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Localization of Mobile Robot using Particle Filter

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Abstract: Localization is an important topic in mobile robots. It is essential that a mobile robot plans movement and reaches goals. In this paper, we described self-localization technique for mobile robot based on particle filtering in active beacon system. The basic method is to estimate value of position and heading of mobile robot using ultrasonic sensor as particle filter is used to eliminate process and measurement noise. Several variants of the particle filter such as SIR and RPF are introduced. These are discussed and compared with EKF for localization of mobile robot.

Keywords: Mobile robot, particle filter, ultrasonic sensor, localization.

1. INTRODUCTION

Localization of mobile robotics is to determine their position and heading with respect to known locations in the environment. It is one of the most important issues for mobile robot researches since it is essential to mobile robot for long term reliable operation. The most common and basic method of performing localization is through dead-reckoning [1]. This technique integrates the velocity history of the robot over time to determine the change in position from the starting location. However dead-reckoning methods are prone to errors that grow without bound over time because effects of wheel slippage and an incomplete sensor reading cause the mobile robot's intrinsic uncertainties. Other localization systems use beacons placed at known position in the environment [2]. It uses ultrasonic pulses to determine a distance between robot and beacons, and estimate a position of mobile robot. Although the method has large noise randomly, it has advantage that error is not accumulated. Besides method of localization is a laser system [3]. It is very precise, but considerably expensive than ultrasonic sensor

Kalman filter (KF) is used to eliminate system noise and measurement noise. Kalman filter is an estimator for linear-quadratic problem, which is the problem of estimating the instantaneous state of a linear dynamic system perturbed by white noise [4]. The resulting estimator is statistically optimal with respect to any quadratic function of estimation error. Many dynamic systems and sensor are not absolutely linear. It is used to Extended Kalman filter (EKF) in nonlinear system. The main feature of Extended Kalman filter is that it linearizes the nonlinear functions in the state dynamic and measurement models. Its estimate value is approximated. Kalman Filter assumes that noise is Gaussian distribution. However real data can be very complex, typically involving elements of non-Gaussianity, high dimensionality and nonlinearity, whose conditions usually preclude analytic solutions.

Sequential Monte Carlo (SMC) methods are a set of simulation-based methods which provide a convenient and attractive approach to computing the posterior distributions. Particle filtering is one of SMC methods [5]. They perform sequential Monte Carlo estimation based on point mass (or "particle") representation of probability densities. These methods have the great advantage of not being subject to any linearity or Gaussiantiy constraints on the model, in addition they also have applealing convergence properties.

In this paper, we proposed self-localization method for mobile robot using particle filtering in active beacon system. The paper is structured as follows. In Section 2, a brief formulation of the problem of robot localization based on active beacon system is given. In Section3 and 4, we demonstrated EKF and particle filter. Section 5 describes the localization algorithm using particle filtering. The experimental results that we have achieved with simulation and real data are described in Section 6. Finally, Section 7 summarizes the paper and gives some concluding remarks.

2. LOCALIZATION IN ACTIVE SYSTEM

The components of the active beacon system are shown in Fig.1 [6]. There are ultrasonic beacons and receivers. The beacons are placed on the wall or around the edges of the environment. The receiver is fixed on mobile robot.

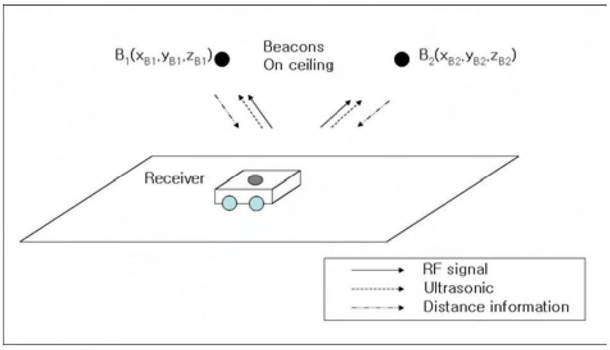

Fig.1 Localization in active beacon system.

The receiver transmits RF signal and ultrasonic to beacons. Upon receipt of receiver RF signal, beacons then listen for the corresponding ultrasonic pulse. When this pulse arrives, each beacon obtains a distance estimate from the receiver by taking advantage of the difference in propagation speed between RF and ultrasound. And beacon send distance information to the mobile robot with RF. So, Mobile robot can estimate its position using triangulation. The number of beacon can be adjusted to size of environment. But beacons must listen to receiver signal more than two at the least, assuming that mobile robot moves on the plane.

