A value of θ which produces interesting results in this case is $\theta = 135°$. A typical z_o then is $-0.1 \pm j1.1$, for which $2\epsilon = 0.1414$. The δ_1 and δ_2 values from (37) are $\delta_1 = 0$ and $\delta_2 = \sqrt{2}$, which reduce (18) to

$$y\left[y^2 + x(x-2\epsilon) - 1.414\epsilon\right] = 0. \tag{38}$$

Equation (38) is satisfied if

$$y = 0 \quad \text{or} \quad y^2 + (x-\epsilon)^2 = \epsilon(\epsilon + 1.414). \tag{39}$$

The circle characteristic of (39) is evident. The radius of the actual loci "circle" in Fig. 3 varies but slightly from the value of 0.324 obtained from (39) with $2\epsilon = 0.1414$.

V. Conclusion

Quasi-dipoles are pole–zero pairs embedded in suitable approximations to linear phase fields. Closed-form solutions are obtained for root locus branches in the neighborhood of quasi-dipoles. The solutions are in the form of roots of cubic equations. For practical problems in which neighboring pole–zero pairs exist, the quasi-dipole approximation can be used to study the neighboring locus branch; this can be done rapidly for a wide range of quasi-dipole orientations because iterative root-finding methods are not employed. The examples given here suggest that quasi-dipole approximations can be used over a wide range of pole–zero displacements and orientations.

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An Optimal Proportional-Plus-Integral/Tracking Control Law for Aircraft Applications

TAHM SADEGHI AND MICHAEL WOZNY

Abstract —**An optimal proportional-plus-integral/tracking control law is formulated. The control law has a command augmentation system configuration suitable for implementation on a digital computer on-board an aircraft. The proposed configuration offers the flexibility for choosing a feedforward matrix incorporating a set of additional control elements and for shaping the transient response without affecting the steady-state tracking property. Assuming the system is open-loop stable, then in the presence of a "jam" the disengaged system will maintain the steady-state tracking property which is desirable for aircraft continuing their mission.**

Introduction

With the proven record for implementation of the command augmentation systems (CAS) over the conventional stability augmentation system (SAS) [1], the exploitation of the digital-fly-by-wire (DFBW)/software redundancy management (SRM) concepts, and the innovations in aerodynamic structures [2] (e.g., chin fin, variable incidence wing panels, etc.) and control elements [2] (e.g., integrated maneuvering nozzles), the need for reconfiguration of the flight control system (FCS) of a tactical fighter aircraft in the presence of battle damage has become an important issue in future designs. The reconfiguration of FCS increases the survivability and recoverability of battle damaged aircraft and lends itself to a self-repairing FCS concept [6].

Various approaches have been used to investigate the reconfiguration of flight control systems [3], [5], [7]. Case studies were conducted [3] using the entire eigenstructure techniques [4] to develop control laws that would compensate for the loss of the right aileron or the left horizontal stabilizer, for example, using the A-7D DIGITAC II aircraft data. Parallel to this effort, analytical redundancy through the design and implementation of diagnostics filters [5] which allow regrouping of sensors to generate lost information, was also evaluated. Another approach used is the multimode and decoupled six-degree-of-freedom (6-DOF) flight control systems [7] employing additional control surfaces such as canard and differential horizontal tails. This approach allows regrouping of control surfaces through substitution of parallel modes in order to compensate for battle damage. Addressing the problems associated with the survivability and recoverability of damaged aircraft, the preceding approaches can be categorized under the concept of self-repairing flight control systems. The self-repairing flight control system concept, as defined in [6], is to detect, isolate, and recover from failed or damaged elements in the flight control system.

Where each of the preceding approaches has significant impact on increasing the rates of survivability and recoverability of tactical aircraft, a shortcoming appears in the lack of proper integration and utilization of available technologies and techniques. To this end a new control system design technique has been proposed in this note which integrates the concepts behind the optimal, proportional-plus-integral/tracking (OPIT) control law and the command augmentation system (CAS) suitable for implementation on a DFBW/SRM computer on-board a tactical aircraft. The OPIT/CAS technique suggests utilizing a set of innovative control elements in conjunction with the conventional control surface set. It provides the capability of disengaging these two sets while maintaining the flying characteristics required to continue the aircraft mission.

