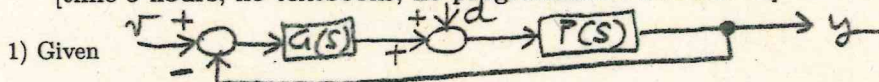


CONTROL SYSTEMS - 5/2/2019 (B)

[time 3 hours; no textbooks; no programmable calculators]

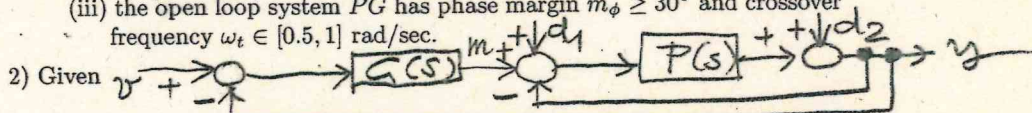


with  $P(s) = \frac{2s+1}{s^2}$  design a 1-dimensional controller  $G(s)$ , if any, such that

- (i) the feedback system  $W(s) = \frac{PG(s)}{1+PG(s)}$  is asymptotically stable (use the Nyquist criterion with reasonable approximations for the Bode plots) and its steady state output response  $y_{ss}$  to constant disturbances  $d$  is 0,

(ii)  $|j\omega G(j\omega)|_{dB} \leq -8 \text{ dB } \forall \omega$

- (iii) the open loop system  $PG$  has phase margin  $m_\phi \geq 30^\circ$  and crossover frequency  $\omega_c \in [0.5, 1]$  rad/sec.



with  $P(s) = \frac{2s+3}{s^3-3s^2-3}$ , determine, if any, a 2-dimensional controller  $G(s)$  such that the given feedback system has the following properties:

- i) it is asymptotically stable with poles having negative real part  $\leq -1$
- ii) the steady state output response to constant disturbances  $d_1$  is 0
- ii) the absolute value of the steady state output response to unit ramp disturbances  $d_2$  (i.e.  $d_2 = t$ ) is  $\leq 0.1$ .

- 3) Given  $P(s) = \frac{(s+1)^2}{(s^2+1)s}$  draw the root locus of  $P$  and design, if any, a controller  $G_1(s) = K$  such that the closed-loop system  $W(s) = \frac{PG_1(s)}{1+PG_1(s)}$  is asymptotically stable. With  $G_1(s) = \frac{1}{s-1}$  draw the root locus of  $PG_1$  (help: the positive root locus has the singular points  $s \approx 0.2 \pm 0.6j$  for  $k \approx 0.2$  and  $s \approx 0.4$  for  $k \approx 0.1$ ; the negative root locus has the singular point  $s \approx -2.4$  for  $k \approx -28$ ). Design, if any, a controller  $G_2(s) = K$  such that the closed-loop system  $W(s) = \frac{PG_1G_2(s)}{1+PG_1G_2(s)}$  is asymptotically stable.

4) Given the system

$$\dot{x} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} d + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u, \quad y = \begin{pmatrix} 0 & 1 \end{pmatrix} x, \quad (1)$$

with state  $x \in \mathbb{R}^2$ , control  $u$ , controlled output  $y$ , disturbance  $d = Dt$ , with unknown  $D \in \mathbb{R}$ , and reference input  $v = e^{-t}$ , find, if possible, a controller such that the closed loop system is asymptotically stable and its steady state output response  $y_{ss}$  is  $v$ .