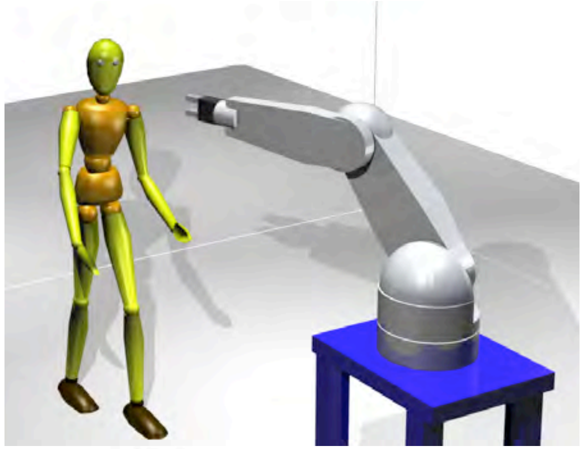
Stiffness estimation for flexible transmissions

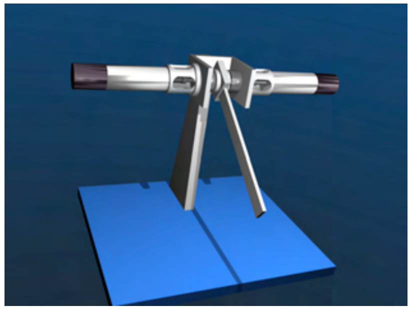
Fabrizio Flacco
Alessandro De Luca
Dipartimento di Informatica e Sistemistica
SAPIENZA Università di Roma

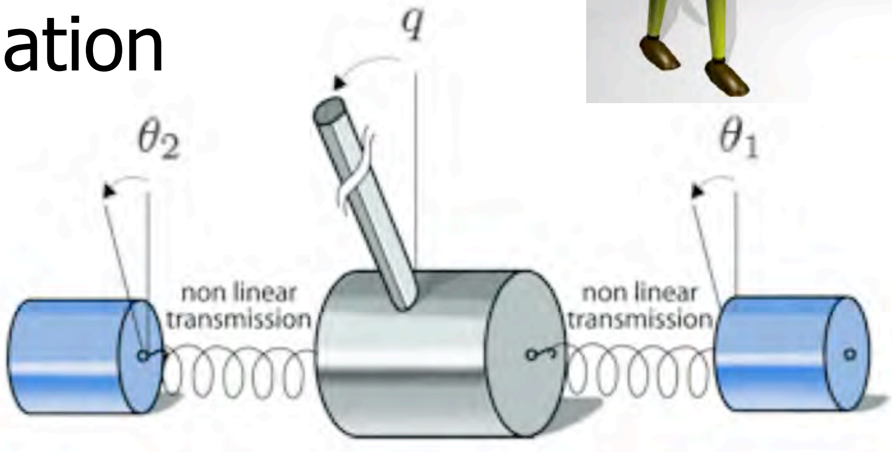
Motivation

Why we need to know the stiffness of a transmission ?

- Safety in pHRI
- Bio-inspired actuators (VSA)
- Feedback linearization control
- Gravity cancellation







Stiffness

Classical flexible joint model

$$
\begin{aligned}
M\ddot{q} + D_q\dot{q} + \tau_{e_{TOT}} + g(q) + \tau_k &= 0 \\
B_1\ddot{\theta}_1 + D_{\theta_1}\dot{\theta}_1 - \tau_{e_1}(\phi_1) &= \tau_1 \\
&\vdots \\
B_m\ddot{\theta}_m + D_{\theta_m}\dot{\theta}_m - \tau_{e_m}(\phi_m) &= \tau_m
\end{aligned}
$$

where $\tau_{e_{TOT}} = \Phi\left(\tau_{e_1}, \tau_{e_2}, \ldots, \tau_{e_m}\right),$

stiffness is defined as

$$\sigma(\phi) = \frac{\partial \tau_e(\phi)}{\partial \phi}\,.$$

$$\phi = q - \theta$$

Our idea

- Make our observation on motor side
- Use a residual-based method to estimate the stiffness of the transmission
- Compute the total stiffness applying transmission arrangement rules

