Proceedings of the American Control Conference
Chicago, Illinois • June 2000

Control of convey-crane based on passivity.*

Joaquín Collado[+×] , Rogelio Lozano[+] and Isabelle Fantoni[+]
[×]Universidad Autónoma de Nuevo León
Facultad de Ingeniería Mecánica y Eléctrica
Apdo. Postal F-30
66451 San Nicolás, N.L. México
e-mail: jcollado@ccr.dsi.uanl.mx
[+]Heudiasyc UMR 6599 CNRS - UTC
Centre de Recherche de Royallieu
BP 20.529 - 60205 Compiegne France
e-mail: rlozano@hds.utc.fr
e-mail: isabelle.fantoni@hds.utc.fr

Abstract

Based on the recent results of control of the inverted pendulum on a cart, we propose a control law of a convey-crane which transports a load suspended on a cart from a given point to another, minimizing its oscillations. The control law derived is based on the passivity property of the system. Some simulations are presented.

1 Introduction.

The inverted pendulum on a cart has originated many contributions, its objective is the stabilization of the unstable equilibrium point, see for example [1], [2], [8] , [9], [10] and [13]. Much less effort has attracted the problem of asymptotic stabilization of the lower equilibrium point. The problem of control of a convey crane presented in this paper is: given the cart in some initial position, bring it to the origin keeping the oscillations of the suspended mass as small as possible. The system dynamics corresponds exactly to the equations of the inverted pendulum on a cart, but now the point of interest is the lower equilibrium point. One possible application of this system is a convey-crane carrying a heavy load from one starting point to another keeping oscillations small. This system is underactuated and is not input-output linearizable, [5] or [6]. The stability analysis is carried out using Lyapunov techniques and the stabilization control law is based on the original work of Spong and Praly [12] and further developments in Lozano and Fantoni [9]. The present approach takes advantage of the passivity of the model. The nonlinearities are not canceled and the control law may be interpreted as adding a nonlinear damping to the system dynamics. The performance of the control law is shown in simulations.

*This work was elaborated while first author was in a sabbatical year at HEUDIASYC Laboratory at Compiegne, France. He was partially supported by Conacyt.

2 Model.

Consider the convey-crane system as shown in the figure 1, where M is the mass of the cart, m the mass of the pendulum with the load of the crane, θ the angle that the pendulum makes with the vertical and l the length of the rod. We will assume, as in the case of the inverted pendulum, that the masses are concentrated at their geometrical center, no mass of the pole of constant length l .

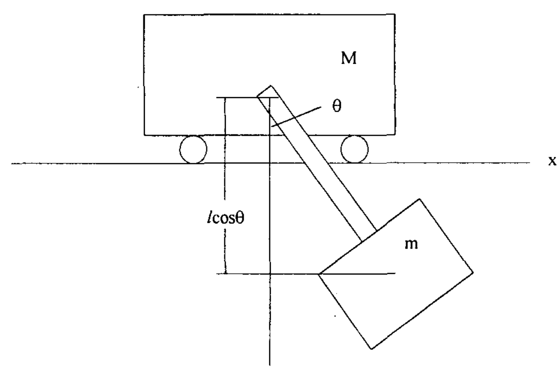

Figure 1:

The equations may be obtained by standard Euler-

Lagrange methods or applying Newton's second law. The system dynamics may be described by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \qquad (1)$$

where

$$q = \begin{bmatrix} x \\ \theta \end{bmatrix}; \qquad M(q) = \begin{bmatrix} M+m & -ml\cos\theta \\ -ml\cos\theta & ml^2 \end{bmatrix} \qquad (2)$$

$$C(q,\dot{q}) = \begin{bmatrix} 0 & m\,l\,\sin\theta\,\dot{\theta} \\ 0 & 0 \end{bmatrix} \qquad (3)$$

$$G(q) = \begin{bmatrix} 0 \\ m\,g\,l\sin\theta \end{bmatrix}; \text{ and } \qquad \tau = \begin{bmatrix} f \\ 0 \end{bmatrix} \qquad (4)$$

The above model corresponds to the model used for the inverted pendulum on a cart, replacing $\theta \to \theta + \pi$ see [9] or [7]. Notice that $M(q)$ is symmetric and positive definite, since the parameters M, m, l are positive;

$$\begin{aligned} \det[M(q)] &= (M+m)\,m\,l^2 - (-m\,l\cos\theta)^2 \\ &= M\,m\,l^2 + m^2 l^2 \sin^2\theta > 0 \end{aligned} \qquad (5)$$

A second well known property is that the parameters of the model are such that the matrix

$$\dot{M}(q) - 2C(q,\dot{q}) = \begin{bmatrix} 0 & -m\,l\sin\theta\,\dot{\theta} \\ m\,l\sin\theta\,\dot{\theta} & 0 \end{bmatrix} \qquad (6)$$

is skew-symmetric, property required to establish the passivity of the model. Recall [4] that for any skew symmetric matrix A, $x^T A x = 0$.

