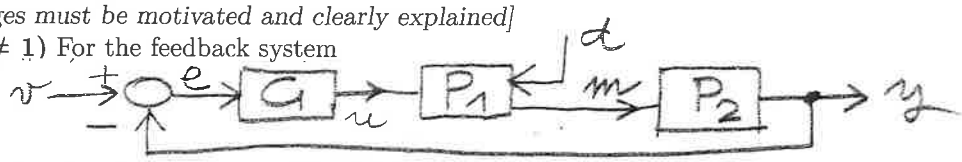
CONTROL SYSTEMS (A) - 3/2/2023

[time 3 hours; no textbooks; no programmable calculators; all the mathematical passages must be motivated and clearly explained]

Ex. # 1) For the feedback system



where $\mathbf{P}_1 : \dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}, \ \mathbf{m} = C\mathbf{x} + \mathbf{u} + \mathbf{d}$ with

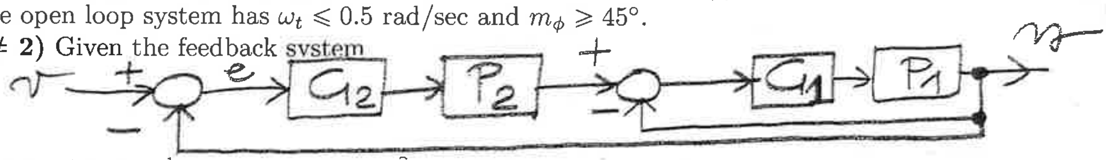
$$A = \begin{pmatrix} 0 & 1 \\ 0 & -2 \end{pmatrix}, \ B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 \end{pmatrix},$$

and $\mathbf{P}_2 = \frac{s-2}{(s+1)^2}$, design a controller $\mathbf{G}(s)$ with minimal dimension in such a way that:

(i) the feedback system is asymptotically stable (use Nyquist criterion as stability test) with steady-state error response $|\mathbf{e}_{ss}(t)| \leqslant 0.1$ to inputs $\mathbf{v}(t) = t$ and steady-state output response $\mathbf{y}_{ss}(t) = 0$ to constant disturbances $\mathbf{d}(t)$.

(ii) the open loop system has $\omega_t \leqslant 0.5$ rad/sec and $m_\phi \geqslant 45°$.

Ex. # 2) Given the feedback system



with $\mathbf{P}_1(s) = \frac{1}{s(s-2)}$ and $\mathbf{P}_2(s) = \frac{s-2}{s+3}$, design the controllers $\mathbf{G}_1(s)$ and $\mathbf{G}_2(s)$ in such a way that:

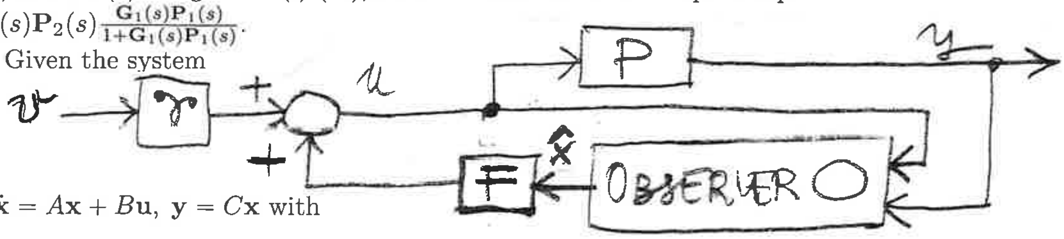
(i) $\mathbf{G}_1(s)\mathbf{G}_2(s)$ has dimension $\leqslant 2$, $\mathbf{G}_1(s)$ is proper and $\mathbf{G}_2(s)$ is strictly proper

(ii) the poles of the internal feedback system $\frac{\mathbf{G}_1(s)\mathbf{P}_1(s)}{1+\mathbf{G}_1(s)\mathbf{P}_1(s)}$ are all in $\mathbb{C}^-$ with real part $\leqslant -1$

(iii) the feedback system is asymptotically stable with steady-state error $\mathbf{e}_{ss}(t) = 0$ to constant inputs $\mathbf{v}(t)$.

With $\mathbf{G}_1(s)$ and $\mathbf{G}_2(s)$ designed in (i)-(iii), draw the root locus of the open loop system $\mathbf{G}_2(s)\mathbf{P}_2(s)\frac{\mathbf{G}_1(s)\mathbf{P}_1(s)}{1+\mathbf{G}_1(s)\mathbf{P}_1(s)}$.

Ex. # 3) Given the system



where $\mathbf{P} : \dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}, \ \mathbf{y} = C\mathbf{x}$ with

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 \end{pmatrix},$$

design $\gamma \in \mathbb{R}$, $F \in \mathbb{R}^{1\times 2}$ and the state observer O in such a way that the system is asymptotically stable with steady-state output response $\mathbf{y}_{ss}(t) = 1$ to inputs $\mathbf{v}(t) = 1$ and with all eigenvalues in -2 (*Hint*: here the state observer O is a dynamical system which computes an asymptotic estimate $\hat{\mathbf{x}}$ of the state $\mathbf{x}$ of $\mathbf{P}$ from $\mathbf{y}$ and $\mathbf{u}$).