Residual

Generalized momentum of the motor $p = B\dot{\theta}$

Residual

$$r_e = K_I \left(p + D_\theta \theta - \int_0^t \tau + r_e ds \right)$$

where $K_I > 0$ is a free design parameter, and for $r_e(0) = 0$ a system initially at rest

it is easy to check that the residual satisfies

$$\dot{r}_e = K_I \left(\tau_e - r_e \right)$$

resulting in a fist-order, stable filter of the unknown flexible torque of the transmission

Model-Based Estimator

Flexible torque is a function of transmission deformation and a number of constant parameters

$$\tau_e(\phi) = f(\phi, \beta)$$

Dataset of experimental values $y_{(v)} = \left(\phi_{(v)}, r_{e_{(v)}}\right)$

The torque estimation error will be

$$\Delta r_{e_{(v)}} = r_{e_{(v)}} - f\left(\phi_{(v)}, \beta^{(k)}\right)$$

We would like to minimize

$$\sum_{v=1}^{s} \Delta r_{e_{(v)}}^2$$

Model-Based Estimator

Compute the jacobian

$$J_{(v)} = \left[\frac{\partial f\left(\phi_{(v)}, \beta^{(k)}\right)}{\partial \beta_1} \cdots \frac{\partial f\left(\phi_{(v)}, \beta^{(k)}\right)}{\partial \beta_n} \right]$$

The parameter correction will be

$$\Delta\beta = \left(\mathbf{J}^T\mathbf{J}\right)^{-1}\mathbf{J}^T\Delta\mathbf{r}_e$$

New estimation of the parameters

$$\beta \approx \beta^{(k+1)} = \beta^{(k)} + \Delta\beta$$

The parameters of the flexible torque model will be also the parameters present in the stiffness model, since

$$\sigma(\phi) = \frac{\partial \tau_e(\phi)}{\partial \phi} = \frac{\partial f(\phi, \beta)}{\partial \phi} = g(\phi, \beta)$$

Second-order Residual

We take into account the second-order residual

$$r_\sigma = K_2 \left[p + D_\theta \theta - \int_0^t \tau + \int_0^t r_\sigma \, dt_1 dt_2 \right] - K_1 \int_0^t r_\sigma \, ds$$

Its derivates are

$$\dot{r}_\sigma = K_2 \left[\dot{p} + D_\theta \dot{\theta} - \tau - \int_0^t r_\sigma \, dt_2 \right] - K_1 r_\sigma$$

$$\ddot{r}_\sigma = K_2 \left[\dot{\tau}_e(\phi) - r_\sigma \, dt_2 \right] - K_1 \dot{r}_\sigma,$$

Second-order stable filter of

$$\dot{\tau}_e(\phi) = \frac{\partial \tau_e(\phi)}{\partial \phi} \dot{\phi} = \sigma(\phi) \dot{\phi}$$

Black-Box Estimator

The stiffness could be obtained from the residual as

$$\sigma(\phi) = \frac{r_\sigma}{\dot{\phi}}$$

which is singular if there is no velocity of transmission deformation

We apply thus the regressor

$$\begin{aligned}\hat{\sigma}(k+1) &= \hat{\sigma}(k) + K_p \left(r_\sigma \dot{\phi} - \hat{\sigma}(k) \dot{\phi}^2 \right) \\ &\simeq \hat{\sigma}(k) + K_p \dot{\phi}^2 \left(\sigma(\phi) - \hat{\sigma}(k) \right)\end{aligned}$$

The estimated stiffness converge to the real stiffness with a weighted proportional factor $K_p \dot{\phi}^2$

Black-Box Estimator

Our regressor can thus become unstable

Rewriting the regressor as

$$\hat{\sigma}(k+1) \simeq \left(1 - K_p\dot{\phi}^2\right)\hat{\sigma}(k) + \sigma(\phi)K_p\dot{\phi}^2$$

we check that regressor is asymptotically stable if

$$\left|1 - K_p\dot{\phi}^2\right| < 1 \rightarrow K_p\dot{\phi}^2 \in [0, 2)$$

To avoid unstable behaviors we use a simple stability recovery algorithm

$$\text{if } K_p\dot{\phi}^2 > F_{max} \;\Rightarrow\; K_p = \frac{F_{max}}{\dot{\phi}^2}$$

