A Convex Optimization Approach for a Class of Nonconvex Quadratic Estimation Problems

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Many problems in data fitting and estimation which arise in a variety of scientific disciplines such as: signal processing, statistics, physics, economy and medicine, require to solve an over-determined system of linear equations where the data (matrix and right hand side) is contaminated by noise.

A common methodology to model this situation and get a stabilized solution, is via the so-called Regularized Total Least Squares (RTLS) approach which leads to formulate the estimation problem as one of minimizing the ratio of two convex quadratic functions subject to a quadratic constraint. The specific form of the (RTLS) problem involving the ratio of two special convex functions does not appear to play any advantageous role toward its solution, and the key difficulty, namely its *nonconvexity* remains. As a result, several well known methods devised to solve it are not guaranteed to converge to a global optimum, and at best are proven to converge to a point satisfying first order necessary optimality conditions.

This talk first briefly reviews such class of estimation problems and then focuses on the more general nonconvex problem of minimizing the ratio of two (possibly indefinite) quadratic functions (RQ) subject to a degenerate ellipsoid. We prove that under a certain mild assumption on the problem's data, problem (RQ) admits an exact semidefinite programming relaxation, and an optimal solution of the original problem (RQ) can be extracted from the optimal solution of this semidefinite formulation. Building on this hidden convexity property of problem (RQ), we then study a simple and efficient iterative procedure for solving problem (RQ) which is proven to globally converge superlinearly. Moreover, it is shown that this algorithm produces an ε-global optimal solution of (RQ) in no more than $O(\kappa\sqrt{\log \varepsilon^{-1}})$ iterations, for some positive constant κ given in terms of the problem's data. This result also provides a theoretical justification to some recent successful computational results reported in the literature in the special context of (RTLS). Numerical experiments will illustrate the attractiveness of the algorithm.

This talk is based on a joint work with Amir Beck.