Finally the potential energy associated to the pendulum, may be defined as $P = mgl(1-\cos\theta)$. With this definition, P and $G(q)$ are related by

$$G(q) = \frac{\partial P}{\partial q} = \begin{bmatrix} 0 \\ m\,g\,l\sin\theta \end{bmatrix} \qquad (7)$$

3 Passivity of the system.

The total energy of the system, i.e., the sum of the kinetic energy of the two masses and the potential energy of the pendulum is given by

$$\begin{aligned} E &= \tfrac{1}{2}\dot{q}^T M(q)\dot{q} + P(q) \\ &= \tfrac{1}{2}\dot{q}^T M(q)\dot{q} + m\,g\,l\,(1-\cos\theta) \end{aligned} \qquad (8)$$

Using (1)-(4) and (6)-(7) we may calculate the derivative of the energy E as:

$$\begin{aligned} \dot{E} &= \dot{q}^T M(q)\ddot{q} + \tfrac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} + \dot{q}^T G(q) \\ &= \dot{q}^T\left(-C\dot{q} - G + \tau + \tfrac{1}{2}\dot{M}(q)\dot{q}\right) + \dot{q}^T G(q) \\ &= \dot{q}^T \tau = \dot{x} f \end{aligned} \qquad (9)$$

Integrating the last relationship from zero to t, we get

$$\begin{aligned} \int_0^t \dot{x} f\, dt' &= E(t) - E(0) \\ &\geq -E(0) \end{aligned} \qquad (10)$$

Which proves that if f is the input and $\dot{x}$ is the output, then the system is passive. Note that for input force zero and restricting $\theta \in [0, 2\pi]$, the system (1) has two subsets of equilibrium; $\left(x, \dot{x}, \theta, \dot{\theta}\right) = (*, 0, 0, 0)$ is a set of stable equilibrium points; and $\left(x, \dot{x}, \theta, \dot{\theta}\right) = (*, 0, \pi, 0)$ correspond to a set of unstable equilibrium points. The minimum energy corresponds to the lower position of the pendulum and equals zero.

The control objective is: given the initial conditions $\left(x(0), \dot{x}(0), \theta(0), \dot{\theta}(0)\right) = (x_0, 0, \theta_0, 0)$ bring this state to the origin with minimum of oscillations; i.e. change the stable set $\left(x, \dot{x}, \theta, \dot{\theta}\right) = (0,0,0,0)$ to be asymptotically stable equilibrium point around some neighborhood of the origin.

4 Damping oscillations control law

Notice that if $\dot{x} = 0$ and $E = 0$ then

$$\frac{1}{2} m\,l^2 \dot{\theta}^2 = m\,g\,l\,(\cos\theta - 1) \qquad (11)$$

this equation has the only solution $\left(\theta, \dot{\theta}\right) = (0, 0)$.

In order to take advantage of the passivity property of the system, let us propose as a Lyapunov function candidate the following:

$$V(q,\dot{q}) = \frac{k_E}{2}E(q,\dot{q}) + \frac{k_x}{2}x^2 \tag{12}$$

where k_E, k_x are strictly positive constants. The Lyapunov function candidate $V(q,\dot{q})$ is positive definite if we restrict $\theta \in [0, 2\pi)$. Differentiating $V(q,\dot{q})$ and using (9) we get

$$\begin{aligned} \dot{V} &= k_E\dot{E} + k_x x\dot{x} \\ &= k_E\dot{x}f + k_x x\dot{x} \\ &= \dot{x}\left(k_E f + k_x x\right) \end{aligned} \tag{13}$$

From system dynamics (1), and from

$$[M(q)]^{-1} = \frac{1}{\det(M(q))}\begin{bmatrix} m\,l^2 & m\,l\cos\theta \\ m\,l\cos\theta & M+m \end{bmatrix} \tag{14}$$

where $\det(M(q)) = m\,l^2\left(M + m\sin^2\theta\right)$ we may get $\ddot{x}$ and it is given by

$$\begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \frac{1}{\det(M(q))}\begin{bmatrix} m\,l^2 & m\,l\cos\theta \\ ml\cos\theta & M+m \end{bmatrix} \bullet \left\{ -\begin{bmatrix} m\,l\sin\theta\dot{\theta}^2 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ m\,g\,l\sin\theta \end{bmatrix} + \begin{bmatrix} f \\ 0 \end{bmatrix} \right\}$$

