NAME, SURNAME AND STUDENT NUMBER (* required fields):

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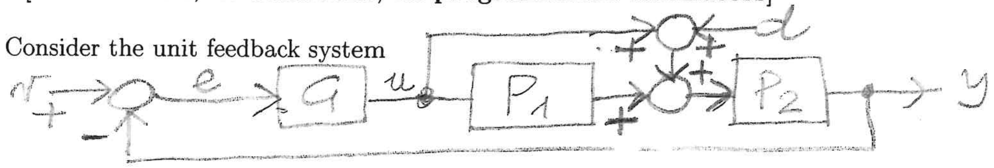
CONTROL SYSTEMS - 16/9/2019

[time 3 hours; no textbooks; no programmable calculators]

1) Consider the unit feedback system

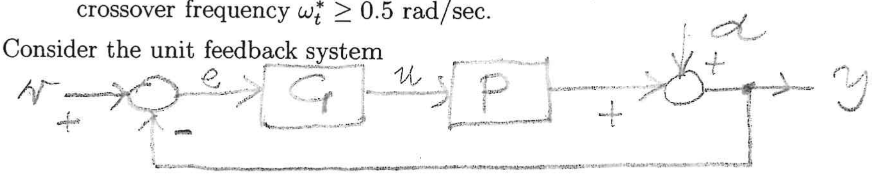


with disturbance d, input v, error e and output y, $P_1(s) = \frac{1}{s(s+2)}$ and $P_2(s) = \frac{s-2}{(s+1)^2}$. Design a controller $G(s)$ such that

(i) the closed-loop system is asymptotically stable (use Nyquist criterion), the steady state output response $y_{ss}(t)$ to constant disturbances $d(t)$ is 0, the steady state error response $e_{ss}(t)$ to unit ramp inputs $v(t) = t$ satisfies $|e_{ss}(t)| \leq 0.1$,

(iii) the open loop system has phase margin $m_\phi^* \geq 45°$ rad/sec and crossover frequency $\omega_t^* \geq 0.5$ rad/sec.

2) Consider the unit feedback system



with disturbance d, input v and output y and

$$P: \dot{x} = Ax + Bu, \; y = Cx,$$

$$A = \begin{pmatrix} 0 & 2 \\ -1 & -3 \end{pmatrix}, \; B = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \; C = \begin{pmatrix} -1 & -2 \end{pmatrix}.$$

Design a controller $G(s)$ with minimal dimension such that for the closed-loop system:

(i) the steady state output response $y_{ss}(t)$ to constant disturbances $d(t)$ and sinusoidal disturbances $d(t) = \sin(t)$ is 0,

(ii) all the eigenvalues have real part ≤ -0.3.

Draw the root locus for $PG(s)$ and find all the real values of the gain K for which the system $W(s) = \frac{KPG(s)}{1+KPG(s)}$ is asymptotically stable.

3) Given $\dot{x}(t) = Ax(t)$, with $A = \begin{pmatrix} -3 & 0 \\ 1 & 2 \end{pmatrix}$, find the initial condition $x(0) = x_0$ for which at $t_f = 2$ sec we have $x(t_f) = \begin{pmatrix} 0 & 1 \end{pmatrix}^T$. Determine $\lim_{t\to+\infty} \|x(t)\|$ for the solution $x(t)$ starting from x_0 and say if there are any solutions $x(t)$ such that $\lim_{t\to+\infty} x(t) = 0$.