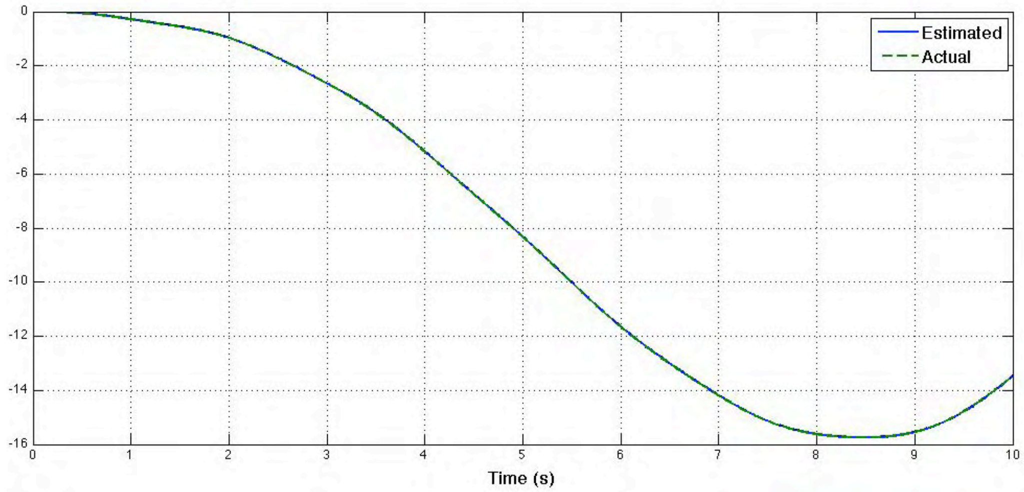
Simulation 1: Constant stiffness

Joint with a constant stiffness transmission

$$\begin{aligned} M\ddot{q} + D_q\dot{q} + K\phi + g(q) &= 0 \\ B\ddot{\theta} + D_\theta\dot{\theta} - K\phi &= \tau \end{aligned}$$