From the above equation, we get

$$\ddot{x} = \frac{1}{M + m\sin^2\theta}\left[-m\sin\theta\left(l\,\dot{\theta}^2 + g\cos\theta\right) + f\right] \tag{15}$$

and

$$\ddot{\theta} = \frac{1}{l\,(M + m\sin^2\theta)}\left[-\sin\theta\left((M+m)\,g + m\,l\,\cos\theta\dot{\theta}^2\right) + \cos\theta f\right] \tag{16}$$

For sake of simplicity we will consider $M = m = l = 1$, then we propose the control law such that:

$$(k_E\,f + k_x x) = -\gamma\dot{x} \tag{17}$$

for some $\gamma > 0$ which gives us

$$\dot{V} = -\gamma\dot{x}^2 \tag{18}$$

The explicit control law defined by (17) is

$$f = -\frac{1}{k_E}\left(k_x x + \gamma\dot{x}\right) \tag{19}$$

The control law (19) guarantees $\dot{V} = -\gamma\dot{x}^2$ which is negative semidefinite, therefore the closed loop is stable [7].

4.1 Asymptotic Stability Analysis.

Using the Invariance principle of LaSalle, we will prove the following:

Theorem 1 *The closed loop system given by equations (1) and (19) has the origin asymptotically stable for all points in* $\mathbf{R}^4 \setminus \{(0,0,\pi,0)\}$

Proof.

From $\dot{V} = -\gamma\dot{x}^2$ negative semidefinite, it remains to be proven that the whole state converges to zero. This follows from $\dot{x} \to 0$ which implies $x \to a$ which we assume different from zero. Then $f \to -\frac{k_x}{k_E}a$ different from zero, which leads to a contradiction, see [9] for details. Then $x \to 0$ and $f \to 0$. Now making $\dot{x}, x$ and f zero in (15) and (16) we get

$$\begin{aligned} \sin\theta\left(l\dot{\theta}^2 + g\cos\theta\right) &= 0 \\ \sin\theta\left((M+m)\,g + ml\cos\theta\dot{\theta}^2\right) &= 0 \end{aligned} \tag{20}$$

these equations have solutions $\left(\theta, \dot{\theta}\right) = (n\,\pi, 0)$ ■

5 Simulation results

For comparison reasons, we obtained from (15) and (16) a linearized model of the convey crane around its lower equilibrium point and with a force $f = 0$.

$$\dot{z} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{(M+m)\,g}{l\,M} & 0 \end{bmatrix} z + \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ \frac{1}{l\,M} \end{bmatrix} f$$

we use full state feedback $f = -k\,z$ on the linearized model with $k = \begin{bmatrix} 3 & 3.69 & .71 & -.87 \end{bmatrix}$. Simulations were performed using SIMULINK, we considered $M = 1$, $m = 1$, $l = 1$ and $g = 9.8\ m/s^2$, and the initial conditions are $\left(x(0), \dot{x}(0), \theta(0), \dot{\theta}(0)\right) = (-5, 0, -\pi/4, 0)$. The parameters of the control law

(19) were $k_E = 1$, $k_x = 3$ and $\gamma = 4.3$. Figure 2 shows the angle θ for the original and linearized models with their respective controllers, Figure 3 shows the position of the cart x. Figure 4 and Figure 5 show the angle θ and the position of the cart x, now for the initial conditions $(-5, 0, 0, 0)$. In these two cases and for different initial conditions, the proposed controller outperforms the linearized controller, moreover stability region for the proposed controller is almost all $\mathbf{R}^4$ and stability region for linearized system is a neigbhorhood around the origin.

6 Concluding Remarks

We presented a control law for the convey crane model which is similar to the inverted pendulum on a cart, considering the lower equilibrium point as the control objective. We proved asymptotic stability of the proposed control law based on a Lyapunov function which is based on the energy of the system. The convergence analysis was completed using the LaSalle's Invariance Theorem. Simulations show that the region of attraction is practically the whole state space.

References

[1] K.J. Åström and K. Furuta. "Swinging up a pendulum by energy control". In IFAC'96, Preprints 13th World Congress of IFAC, vol. E, pp. 37-42; San Francisco, California, 1996.

[2] C.C. Chung and J. Hauser. "Nonlinear control of a swinging pendulum". Automatica, Vol. 31, No. 6, pp. 851-862, 1995.

