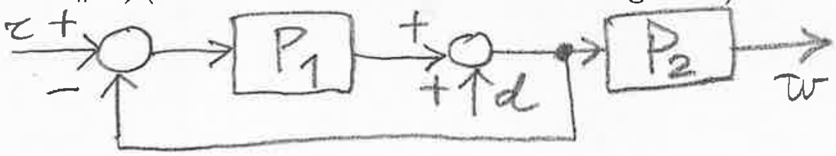
CONTROL SYSTEMS (a) - 8/1/2024

[time 3 hours; no textbooks or notes; no programmable calculators; all the mathematical passages must be motivated and clearly explained]

Ex. # 1) Given $\mathbf{P}(s) = -\frac{10}{s(s+1)}$ design a controller $\mathbf{G}(s)$ such that
(i) the feedback system $\frac{\mathbf{G}(s)\mathbf{P}(s)}{1+\mathbf{G}(s)\mathbf{P}(s)}$ is asymptotically stable (use Nyquist criterion for assessing stability) with steady-state error $|\mathbf{e}_{ss}(t)| \leqslant 0.01$ to inputs $\mathbf{v}(t) = t$
(ii) the open-loop system $\mathbf{G}(s)\mathbf{P}(s)$ has crossover frequency $\omega_t^* = 10$ rad/sec and phase margin $m_\phi^* \geqslant 50°$
(iii) $\mathbf{G}(s)$ has minimal dimension and $|\mathbf{G}(j\omega)|_{dB} \leqslant 40$ dB for all $\omega \geqslant 0$.

Ex. # 2) Given $\mathbf{P}(s) = \frac{(s+\frac{2}{3})(s+2)}{(s-2)^2(s+3)}$
(i) draw accurately the root locus of $\mathbf{P}$, using the Routh table for determining the crossing points of the imaginary axis (there is a singular point at $s = -\frac{4}{3}$).
(ii) design a controller $\mathbf{G}(s) = K$ such that the feedback system $\frac{\mathbf{G}(s)\mathbf{P}(s)}{1+\mathbf{G}(s)\mathbf{P}(s)}$ is asymptotically stable with all negative real poles
(iii) design a one-dimensional controller $\mathbf{G}(s)$ such that the feedback system $\frac{\mathbf{G}(s)\mathbf{P}(s)}{1+\mathbf{G}(s)\mathbf{P}(s)}$ is asymptotically stable with all real poles < -2
(iv) design a two-dimensional controller $\mathbf{G}(s)$ such that the feedback system $\frac{\mathbf{G}(s)\mathbf{P}(s)}{1+\mathbf{G}(s)\mathbf{P}(s)}$ is asymptotically stable with all real poles < -4
(v) design a two-dimensional controller $\mathbf{G}(s)$ such that the feedback system $\frac{\mathbf{G}(s)\mathbf{P}(s)}{1+\mathbf{G}(s)\mathbf{P}(s)}$ is asymptotically stable with all real negative poles and steady-state error $\mathbf{e}_{ss}(t) = 0$ to sinusoidal inputs $\mathbf{v}(t) = \sin(t)$.

Ex. # 3) (**also for students with 3-credits integration**) Given



with $\mathbf{P}_1(s) = \frac{s+1}{s+2}$ and $\mathbf{P}_2(s) = \frac{1}{s+1}$
(i) determine a state space realization S with two states $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2)^\top$, input $\mathbf{r}$, disturbance $\mathbf{d}$ and output $\mathbf{w}$.
For S with $\mathbf{d} = 0$:
(ii) compute the set of reachable states from the origin with control $\mathbf{r}$; determine if $x_f = (1, 1 + \frac{1}{e})^\top$ is reachable from $x_0 = (0,1)^\top$ and, if yes, compute the corresponding control input function $\mathbf{r}(t)$
(iii) (**only for students with 3-credits integration**) find the controllable and observable subsystems.