \{I_2, \ldots, I_n\}$. Thus $\{I_1\} \triangle \cdots \triangle \{I_n\} \subseteq \{I_1, \ldots, I_n\}$.

The result now easily follows.

$$A \triangle B = \{A_1, \ldots, A_n\} \triangle \{B_1, \ldots, B_m\} =$$

$$= (\{A_1\} \cup \cdots \cup \{A_n\}) \triangle (\{B_1\} \cup \cdots \cup \{B_m\}) =$$

$$= \{A_1\} \triangle \cdots \triangle \{A_n\} \triangle \{B_1\} \triangle \cdots \triangle \{B_m\} = A \cup B$$

□

Another interesting property of associative arbitrations satisfying 1-7 concerns the arbitration of singletons (sets with only one element).

Theorem 11 *Let $\leq$ be a relation such that $I \leq J$ iff $I \in \{I\} \triangle \{J\}$. As a consequence, whenever $\triangle$ satisfies 1-7 and (assoc), the relation $\leq$ is transitive (and, obviously, reflexive and total).*

Example An associative arbitration is

$$A \triangle B = \begin{cases} A \cap B & \text{if } A \cap B \neq \emptyset \\ min(A \cup B, \leq) & \end{cases}$$

where $\leq$ is a reflexive, transitive, and total order on interpretations.

6 Concluding Remarks

In this paper we presented a new approach to the problem of combining possibly inconsistent information coming from different sources each of them equally reliable. The proposed operator captures a form of revision not encompassed by the AGM postulates. While the proposed system is original, in [10] Revesz proposes a first formalization of arbitration. However, his definition is introduced via a new operator called *model fitting*. Postulates are proposed for this operator, not for arbitration.

As it turns out, our proposal is radically different from Revesz's one. While Revesz's arbitration satisfies postulates 1-2 and 5, it does not satisfy 3, 4, 6-8. Furthermore, it does not satisfy associativity, as shown by the following example:

Let $A = x \wedge y$, $B = \neg x \wedge y$ and $C = (x \wedge \neg y) \vee (\neg x \wedge \neg y)$. Using the arbitration operator presented in Revesz's paper, it turns out that $(A \triangle B) \triangle C = (x \wedge y) \vee (x \wedge \neg y) \vee (\neg x \wedge y) \vee (\neg x \wedge \neg y)$ while $A \triangle (B \triangle C) = (x \wedge \neg y) \vee (\neg x \wedge y)$.

While Revesz's arbitration cannot satisfy 6, 7, 8, (monot) and (assoc) without major modifications, it satisfies 3 and 4 if we impose:

$$A \triangle B = \begin{cases} A \cap B & if\ A \cap B \neq \emptyset \\ \text{Revesz's definition} & otherwise \end{cases}$$

An open problem is to extend arbitration to n operands. The definition of postulate for $\triangle(A_1, \ldots, A_n)$ is immediate. In the longer version, we show by induction that, if we assume an extension of postulates 1-8, then knowing $\triangle(\{I_1, J_1\}, \ldots, \{I_n, J_n\})$ is enough to determine $\triangle(A_1, \ldots, A_n)$ for every tuple of sets of models $A_1, \ldots, A_n$.

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