With K=100 Nmm/rad

The torque estimated with the first-order residual is



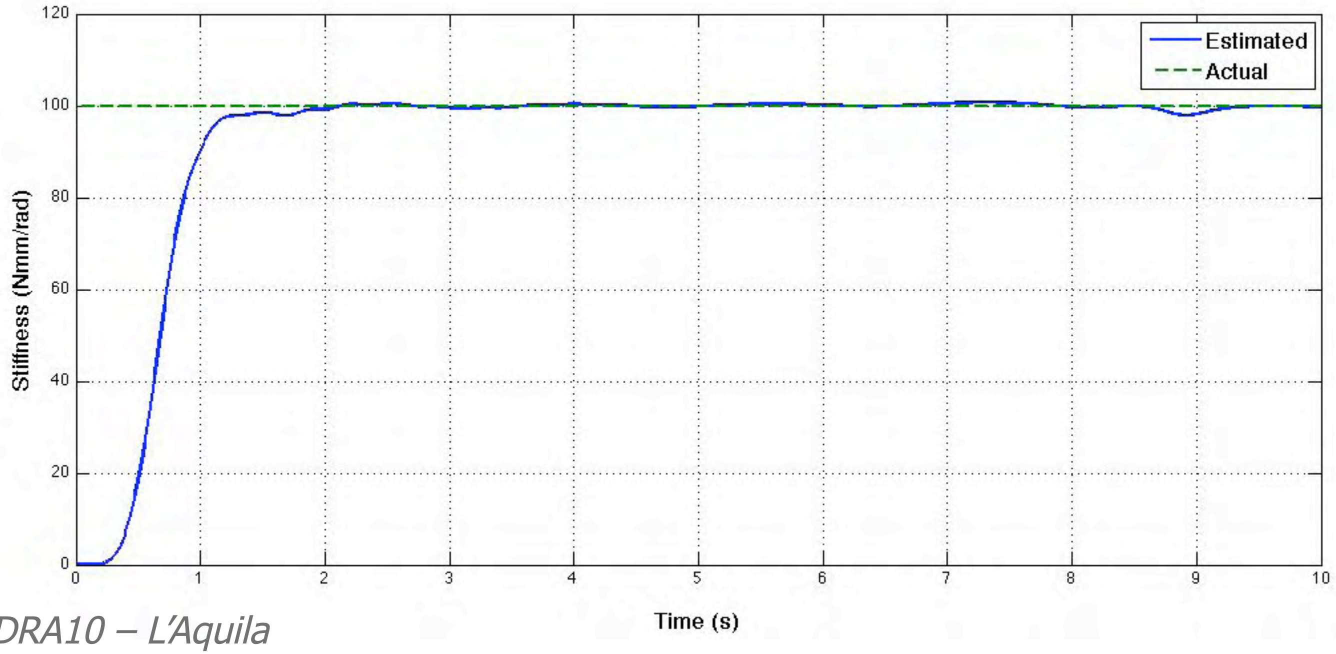
Simulation 1: Constant Stiffness

Considering the model of the elastic torque

$$\tau_e(\phi) = K\phi$$

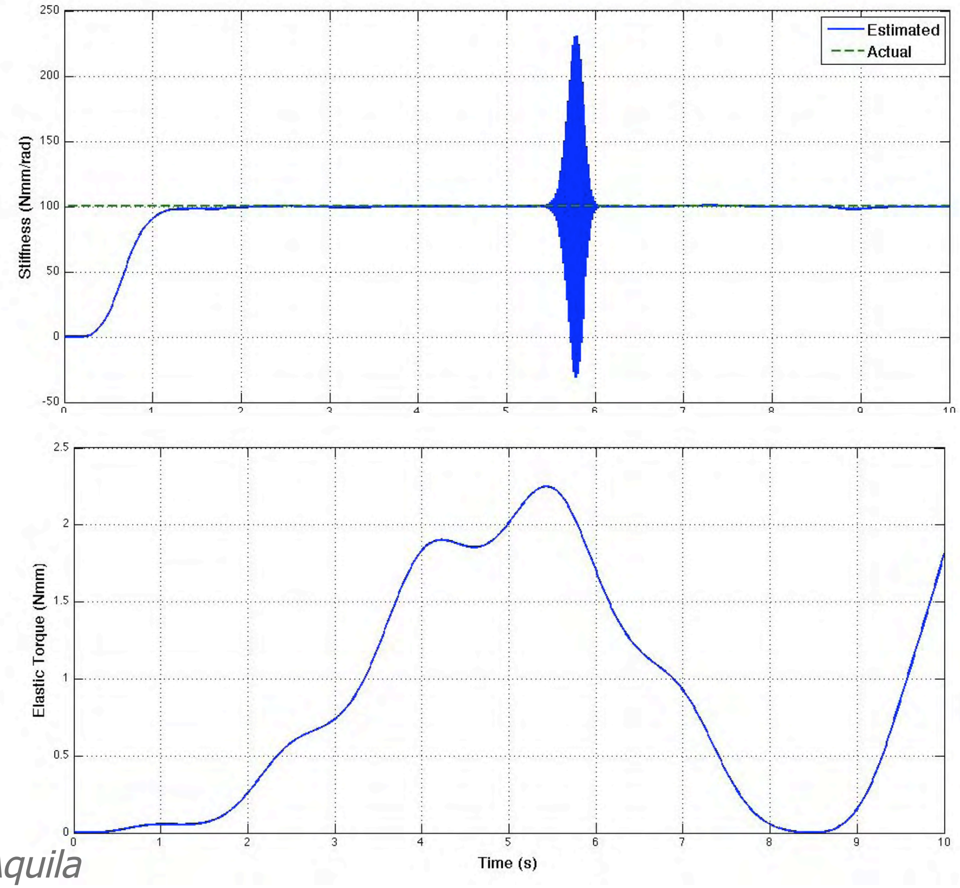
and using the MBE method, we estimate K=99.97

With the second-order residual and the BBE, we obtain



Simulation 1: Constant stiffness

Without the stability recovery algorithm, the BBE method becomes unstable as soon as $K_P \dot{\phi}^2 > 2$

