VTOL Aircraft Control Output Tracking Sensitivity Design

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A method is presented for reducing trajectory sensitivity and achieving robust asymptotic tracking for linear feedback systems when there are parameter perturbations and disturbance inputs. The controller consists of a servocompensator containing the modes of the reference signals and disturbance inputs, a stabilizing feedback loop, and a feedforward compensator. Application of the method to the design of a vertical takeoff and landing (VTOL) aircraft flight control system is discussed. The use of a precompensator allows performance maneuvers such that the aircraft tracks desired trajectories and the feedforward and feedback signals aid in reducing the trajectory sensitivity to variations of parameters due to change in airspeed and to wind gust. Simulation results are presented to show the robust tracking, disturbance rejection, and sensitivity reduction capabilities of the flight control system.

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I. INTRODUCTION

The design of controllers for vertical takeoff and landing (VTOL) aircraft so that its controlled outputs follow desired reference trajectories in large flight regimes despite disturbance inputs is of considerable interest. The theory of robust servomechanisms [1] partially solves the design problem in the sense that although the desired outputs asymptotically track the reference signals, the control system does not maintain desirable response characteristics when the values of the system parameters change. Recently, sensitivity reduction methods for designing controllers when the system parameters are varying have been applied to controller design of aircraft [2–4]. However, the question of output tracking has not been studied in [2], and the uncertainties in the input matrix have not been considered in [3]. A double perfect model following system has been proposed in [4]. However, following this approach a controller of dimension $2n$ for an nth-order plant is required; moreover, the question of disturbance rejection is not considered. Although the adaptive controllers proposed in [5, 6] for controlling VTOL aircraft perform better, one requires on-board computers for the synthesis of the control law.

In this paper an approach combining the robust servomechanism theory of [1] and the sensitivity reduction method of [2] to derive a controller which accomplishes both sensitivity reduction and has output tracking capability for uncertain systems is developed. The output tracking is achieved for a class of reference inputs and disturbance signals in the closed-loop system similar to [1]. The controller includes a servocompensator driven by the error signal, a feedback element, and a feedforward compensator. The servocompensator contains all the modes of the reference and the disturbance inputs. The feedback gain is designed using the nominal parameter values. The proposed method is applied to the design of a VTOL aircraft control system [5]. The chosen controlled outputs are the horizontal and vertical velocities. It is seen that the inclusion of a servocompensator gives a good capability for maneuvering the aircraft and, unlike [2], the steady state errors in the controller outputs are zero even when the wind gust is present. Furthermore, unlike [3], uncertainties in the input matrix are also allowed here.

The organization of the paper is as follows. In Section II the structure of the servocompensator for robust tracking and disturbance rejection is presented. A control law for sensitivity reduction of the composite system consisting of the servocompensator and the plant is obtained in Section III. Application to a VTOL aircraft control system design and simulation results are presented in Section IV and Section V, respectively.

II. ROBUST TRACKING

We shall consider linear time-invariant systems of the form

$$\dot{x}(t) = A(\mu)x(t) + B(\mu)u(t) + Ew(t), \quad x(0) = x_0 \tag{1a}$$

$$y(t) = C(\mu)x(t) + Fw(t) \tag{1b}$$

where x is the n-dimensional state vector, u is the m-dimensional control vector, w is the n_d-dimensional disturbance input, μ is the p-dimensional parameter vector, y is the n_0-dimensional output vector, $A(\mu)$, $B(\mu)$, and $C(\mu)$ are time-invariant matrices whose elements are continuously differentiable with respect to μ, and the dot indicates differentiation with respect to time t.

It is desired that the output $y(t)$ tracks a reference signal $r(t)$. Let the error vector $e(t)$ be given by

$$e(t) = r(t) - y(t). \tag{2}$$

We assume that $w(t)$ and $r(t)$ can be modeled as outputs of certain dynamical systems.

$$\dot{x}_w(t) = A_w x_w(t), \qquad x_w(0) = x_{w0} \tag{3a}$$

$$w(t) = C_w x_w(t) \tag{3b}$$

$$\dot{x}_r(t) = A_r x_r(t), \qquad x_r(0) = x_{r0} \tag{4a}$$

$$r(t) = C_r x_r(t) \tag{4b}$$

where $x_w \in R^{n_w}$, $x_r \in R^{n_r}$, and R^k denotes the Euclidean space of dimension k. It is also assumed that systems (3) and (4) are observable. By choosing A_w and A_r, the reference and disturbance signals of desirable waveforms including polynomial, sinusoidal, and exponential functions are obtained.

In order to achieve robust tracking and disturbance rejection for the given class of reference and disturbance inputs of the form (3) and (4), we introduce a servocompensator of the form [1]

$$\dot{x}_c(t) = A_c x_c(t) + B_c e(t), \quad x_c \in R^{n_c}, \quad x_c(0) \in R^{n_c} \tag{5}$$

$$A_c = \text{diag}[A_{c1}, \ldots, A_{cn_0}]$$

$$B_c = \text{diag}[B_{c1}, \ldots, B_{cn_0}] \tag{6}$$

with

$$A_{ci} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\alpha_q & -\alpha_{q-1} & -\alpha_{q-2} & \cdots & -\alpha_1 \end{bmatrix},$$

$$B_{ci} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \tag{7}$$

where diag denotes a diagonal matrix, $A_{ci} \in R^{q \times q}$, the class of $q \times q$ matrices, $B_{ci} \in R^q$, and ϕ_{wr} is the least common multiple of the minimum polynomials Ψ_w and Ψ_r of A_w and A_r, respectively, given by

$$\phi_{wr} = s^q + \alpha_1 s^{q-1} + \cdots + \alpha_{q-1} s + \alpha_q. \tag{8}$$

Without loss of generality, we assume that the real parts of the roots λ_i of $\phi_{wr}(s) = 0$ are nonnegative.