In active beacon system, system and measurement equations is given as follows

$$r_k = f(r_{k-1}, w_{k-1})$$

$$\begin{bmatrix} x_k \\ y_k \\ \theta_k \end{bmatrix} = \begin{bmatrix} x_{k-1} \\ y_{k-1} \\ \theta_{k-1} \end{bmatrix} + \begin{bmatrix} COS\theta_{k-1} & 0 \\ SIN\theta_{k-1} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} d \\ \Delta\theta_{k-1} \end{bmatrix} + w_{k-1} \quad (1)$$

$$z_k = h(r_k^i, v_k)$$

$$= \begin{bmatrix} (x_k^i - x_{Bi})^2 + (y_k^i - y_{Bi})^2 + (z_k^i - z_{Bi})^2 \\ (x_k^i - x_{Bj})^2 + (y_k^i - y_{Bj})^2 + (z_k^i - z_{Bj})^2 \end{bmatrix} + v_k \quad (2)$$

where r_k and z_k are state and measurement, w_{k-1} and v_k are mutually independent noise, (x_i y_i z_i) is position of beacon. Input (d , $\Delta\theta_{k-1}$) represent displacement of position and heading.

3. PARTICLE FILTERING

Particle filters approximate the posterior densities by swarms of points in the sample space. These points are called 'particles'. The particles each have an assigned weight, and the posterior distribution can then be approximated by a discrete distribution which has support on each of the particles. The probability assigned to each particle is proportional to the weight [5][7] .

In order to present the details of the algorithm, let $\{X_{0:k}^i, w_k^i\}_{i=1}^{N_s}$ denote a random measure that characterizes the posterior probability density distribution, $p\{X_{0:k} \mid Z_{1:k}\}$. $\{X_{0:k}^i, i = 1, \cdots, N\}$ is a set of particles with their associated weights $\{w_k^i, = 1, \cdots, N\}$. The weights are normalized and $\sum_i w_k^i = 1$. N is the number of particle used in the approximation. The posterior probability density distributions can be approximated as

$$p(X_k \mid Z_k) \approx \sum_1^N w^i \delta(X_k - X_k^i) \quad (3)$$

where $\delta(\cdot)$ is the Dirac delta function.

The normalized weights are chosen using the principle of importance sampling. If the samples X_k^i were drawn from an importance density $q(X_k \mid Z_k)$ by the principle of importance sampling.

$$\tilde{w}_k^i \propto \frac{p(X_k^i \mid Z_k)}{q(X_k^i \mid Z_k)} \quad (4)$$

Normalization become

$$w_k^i = \frac{\tilde{w}_k^i}{\sum_{i=1}^N \tilde{w}_k^i} \quad (5)$$

If the importance density is chosen to factorize such that

$$q(X_k \mid Z_k) \triangleq q(x_k \mid X_{k-1}, Z_k) q(X_{k-1} \mid Z_{k-1}) \quad (6)$$

Then one can obtain samples $X_k^i \sim q(X_k \mid Z_k)$ by augmenting each of the existing samples $X_{k-1}^i \sim q(X_{k-1} \mid Z_{k-1})$ with the new state $x_k^i \sim q(x_k \mid X_{k-1}, Z_k)$. To derive the weight update equation, the pdf $p(X_k \mid Z_k)$ is expressed as follow :

$$\begin{aligned} p(X_k \mid Z) &= \frac{p(z_k \mid X_k, Z_{k-1}) p(X_k \mid Z_{k-1})}{p(z_k \mid Z_{k-1})} \\ &= \frac{p(z_k \mid X_k, Z_{k-1}) p(x_k \mid X_{k-1}, Z_{k-1}) p(X_{k-1} \mid Z_{k-1})}{p(z_k \mid Z_{k-1})} \\ &\propto p(z_k \mid x_k) p(x_k \mid x_{k-1}) p(X_{k-1} \mid Z_{k-1}) \end{aligned} \quad (7)$$

By substituting into, the weight update equation can then be shown to be

$$\begin{aligned} w_k^i &\propto \frac{p(z_k \mid x_k^i) p(x_k^i \mid x_{k-1}^i) p(X_{k-1}^i \mid Z_{k-1})}{q(x_k^i \mid X_{k-1}^i, Z_k) q(X_{k-1}^i \mid Z_{k-1})} \\ &= w_{k-1}^i \frac{p(z_k \mid x_k^i) p(x_k^i \mid x_{k-1}^i)}{q(x_k^i \mid X_{k-1}^i, Z_k)} \end{aligned} \quad (8)$$

