Computer Graphics and Visualisation

Geometry for Computer Graphics

Overhead Projection (OHP) Overviews

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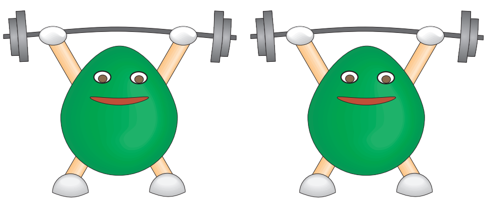

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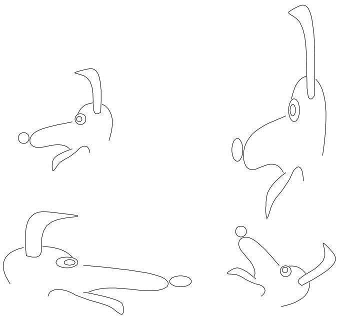
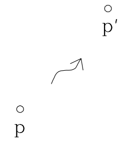
Two-Dimensional Transformations

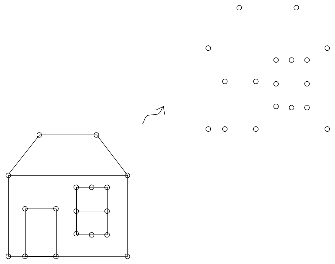


Two-Dimensional Transformations

- Transforming a point

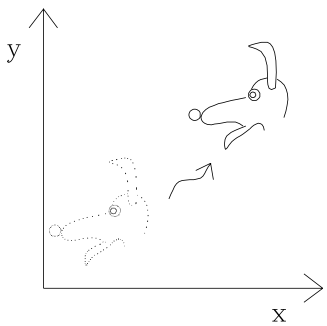


- Transforming an object



Types of Transformation

- Translation

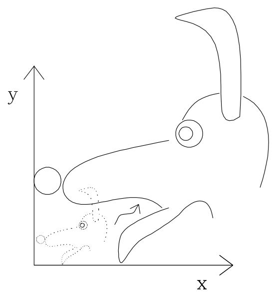


$$x' = x + t_x$$

$$y' = y + t_y$$

Types of Transformation

- Scaling



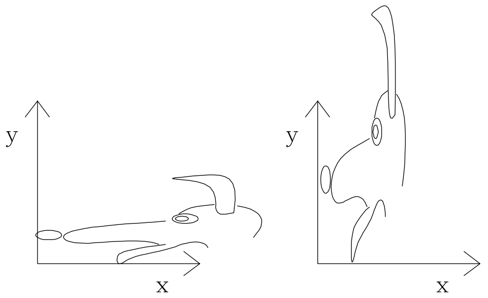
Sx=Sy=3

$$x' = x \cdot s_x$$

$$y' = y \cdot s_y$$

Symmetric vs asymmetric scaling

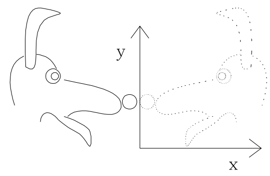
- Symmetric scaling: $s_x = s_y$
- Asymmetric scaling: $s_x > s_y$ or $s_x < s_y$



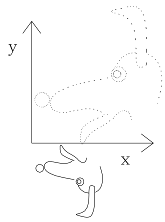
sx > sy   sx < sy

Scaling to achieve reflection

- Reflection in *y* axis: $s_x < 0$



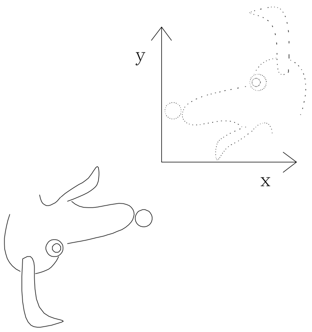
- Reflection in *x* axis: $s_y < 0$



Scaling to achieve reflection

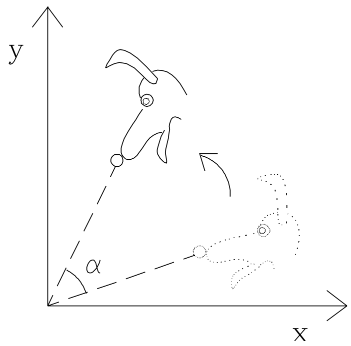
- Reflection in *x* and *y* axes:

  $s_y < 0$ *and* $s_x < 0$

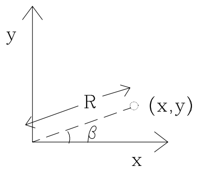


Types of Transformation

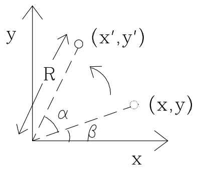
- Rotation



Rotation



$$x = R \cdot \cos\beta$$
$$y = R \cdot \sin\beta$$



$$x' = R \cdot \cos(\alpha + \beta)$$
$$y' = R \cdot \sin(\alpha + \beta)$$

Rotation

$$x' = R \cdot \cos(\alpha + \beta)$$
$$y' = R \cdot \sin(\alpha + \beta)$$

Expanding the formulae for $\cos(\alpha+\beta)$ and $\sin(\alpha+\beta)$:

