CONTROL SYSTEMS - 9/9/2021

[time 2 hours and 30 minutes; no textbooks; no programmable calculators]

Ex. # 1) Given the process $\mathbf{P}(s) = \frac{-5}{s(1+s)}$, design a minimal dimensional controller $\mathbf{G}(s)$ such that

(i) the feedback system $\mathbf{W}(s) = \frac{\mathbf{PG}(s)}{1+\mathbf{PG}(s)}$ is asymptotically stable (check with Nyquist criterion) with steady-state error $|\mathbf{e}_{ss}(t)| \leqslant 0.02$ to ramp inputs $\mathbf{v}(t) = t$,

(ii) $20log_{10}|\mathbf{G}(j\omega)| \leqslant 36dB$ for all $\omega > 0$,

(iii) the open-loop system $\mathbf{PG}$ has crossover frequency $\omega_t \geqslant 8$ rad/sec and phase margin $m_\phi \geqslant 50°$.

For the feedback system $\mathbf{W}(s)$ calculate the steady-state error to an input $\mathbf{v}(t) = \sin t$.

Ex. # 2) Given the process $\mathbf{P}$ described by:

$$\begin{aligned} \dot{\mathbf{x}}_1 &= \mathbf{x}_2 \\ \dot{\mathbf{x}}_2 &= -4.5\mathbf{x}_1 - 4.5\mathbf{x}_2 + \mathbf{u} \\ \mathbf{y} &= 1.5\mathbf{x}_1 + \mathbf{x}_2 \end{aligned} \tag{1}$$

design a controller $\mathbf{G}(s)$ such that the feedback system $\mathbf{W}(s) = \frac{\mathbf{PG}(s)}{1+\mathbf{PG}(s)}$

- has eigenvalues with real part less or equal than -1;

- has zero steady-state response to additive in the output disturbances $\mathbf{d}(t) = 1$.

Draw the root locus of $\mathbf{PG}(s)$ (use Routh criterion).

Ex. # 3) Determine the reachable states (with control $\mathbf{u}$) from a generic initial state $\mathbf{x}_0$ for

$$\begin{aligned} \dot{\mathbf{x}}_1 &= \mathbf{u} + \mathbf{x}_1 \\ \dot{\mathbf{x}}_2 &= 0, \end{aligned} \tag{2}$$

and if possible a control law $\mathbf{u}(t)$ which steers the system' state from the initial state $\mathbf{x}_0 = (0,1)^\top$ into $\mathbf{x}_f = (1,1)^\top$. Also, determine the indistinguishable states from the output $\mathbf{y} = \mathbf{x}_2$. If $\mathbf{y} = (\mathbf{x}_1, \mathbf{x}_2)^\top$ how the set of indistinguishable states from the output $\mathbf{y}$ becomes?