The choice of proposal distribution is one of the most critical design issues for successful particle filter. Two of those critical reasons are as follows: samples are drawn from the proposal distribution, and the proposal distribution is used to evaluate important weights. Accordingly, the support of proposal distribution must include the support of true posterior distribution. In addition, it must include most recent observations. The following equation is chosen to minimize variance of noise, w_k .

$$q(x_k \mid X_{k-1}, Y_k) = p(x_k \mid X_{k-1}, Y_k) \quad (9)$$

Although it does not include most recent observation - because it is easy to implement – the most common choice is the prior distribution as follows:

$$q(x_k \mid X_{k-1}, Y_k) = p(x_k \mid x_{k-1}) \quad (10)$$

By substituting equation in equation, the weight update equation is simplified as follow:

$$\begin{aligned} w_k^i &\propto w_{k-1}^i \frac{p(z_k \mid x_k^i) p(x_k^i \mid x_{k-1}^i)}{q(x_k^i \mid X_{k-1}^i, Z_k)} \\ &= w_{k-1}^i p(z_k \mid x_k^i) \end{aligned} \quad (11)$$

Particle filter described above is called the Sequential Importance Sampling (SIS). SIS has a problem is that the discrete random measure degenerate quickly. In practical terms this means that after a certain number of recursive steps, all but one particle will have negligible. Degeneracy can be reduced by using good importance sampling function and resampling. Resampling is a scheme to eliminate particles small weights and to concentrate and replicate on particles with large weights. It is involves a mapping of random measure

$\{x_k^i, w_k^i\}$ into a random measure $\{x_k^{i*}, 1/N\}$ with uniform weights [8]. The method is Sampling Importance Sampling (SIR).

Although the SIR reduces the effect of degeneracy, it introduces other practical problem. This leads to a loss of diversity among the particles. This arises due to the fact that in the resampling stage, samples are drawn from a discrete distribution rather than a continuous one. It may lead to that all particles occupy the same point in the state spare, giving a poor representation of the posterior density. The technique that attempt to improve the problem is regularized particle filter (RPF) based on the regularization step [7].The RPF resamples particle from a continuous approximation of the posterior particle distribution:

$$p(x_k \mid Z_k) \approx \sum_{i=1}^{N} w_k^i K_h(x_k - x_k^i) \tag{12}$$

where

$$K_h(x) = \frac{1}{h^{n_x}} K(\frac{x}{h})$$

is the rescaled Kernel density, h>0 is the Kernel bandwidth, n is the dimension of the parameter vector x, and w_k^i are normalized weights. The kernel and bandwidth are chosen so as to minimize the mean integrated square error between true posterior density and the corresponding regularized empirical representation. In the special case of an equally weighted sample, the optimal choice of the kernel is the Epanechnikov kernel,

$$K_{opt} = \begin{cases} \dfrac{n+2}{2c_n}(1-\|X\|^2) & if \|X\| < 1 \\ 0 & otherwise \end{cases} \tag{14}$$

where c_n is the volume of the unit hypersphere in $\mathbb{R}^n$. When the underlying density is Gaussian with a unit covariance matrix, the optimal choice for the bandwidth is

$$h_{opt} = A \cdot N^{-\frac{1}{n+4}} \quad \text{with} A = \left[8c_n^{-1}(n+4)(2\sqrt{\pi})^n\right]^{\frac{1}{n+4}} \tag{15}$$

4. LOCALIZATION ALGORITHM

In active beacon system, localization of mobile robot is realized as following algorithm using regularized particle filter:

Step 1. Initialization – Randomly generate an initial pose of mobile robot, $r_1^i = (x_1^i, y_1^i, \theta_1^i)$, $i = 1, \ldots, N$, in location space. It is assumed that a mobile robot move on the plane.

Step 2. Prediction : Each pose is passed through the system model to obtain samples from the prior.

$$r_k^i \sim q(r_k \mid r_{k-1}^i, z_k) = p(r_k \mid r_{k-1}^t) \tag{16}$$

For it is easy to implement, proposal function is chosen by $p(r_k \mid r_{k-1}^t)$.

Step 3. Update : On receipt of the measurement z_k, evaluate the likelihood of each prior sample and obtain a normalized weight for each sample.

$$\tilde{w}_k^i \propto \tilde{w}_{t-1}^i \frac{p(z_k \mid r_k^i) p(r_k^i \mid r_{k-1}^i)}{q(r_k^i \mid r_{k-1}^i, z_k)} = p(z_k \mid r_k^i) \tag{17}$$

Normalize the importance weights

$$w_k^i = \tilde{w}_{t-1}^i / \sum_j^N \tilde{w}_{t-1}^j \tag{18}$$

And for regularization step, calculate the empirical covariance matrix S_k of $\{x_k^i, w_k^i\}_{i+1}^N$

Step 4. Resampling : Multiply/Suppress samples r_k^i with high/low importance weights w_k^i, respectively. To obtain N random samples new r_k^i approximately distributed according to $p(r_k^i \mid y_k)$. And set $w_k^i = 1/N$.