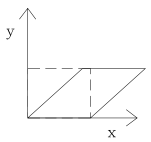
$$x' = R \cdot \cos\alpha \cdot \cos\beta - R \cdot \sin\alpha \cdot \sin\beta$$
$$y' = R \cdot \sin\alpha \cdot \cos\beta + R \cdot \sin\beta \cdot \cos\alpha$$

Substituting for $R \cdot \cos\beta$ and $R \cdot \sin\beta$:

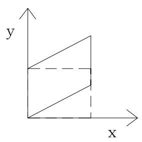
$$x' = x \cdot \cos\alpha - y \cdot \sin\alpha$$
$$y' = x \cdot \sin\alpha + y \cdot \cos\alpha$$

Types of Transformation

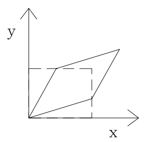
- Shearing



shear in x



shear in y



shear in x and y

$$x' = x + y \cdot a$$
$$y' = y + x \cdot b$$

- ❏ Shear in x: $a \neq 0$
- ❏ Shear in y: $b \neq 0$
- ❏ Shear in x and y: $a \neq 0$ and $b \neq 0$

Matrix Representation of Transformations

- Translate:

$$x' = x + t_x$$
$$y' = y + t_y$$

- Scale:

$$x' = x \cdot s_x$$
$$y' = y \cdot s_y$$

- Rotate:

$$x' = x \cdot \cos\alpha - y \cdot \sin\alpha$$
$$y' = x \cdot \sin\alpha + y \cdot \cos\alpha$$

Matrix Representation of Transformation

- Shear:

$$x' = x + y \cdot a$$
$$y' = y + x \cdot b$$

- In general:

$$x' = a \cdot x + b \cdot y + c$$
$$y' = d \cdot x + e \cdot y + f$$

Matrix Representation of Transformations

$$x' = a \cdot x + b \cdot y + c$$
$$y' = d \cdot x + e \cdot y + f$$

Using matrices:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ d & e \end{bmatrix} \bullet \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ f \end{bmatrix}$$

Include *a* - *f* in one matrix:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Matrix Representation of Transformations

$$x' = a \cdot x + b \cdot y + c$$
$$y' = d \cdot x + e \cdot y + f$$

Using a square matrix:

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Matrix Representation of Transformations

- Translate: $\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$

- Scale: $\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

- Rotate: $\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Matrix Representation of Transformations

- Shear: $\begin{bmatrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- Identity matrix:

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x' = x$$
$$y' = y$$
$$w' = 1$$

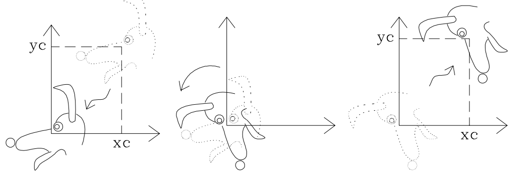
Combining Transformations

Can perform complex transformations by combining simple ones.

For example, rotating an object about its centre:

Combining Transformations

1. Translate by ($-x_c$, $-y_c$)
2. Rotate about the origin
3. Translate by (x_c, y_c)



1 2 3

Combining Transformations

Rotating an object about its centre:

1. Translate by ($-x_c$,$-y_c$):

$$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -x_c \\ 0 & 1 & -y_c \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2. Rotate about the origin:

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

Combining Transformations

3. Translate by (x_c, y_c):

$$\begin{bmatrix} x_3 \\ y_3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & x_c \\ 0 & 1 & y_c \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}$$

Total transformation is:

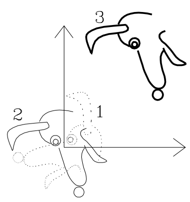
$$\begin{bmatrix} 1 & 0 & x_c \\ 0 & 1 & y_c \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} 1 & 0 & -x_c \\ 0 & 1 & -y_c \\ 0 & 0 & 1 \end{bmatrix}$$

Ordering Transformations

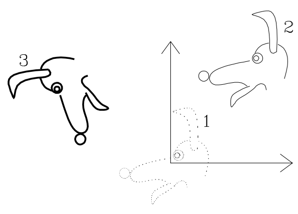
- Matrix multiplication is NOT commutative, $\mathbf{M}_1 \bullet \mathbf{M}_2 \neq \mathbf{M}_2 \bullet \mathbf{M}_1$
- Order of transformations is important

