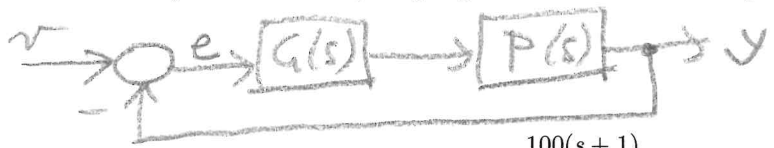
CONTROL SYSTEMS - 30/6/2020

[time 2 hours and 30 minutes; no textbooks; no programmable calculators]

1) Consider



with input v, error e, output y and process $P(s) = \frac{100(s+1)}{(s+5)(s^2+12s+20)}$.
Design a controller $G(s)$ in such a way that

(i) the closed-loop system is asymptotically stable (use Nyquist criterion with approximate Bode plots) with steady state error $e_{ss}(t)$ to $v(t) = t$ such that $|e_{ss}(t)| \leq 0.04$,

(ii) the open loop system $PG(s)$ has crossover frequency $\omega_t \in [3, 6]$ rad/sec and phase margin $m_\phi \geq 50°$.

2) Consider

with input v, error e, output y and $P(s) = \frac{s+2}{s^2+1}$. Draw the root locus of $P(s)$ and design a controller $G(s)$ with minimal dimension such that

(i) the closed-loop system is asymptotically stable with steady state error $e_{ss}(t) = 0$ to inputs $v(t) = \delta_{-1}(t)$.

With the same structure of $G(s)$ check if it is possible to obtain closed-loop poles with the same real part $-\alpha < 0$ and determine this value α.
Finally, design a strictly proper controller $G(s)$ satisfying (i).

3) Given the system $\dot{x} = Ax + Bu$, $y = Cx$, where

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 1 \end{pmatrix} \qquad (1)$$

decompose the system into controllable and uncontrollable subsystems and discuss the stability of these subsystems. Determine if there exists $t \geq 0$ such that $e^{At}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.