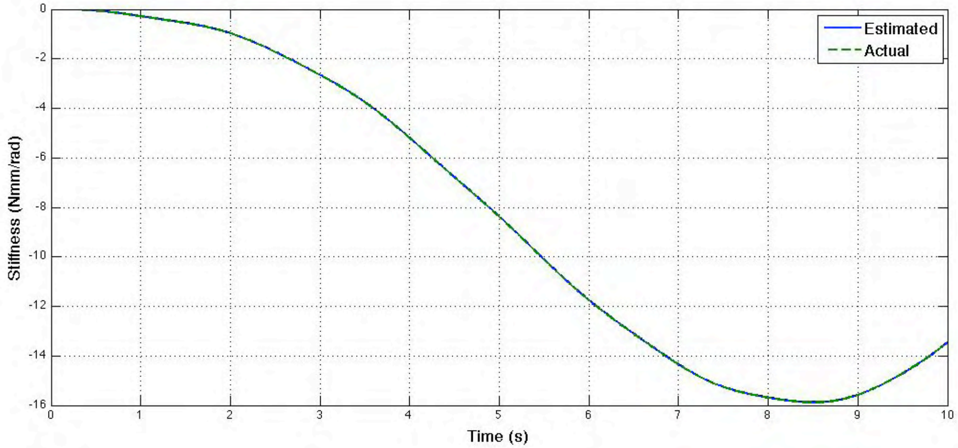


Simulation 2: Nonlinear stiffness

Nonlinear transmission with flexible torque $\tau_e(\phi) = K_a\phi + K_b\phi^3$

Where $K_a = 100$ and $K_b = 500$

The first-order residual is



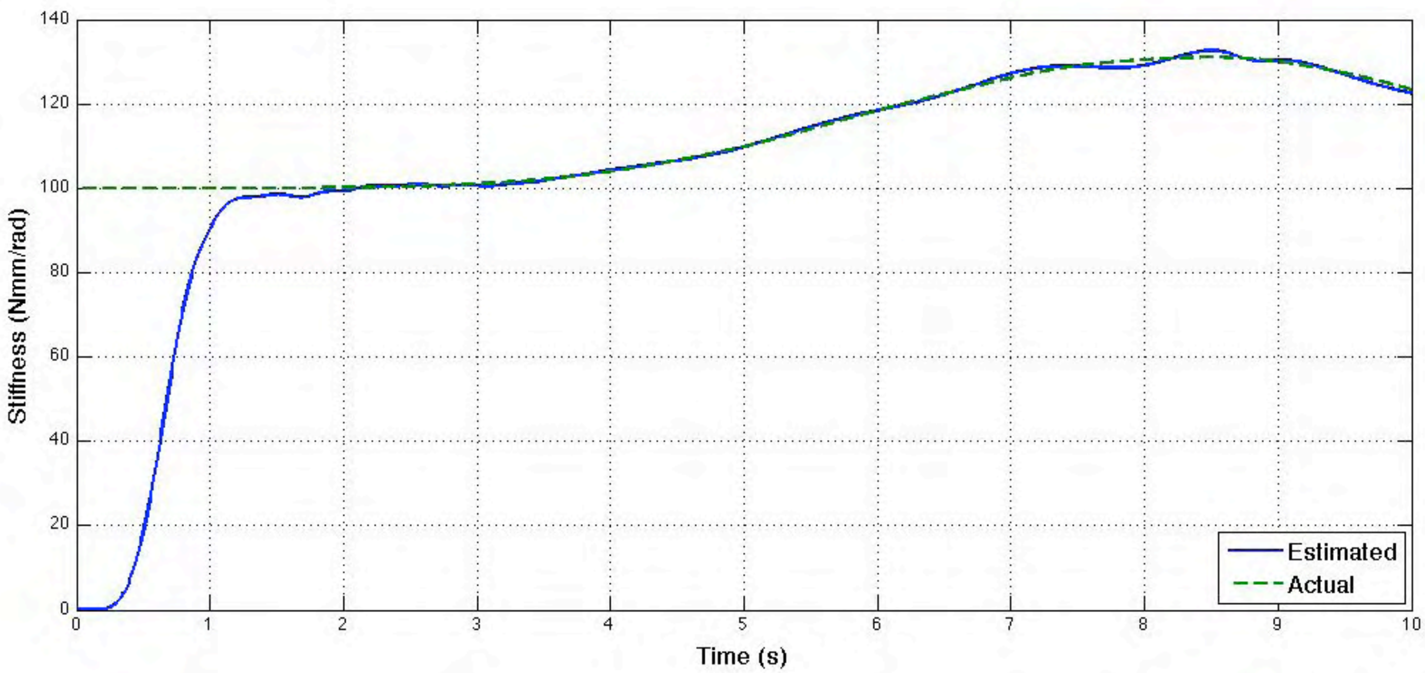
With the MBE, we obtain $K_a = 99.78$ and $K_b = 511.4$

and for the stiffness model

$$\sigma(\phi) = K_a + 3K_b\phi^2$$

Simulation 2: Nonlinear stiffness

The online estimation of the stiffness with the second-order residual and the BBE is