OPIT Control Law

Assuming the new aerodynamic structures and innovative control elements introduced by Rosenthal [2] were to be implemented on a fighter aircraft, then various configurations can be considered for operation in the presence of failure modes. For the sake of deriving the OPIT control law [8] for such an aircraft, conventional operation (e.g., using elevator, aileron, and rudder) versus full operation (e.g., using canard, chin fin, and integrated maneuvering nozzles in addition to the preceding control elements) are considered. Starting from the 6-DOF nonlinear model of the fully operational configuration, the linearized and time-invariant model at some operating point can be obtained and represented by

$$\dot{X}(t) = AX(t) + B_c U_c(t) + B_i U_i(t) \tag{1}$$

and

$$Y(t) = CX(t) \tag{2}$$

where $X(t) \in R^n$ is the state vector, $U_c(t) \in R^m$ is the conventional control vector, $U_i(t) \in R^m$ is the innovative control vector, and $Y(t) \in R^m$ is the observation vector. A, B_c, B_i, and C are constant matrices of appropriate dimensions and $n > m$. It is assumed that (A, B_c) and (A, B_i) are controllable pairs and (A, C^T) is observable.

To derive the OPIT control law, the innovative control elements are eliminated from (1). The desired control law is obtained by minimizing the quadratic cost function:

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$$J_1(F_x, F_u) = 0.5\int_0^{\infty}\left[(Y-Y_r)^T Q(Y-Y_r) + U_c^T R U_c + \dot{U}_c^T S \dot{U}_c\right] dt$$

$$Q = Q^T > 0, \qquad R = R^T > 0, \qquad S = S^T > 0 \tag{3}$$

subject to the plant equations (1)–(2).

The control law has the form

$$\dot{U}_c(t) = -F_x X(t) - F_u U_c(t) + B_r Y_r \tag{4}$$

where Y_r is the step function reference input vector to be tracked.

The following definitions define the state reference, the state error, and the augmented state vectors and will be used to derive the OPIT control law.

Definition 1: The state reference vector is defined as

$$X_r = Q_r C^T\left(C Q_r C^T\right)^{-1} Y_r \tag{5}$$

where the Q_r matrix is such that CQ_rC^T forms a nonsingular matrix. As will be shown later, the Q_r matrix can be selected for coupling the innovative control elements and further shaping of the transient responses independently of the feedback gain matrices and without affecting the steady-state tracking property.

Definition 2: The state error vector is defined as

$$X_e(t) = X(t) - X_r. \tag{6}$$

The state error vector satisfies the plant equation (1)

$$\dot{X}_e(t) = AX_e(t) + B_c U_c(t)$$

or

$$\dot{X}(t) = AX(t) + B_c U_c(t) - AX_r \tag{7}$$

where $-AX_r$ acts as a forcing function in (7).

Definition 3: The augmented state vector is defined as

$$X_a(t) = \left(X_e(t) \quad U_c(t)\right)^T. \tag{8}$$

Therefore, the augmented system equations become

$$\dot{X}_a(t) = A_a X_a(t) + B_a \dot{U}_c(t) \tag{9}$$

and

$$Y(t) = C_a X_a(t) \tag{10}$$

where

$$A_a = \begin{vmatrix} A & B_c \\ 0 & 0 \end{vmatrix}, \qquad B_a = \begin{vmatrix} 0 \\ I \end{vmatrix}, \qquad C_a = |C \quad 0|.$$

Using the preceding definitions, the problem of finding an OPIT control law can be restated in its equivalent standard regulator form.