[3] I. Fantoni, R. Lozano, F. Mazenc and A. Annaswamy, "Stabilization of a two-link robot using an energy approach". ECC 99, Karlsruhe, Germany, Sept. 1999.

[4] R.A. Horn and C.R. Johnson. "*Matrix Analysis*". Cambridge University Press, 1985.

[5] A. Isidori.. "*Nonlinear Control Systems: An Introduction*", 3rd. Ed. Springer-Verlag, 1995.

[6] B. Jakubczyk and W. Respondek. "On the linearization of control systems". Bull. Acad. Polon. Sci. Math., Vol. 28, pp. 517-522, 1980.

[7] H.K. Khalil (1996). "*Nonlinear Systems*". 2nd. Ed., Prentice-Hall, Upper Saddle River, NJ.

[8] Z. Lin, A. Saberi, M. Gutman and Y.A. Shamash. "Linear controller for an inverted pendulum having restricted travel: A high-low gain approach". Automatica, Vol. 32, No. 6, pp. 933-937, 1996.

[9] R. Lozano and I. Fantoni. "Stabilization of the inverted pendulum around its homoclinic orbit". Accepted to Systems and Control Letters, 2000.

[10] F. Mazenc and L. Praly. "Adding integrators, saturated controls, and stabilization for feedforward systems". IEEE Trans. on Autom. Contr., Vol. 41, No. 11, pp. 1559-1578. 1996.

[11] M. Spong. "The swing up control of the Acrobot", 1994 IEEE Int. Conf. on Robotics and Automation, San Diego, CA, 1994.

[12] M. Spong and L. Praly. "Control of underactuated mechanical systems using switching and saturation". Proc. of the Block Island Workshop on Control Using Logic Based Switching, Springer-Verlag, 1996.

[13] Q. Wei, W.P. Dayawansa and W.S. Levine. "Nonlinear controller for an inverted pendulum having restricted travel". Automatica, Vol. 31, No. 6, pp. 841-850, 1995.

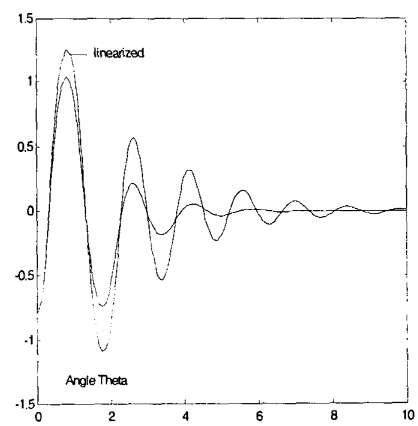

Figure 2: Angle Theta for initial state (-5,0,-45^0,0)

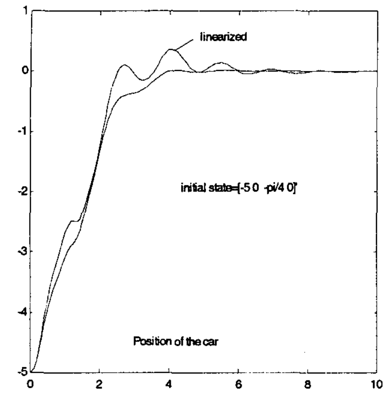

Figure 3: Cart position for initial state (-5,0,-45^0,0)

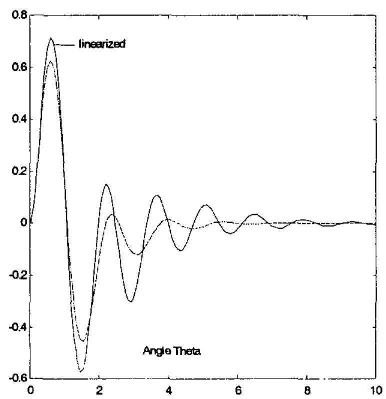

Figure 4: Angle Theta for initial state (-5,0,0,0)

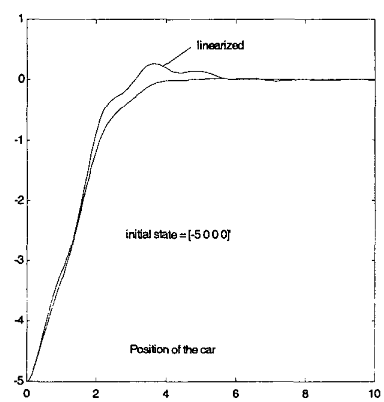

Figure 5: Cart position for initial state (-5,0,0,0)