Simulation 3: VSA-II

In the third simulation we consider the UNIPI VSA-II, which has two motors in antagonistic arrangment and where the elastic torque is

$$\tau_{e_i}(\phi_i) = 2k_i\,\beta(\phi_i)\,\frac{\partial\beta(\phi_i)}{\partial\phi_i}, \qquad i = 1,2$$

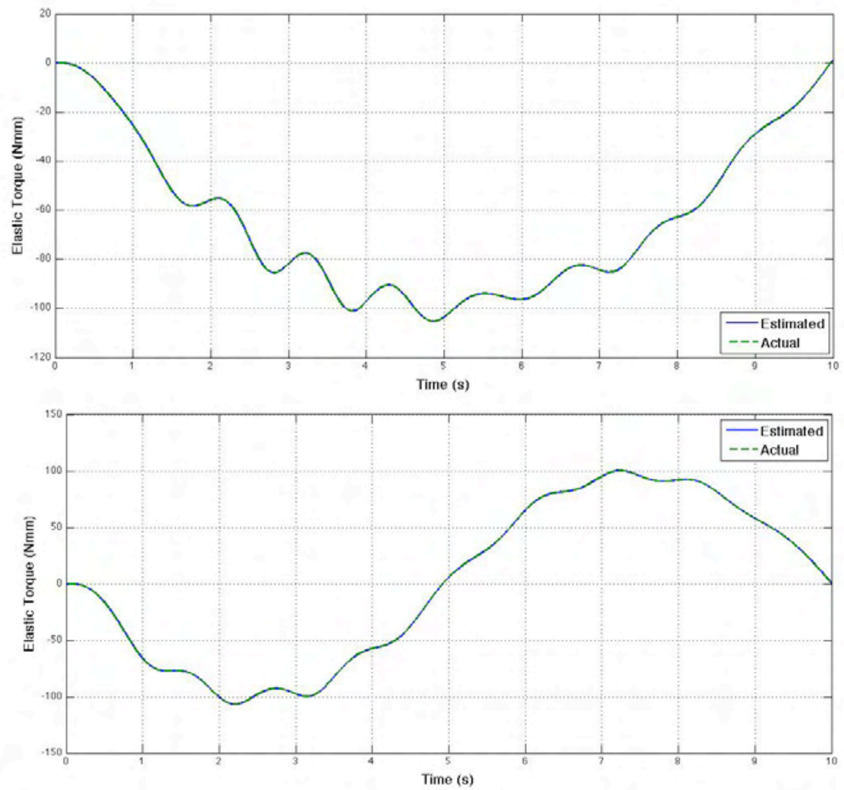
$$\beta(\phi_i) = \arcsin\left(C_i\,\sin\left(\frac{\phi_i}{2}\right)\right) - \frac{\phi_i}{2}, \qquad i = 1,2$$

$$\tau_{e_{TOT}} = \tau_{e_1} + \tau_{e_2}$$

The parameters are k=500 and C=1.75 (for each i)