Equivalent Standard Regulator

Given the system represented by (9), (10), find a control law

$$\dot{U}_c(t) = -F_a X_a(t) \tag{11}$$

by minimizing the quadratic cost function

$$J_2(F_a) = .5\int_0^{\infty}\left(X_a^T Q_a X_a + \dot{U}_c^T R_c \dot{U}_c\right) dt \tag{12}$$

where

$$Q_a = \begin{vmatrix} C^T Q C & 0 \\ 0 & R \end{vmatrix}$$

and $R_c = S$ The solution to this problem is obtained by solving the algebraic Riccati equation

$$A_a^T P_a + P_a A_a - P_a B_a R_c^{-1} B_a^T P_a + Q_a = 0 \tag{13}$$

and has the form

$$\dot{U}_c(t) = -R_c^{-1} B_a^T P_a X_a(t). \tag{14}$$

Substituting (6) for $X_a(t)$ in (14), the control law becomes

$$\dot{U}_c(t) = -F_x X(t) - F_u U_c(t) + F_x X_r, \tag{15}$$

where F_x and F_u are appropriate partition matrices from $R_c^{-1}B_a^T P_a$. Substituting (5) for X_r in (15), the closed-loop system takes the form:

$$\left(\dot{X}(t) \quad \dot{U}_c(t)\right)^T = \left(A_a - B_a F_a\right)\left(X(t) \quad U_c(t)\right)^T + B_r Y_r \tag{16}$$

where

$$B_r = \begin{vmatrix} -A \\ \hline +F_x \end{vmatrix} Q_r C^T\left(C Q_r C^T\right)^{-1}. \tag{17}$$

The block diagram for the closed-loop system is shown in Fig. 1. The diagram shows a direct pilot command link (a new CAS configuration) to the airframe, which is through the upper partition of matrix B_r shown by a dotted line. However, what is available for implementation of the CAS link is the control coupling matrix B_i (1). Therefore, the weighting matrix Q_r must be selected so that:

$$B_i = -AQ_r C^T\left(C Q_r C^T\right)^{-1}. \tag{18}$$

Since the OPIT control law has been derived independently of this weighting matrix, then Q_r can be chosen to satisfy (18) which has n^2 unknowns and nm equations ($n > m$). The additional degree of freedom (i.e., selection of $n(n-m)$ elements) in defining Q_r can be used to further shape the transient response of the system without affecting the steady-state tracking property. The equivalent CAS link for coupling the innovative control elements through matrix B_i is shown by a solid line (replacing the dotted CAS link) in Fig. 1.

The selection of matrix Q_r can be performed, for example, by using a computer-aided design approach [8]. Assuming matrix A is nonsingular (e.g., system is open-loop stable), then (18) becomes a set of underdetermined linear equations in elements of matrix Q_r; that is,

$$\left(A^{-1}B_i C + 1\right)Q_r C^T = 0. \tag{19}$$

Jam/Disengage Condition

In the presence of a jamming condition (e.g., failure of control surfaces, actuators, sensors, etc.), an appropriate SRM can disengage the feedback loops and the feedthrough link at the indicated place shown in Fig. 1. Under such conditions, the pilot flies the aircraft through the CAS link. The model of the disengaged system can be written as

$$\dot{X}_d(t) = AX_d(t) - AQ_r C^T\left(C Q_r C^T\right)^{-1} Y_r \tag{20}$$

and

$$Y_d(t) = CX_d(t). \tag{21}$$

Assuming the open-loop system is stable, the steady-state gain matrix is evaluated as

$$G(s)|_{S=0} = C(SI-A)^{-1}|_{S=0}(-A)Q_r C^T\left(C Q_r C^T\right)^{-1}$$

or

$$G(0) = C(-A)^{-1}(-A)Q_r C^T\left(C Q_r C^T\right)^{-1} = I. \tag{22}$$

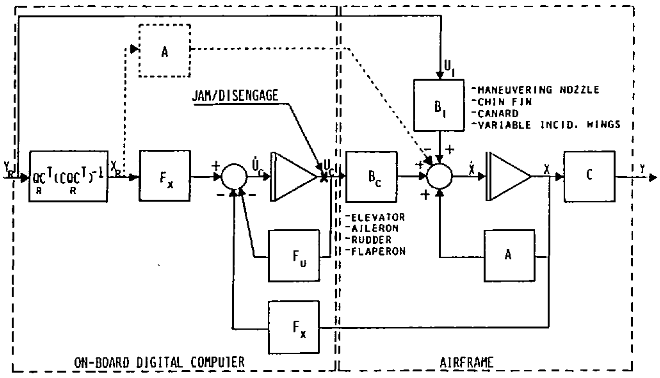

Fig. 1. OPIT/CAS control law.