Ordering Transformations

- Rotate then translate



- Translate then rotate



Ordering Transformations

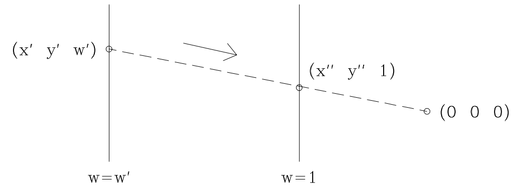
- POSTCONCATENATE $\mathbf{M}_2$ with $\mathbf{M}_1$:
  $\mathbf{p}' = \mathbf{M}_2 \bullet \mathbf{M}_1 \bullet \mathbf{p}$ ($\mathbf{M}_1$ applied first)
- PRECONCATENATE $\mathbf{M}_2$ with $\mathbf{M}_1$:
  $\mathbf{p}' = \mathbf{M}_1 \bullet \mathbf{M}_2 \bullet \mathbf{p}$ ($\mathbf{M}_2$ applied first)
- Premultiply $\equiv$ Postconcatenate
- Postmultiply $\equiv$ Preconcatenate

Homogeneous Coordinates

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Point in **2D** space expressed in **3D homogeneous** coordinates
- If bottom row of matrix is [0 0 1], $w' = 1$
- If $w' \neq 1$ project point (x', y',w') onto plane w =1 by **homogeneous division** (using the origin as the centre of projection)

Homogeneous Coordinates



- Real world coordinates are x'' and y'' where

$$x'' = x'/w'$$
$$y'' = y'/w'$$

Homogeneous Coordinates

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

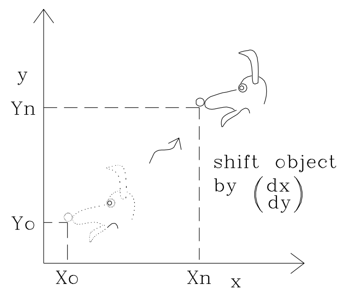
Perform homogeneous division to get “real world” coordinates:

$$x'' = x'/w' = x/4$$
$$y'' = y'/w' = y/4$$

Effect is OVERALL scaling.

Object vs Axis Transformation

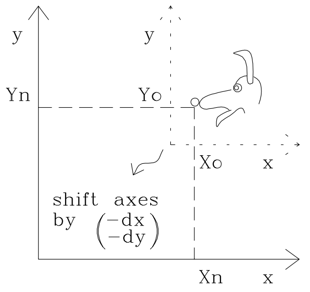
Object transformation



- Object transformed
- Axes fixed

Object vs Axis Transformation

Axis transformation

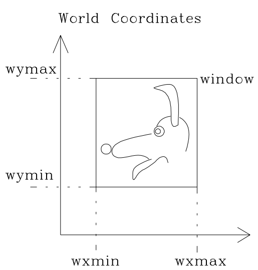


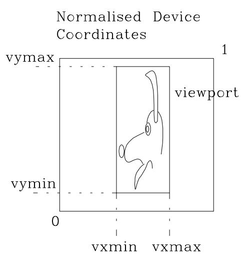
- Object fixed in space
- Axes transformed

Object vs Axis Transformation

- Object translation by (d_x, d_y)
  - ≡ Axis translation by ($-d_x$, $-d_y$)
- Object scale by (s_x, s_y)
  - ≡ Axis scale by ($1/s_x$, $1/s_y$)
- Object rotation by α
  - ≡ Axis rotation by $-\alpha$

Normalization Transformation in GKS





1. Translate bottom left hand corner of window to origin:

$$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -w_{xmin} \\ 0 & 1 & -w_{ymin} \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Normalization Transformation in GKS

2. Scale window to size of viewport:

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

where

$$s_x = \frac{v_{xmax} - v_{xmin}}{w_{xmax} - w_{xmin}}$$

$$s_y = \frac{v_{ymax} - v_{ymin}}{w_{ymax} - w_{ymin}}$$

Normalization Transformation in GKS

3. Translate to bottom left hand corner of viewport:

$$\begin{bmatrix} x_3 \\ y_3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & v_{xmin} \\ 0 & 1 & v_{ymin} \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix}$$

Normalization transformation is:

$$\begin{bmatrix} 1 & 0 & v_{xmin} \\ 0 & 1 & v_{ymin} \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} 1 & 0 & -w_{xmin} \\ 0 & 1 & -w_{ymin} \\ 0 & 0 & 1 \end{bmatrix}$$

Two-Dimensional Transformations

- Different types: translation, scaling, rotation, shearing.
- Object vs axis transformations
- Matrix representation

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Combine transformations by multiplying matrices
- Homogeneous division to get "real world" coordinates

$$x'' = x'/w', y'' = y'/w'$$

Three-Dimensional Transformations

Three-Dimensional Transformations

- For manipulating pictures (as in 2D)
- Help us to understand 3D shape
- Right-handed coordinate system