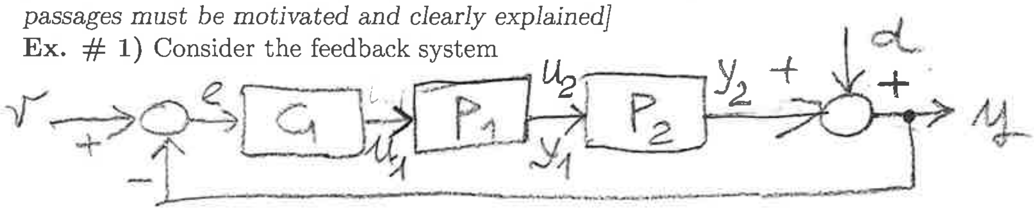
CONTROL SYSTEMS - 24/3/2023

[time 3 hours; no textbooks; no programmable calculators; all the mathematical passages must be motivated and clearly explained]

Ex. # 1) Consider the feedback system



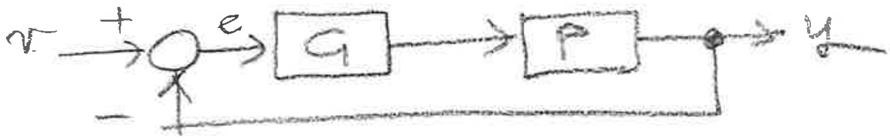
with $\mathbf{P}_1 : \dot{\mathbf{x}}_1 = -\mathbf{x}_1 + \mathbf{u}_1$, $\mathbf{y}_1 = \mathbf{x}_1$ and $\mathbf{P}_2 : \dot{\mathbf{x}}_2 = \mathbf{x}_2 + \mathbf{u}_2$, $\mathbf{y}_2 = \mathbf{x}_2$. Design a one dimensional controller $\mathbf{G}(s)$ such that

(i) the closed-loop system is asymptotically stable (use Nyquist criterion) with steady-state output response $|\mathbf{y}_{ss}(t)| \leqslant 1$ to constant disturbances $\mathbf{d}(t) = \delta^{(-1)}(t)$,

(ii) $|\mathbf{G}(j\omega)|_{dB} \leqslant 26$ dB for all ω,

(iii) the open loop system has phase margin m_φ as large as possible.

Ex. # 2) Consider the feedback system



with $\mathbf{P}(s) = \frac{1}{s+2}$. Design a controller $\mathbf{G}(s)$, with dimension $\leqslant 3$, in such a way that

- the closed-loop system is asymptotically stable with steady-state error $\mathbf{e}_{ss}(t) \equiv 0$ to constant and sinusoidal (with unit frequency) inputs $\mathbf{v}(t)$
- all the closed-loop poles have real part $\leqslant -0.3$.

Draw the root locus of the open loop system $\mathbf{GP}$ (using the Routh table). By means of the analysis of the root locus, explain if (and why) it is possible (or not) to decrease without limit the negative real part of the closed-loop poles by simply increasing the controller gain K.

Ex. # 3) Given the system

$$\dot{\mathbf{x}} = A\mathbf{x} + B(\mathbf{u} + \alpha\mathbf{d}), \mathbf{y} = C\mathbf{x},$$
$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 \end{pmatrix} \tag{1}$$

determine a controller $\mathbf{u} = F\mathbf{x}$ and the values of $\alpha \in \mathbb{R}$ such that the closed-loop system is asymptotically stable with eigenvalues $\{-1, -2\}$ and steady-state output response $|\mathbf{y}_{ss}(t)| \leqslant 0.1$ to constant disturbances $\mathbf{d}(t) = 1$. Calculate for the closed-loop system the steady-state output response $\mathbf{y}_{ss}(t)$ to a sinusoidal disturbance $\mathbf{d}(t) = \sin(t)$.