Equation (22) shows that the disengaged system has the steady-state tracking property which is required for the aircraft to continue its mission.

Conclusion

The optimal, proportional-plus-integral/tracking control law introduced in this paper has a new command augmentation system configuration which is suitable for the advanced fighter technology integrator aircraft. It allows reconfiguration of control surfaces in the presence of battle damage thus maintaining tracking characteristics.

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Transfer-Function Matrix Realization Using Taylor Series Expansion About a General Point a

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***Abstract*—A procedure for the realization of a transfer function matrix by expanding it about a point a is proposed. The procedure is based on the well-known method by Ho and Kalman and its modification by Bruni *et al.* The procedure is illustrated with the help of an example.**

Considerable interest has recently been shown [1]–[6] in determining a quadruple $[A, B, C, D]$ from a given transfer function matrix $H(s)$. Such a quadruple has been called a realization [1] of the transfer function matrix. A quadruple $[A, B, C, D]$ of real constant matrices is said to be a minimal realization of a real rational matrix $H(s)$ iff $H(s)=C(sI-A)^{-1}B+D$ and A has a minimum dimension. The reason for calling the quadruple a "realization" is that once such a quadruple is obtained, it can be easily implemented on an analog computer. Ho and Kalman [2] provided a realization procedure involving the determination of Markov parameters of the transfer function matrix. Bruni *et al.* [3] used a similar procedure but determined the moments of the transfer function matrix instead of the Markov parameters. The procedure in [3] is preferable to that given in [2] when the data are contaminated with noise. However, this procedure in [3] yields A^{-1}, thus requiring an inversion to obtain the matrix A. A number of papers have appeared in the literature which exploit Markov parameters [4], moments [5], and mixtures of Markov parameters and moments [6]. Such minimal realizations are important since they are controllable as well as observable; thus they may be implemented using a minimal number of operational amplifiers. With the paper by Davidson *et al.* [7], great interest [8], [9] has been shown in expanding the given transfer function matrix about a point $s=a$. The object of this present paper is to propose a realization technique for a given transfer function matrix $H(s)$ by expansion about an arbitrary point $s=a$. The advantage of such a realization is that it is more general than the realizations obtained from moments [3] and Markov parameters [2]. The expansion of a transfer function matrix about $s=0$ (moments) ensures [10] correct steady-state response while expansion of the same about $s=\infty$ (Markov parameters) ensures [11] a correct transient response. The expansion about some arbitrary (finite) point (a in this case) obviously can result in a better overall [9], [11] time response by an appropriate choice for the value of a. Furthermore, by substituting different values of a in the quadruple $[A, B, C, D]$, one can determine a family of realizations. This correspondence, in fact, is an extension of Bruni's procedure about a point a.

Consider an m-input/p-output transfer function matrix

$$H(s)=\frac{T(s)}{\alpha(s)}=\frac{\sum_{i=0}^{n} T_{n-i}s^i}{\sum_{i=0}^{r} \alpha_{r-i}s^i}, \quad \text{and } n<r, \tag{1}$$

where the α_i $(i=0,1,\cdots,r)$ are scalar constants and the T_i $(i=0,1,\cdots,n)$ are $(p\times m)$ constant matrices. The Taylor series expansion of $H(s)$ about a point $s=a$ can be written as

$$H(s)=\sum_{i=0}^{\infty} G_i(a)(s-a)^i, \tag{2}$$

where the $(p\times m)$ matrices $G_i(a)$ are given as

$$G_i(a)=\frac{H^i(a)}{i!} \tag{3}$$

and

$$H^i(a)=\left.\frac{d^i}{ds^i}H(s)\right|_{s=a}.$$

The steps for the proposed algorithm are as follows.

1) Expand the transfer function matrix about $s=a$ using (2).

2) Construct the Hankel matrix S_k [2]–[6], by examining in succession the rank of $S_1, S_2, \cdots$ until rank $S_k=$ rank S_{k+1}, where

$$S_k=\begin{bmatrix} M_0^* & M_1^* & \cdots & M_{k-1}^* \\ M_1^* & M_2^* & \cdots & M_k^* \\ \vdots & & & \\ M_{k-1}^* & M_k^* & \cdots & M_{2k-2}^* \end{bmatrix}$$

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