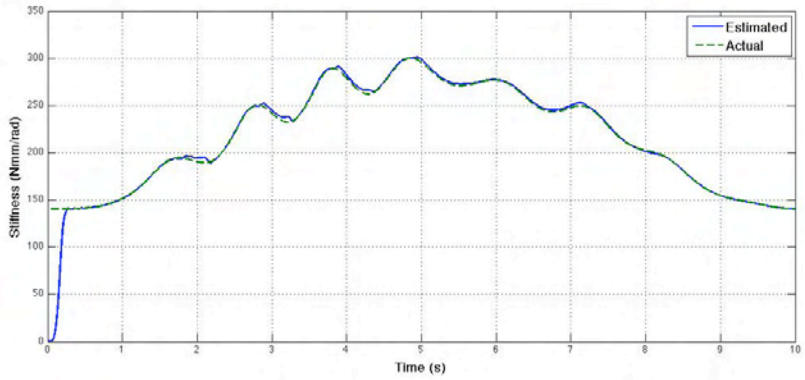
Simulation 3: VSA-II

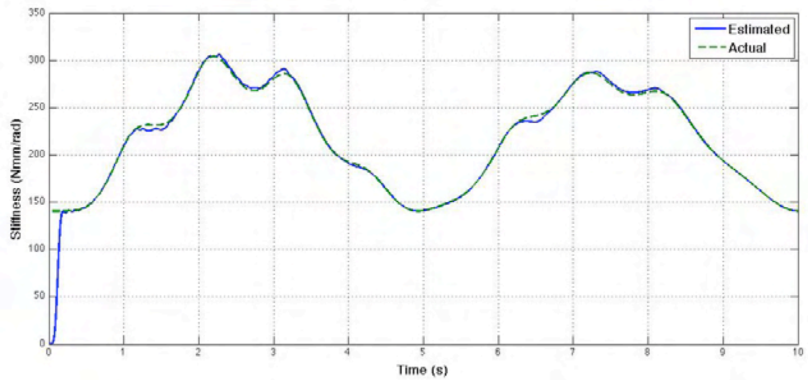
The estimated elastic torque for the two sides, obtained with the first-order residual, are



Simulation 3: VSA-II

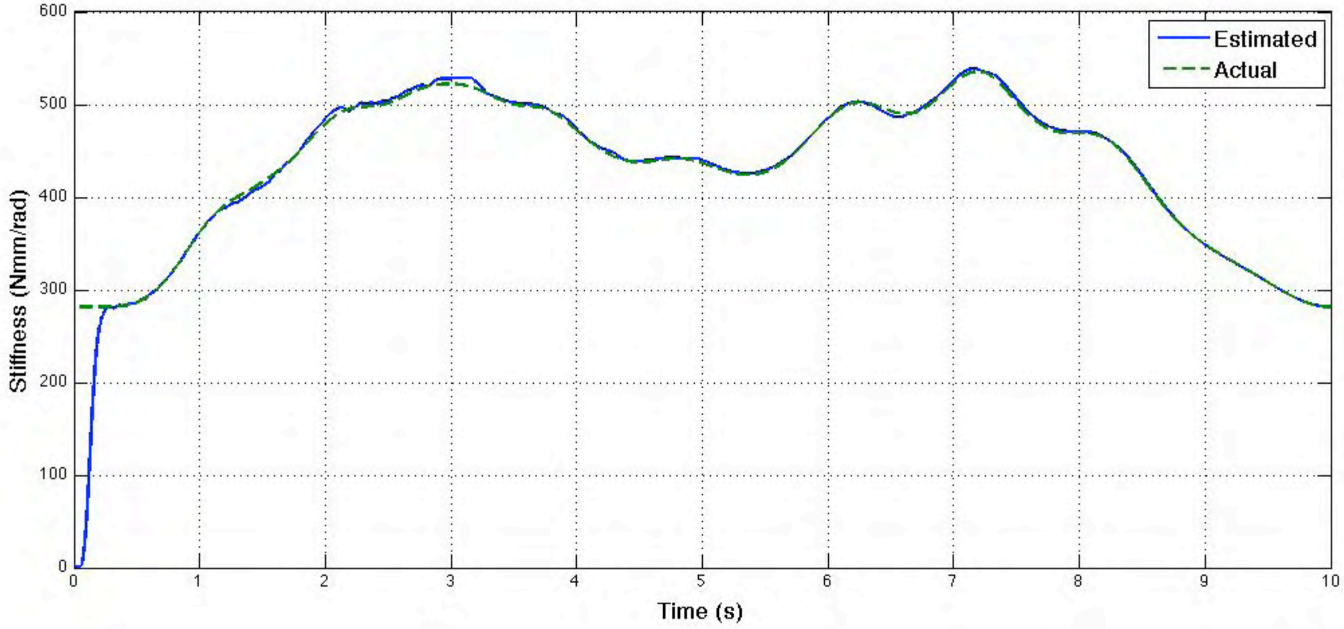
The stiffness estimated with the second-order residual and the BBE are





Simulation 3 : VSA-II

The total stiffness estimated is then the sum:



Conclusion and Future Work

We have presented:

- an offline model-based estimator
- an online black-box estimator

for the stiffness of a flexible transmission

Work in progress:

Experiments using a joint with
a nonlinear flexibility