Step 5. Regularization : Generate $\varepsilon^i \sim K$, Epanechnikov kernel. Compute $r_k^i = r_k^i + hD_k\varepsilon^i$,here h is given Eq. (15) and $D_k D_k^T = S_k$.

Step 6. Increase k and iterate from step 2 to step 6.

Then, the output of the step 5, $(r_k^i, 1/N)$, is set of samples that can be used to approximate the posterior distribution.

5. EXPERIMENTAL RESULT AND PERFORMANCE OF THE SYSTEM

We made an experiment on localization of mobile robot using PF. In experiment, An initial pose of mobile robot, (x_0, y_0, θ_0), is $(50, 20, 0)^T$ where a standard unit is cm. It traveled with a constant speed, 20cm/s. An angle variation is $\Delta\theta = 5^\circ$. For the PF method, 100 particles are allocated to estimate each of the new state.

For experiment of comparison between PF and EKF, it is assuming probability density function for the position location is Gaussian. Covariance of process and measurement noise is assumed through test as followed.

$$P_{w_k} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & pi/180 \end{bmatrix} \quad P_{v_k} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}.$$

The receiver is mounted above the center of the mobile robots. Beacons are attached on the ceiling. The number of beacons is adjusted to size of location space. An interval of beacons is limited because receiver must communicate beacons more than two at the least for triangulation. The distance is affected from performance of sensor. We set it 300cm. In update step, we use two shortest distance of receiver and beacon.

The path taken by the robot during several test runs were recorded on the floor. Fig2 shows the path taken by mobile robot using PF and EKF. The actual and estimated robot trajectories for EKF and PF are described in 60 seconds. The path error is plotted in Fig3 in 300 seconds. We can confirm that the localization with PF method perform more accurate than EKF with naked eye easily. Table 1 represents root mean square error of robot state and operation time of EKF and PF for 300 seconds. Both performance and operation time is more superior in PF.

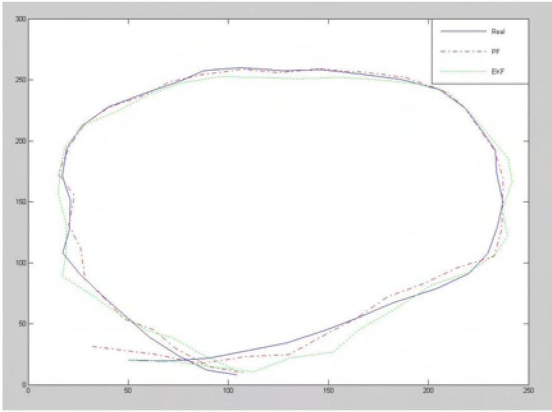

Fig2. Estimation results for the state.

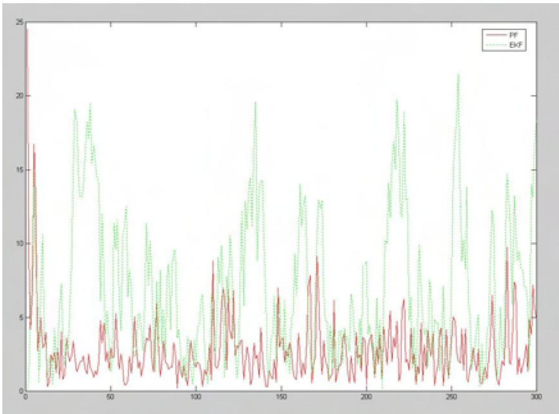

Fig3. Estimation Error.

Table 1 RMS error and Operation Time

	EKF	PF
RMSE	8.524 cm	4.231 cm
Time	7.516 second	3.062 second

In EKF, the first order Tayler approximation is causative of inaccuracy and long time of operation. Otherwise, It was not used a lot of particle in active system with PF. So, operation time is a short term than EKF. The result of experiments indicates the potential of PF for localization in active beacon system.

6. DISCUSSION

The particle filtering is introduced for location of mobile robot into the active beacon system. The simulated results from EKF and PF algorithms are compared. Use of the particle filter in real time is proved effective for estimating the position in paper.

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