We shall be interested in stabilizing control laws of the form

$$u(t) = -Lx(t) - L_c x_c(t). \tag{9}$$

The system (1) and (5) can be written as

$$\begin{bmatrix} \dot{x}(t) \\ \dot{x}_c(t) \end{bmatrix} = \begin{bmatrix} A(\mu) & 0 \\ -B_c C(\mu) & A_c \end{bmatrix} \begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix} + \begin{bmatrix} B(\mu) \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} Ew(t) \\ -B_c Fw(t) + B_c r(t) \end{bmatrix}$$

$$\triangleq \tilde{A}(\mu)\, \tilde{x}(t) + \tilde{B}(\mu) u(t) + v(w,r)$$

$$\tilde{x}(0) = \tilde{x}_0 \tag{10}$$

where $\triangleq$ denotes equality by definition, $\tilde{x} = (x^{\mathrm{T}}, x_c^{\mathrm{T}})^{\mathrm{T}} \in R^{\tilde{n}}$, $\tilde{n} = n + n_c$, T denotes the transpose of a matrix, and

$$\tilde{A}(\mu) = \begin{bmatrix} A(\mu) & 0 \\ -B_c C(\mu) & A_c \end{bmatrix}$$

$$\tilde{B}(\mu) = \begin{bmatrix} B(\mu) \\ 0 \end{bmatrix}$$

$$v(w,r) = \begin{bmatrix} Ew(t) \\ -B_c Fw(t) + B_c r(t) \end{bmatrix}. \tag{11}$$

We assume in the following that the matrix pair (A^*, B^*) is controllable and (C^*, A^*) is observable, where μ^* is the nominal value of μ, $A^* \triangleq A(\mu^*)$, $B^* \triangleq B(\mu^*)$, and $C^* \triangleq C(\mu^*)$.

It has been shown in [1] that if

$$\text{rank} \begin{bmatrix} \lambda I - A^* & B^* \\ -C^* & 0 \end{bmatrix} = n + n_0, \qquad \forall \lambda \in \sigma(A_w) \cup \sigma(A_r) \tag{12}$$

where $\sigma(M)$ denotes the spectrum of a matrix M, the following conditions are then satisfied:

(i) The composite system

$$\dot{\tilde{x}} = \tilde{A}^* \tilde{x} + \tilde{B}^* u \tag{13}$$

is controllable, where $\tilde{A}^* \triangleq \tilde{A}(\mu^*)$; $\tilde{B}^* \triangleq \tilde{B}(\mu^*)$ and, therefore, there exists a control law

$$\mu = -L_x - L_c x_c \tag{14}$$

such that the system (13) and (14) is asymptotically stable.

(ii) In the closed-loop system (10) (with $\mu = \mu^*$) and any stabilizing control (14), asymptotic tracking and disturbance rejection are achieved (i.e., $e(t) \to 0$ as $t \to \infty$ for all $x(0)$, $x_c(0)$, $x_w(0)$, and $x_r(0)$).

(iii) Asymptotic tracking and disturbance rejection are robust with the parameter μ as long as $A(\mu)$, $B(\mu)$, and

$C(\mu)$ are such that the system (10) and (14) with $v(w,r) = 0$ remains stable.

Although the closed-loop system (10) and (14) has desirable output tracking capability as $t \to \infty$, the transient characteristics of the system are not maintained when $\mu \neq \mu^*$ and disturbance inputs are present. Now we are interested in deriving a control law $\mu(t)$ which will achieve sensitivity reduction in the closed-loop system.

III. SENSITIVITY REDUCTION

For a given reference signal $r(t)$, we define a nominal system obtained from (10) of the form

$$\dot{\tilde{x}}^*(t) = \tilde{A}^*\tilde{x}^*(t) + \tilde{B}^*u^*(t) + v(0,r)$$
$$\tilde{x}^*(0) = \tilde{x}_0 \tag{15}$$

by setting $\mu = \mu^*$ and $w \equiv 0$, since $w(t)$ is unknown. We note that the initial conditions of the nominal system (15) and the system (10) consisting of the plant and the servocompensator are equal. Of course, we assume that $x(0)$ is independent of μ and is known.

Consider a control u_1 for the system (10) of the form

$$u_1(t) = -L_1x_1(t,\mu,w) - L_{c1}x_{c1}(t,\mu,w)$$
$$\triangleq -\tilde{L}_1\tilde{x}_1(t,\mu,w) \tag{16}$$

where $\tilde{L}_1 = [L_1, L_{c1}]$, and $\tilde{x}_i(t,\mu,w)$ denotes the trajectory of (10) when $u = u_i$, $i = 1,2$. Here we have used $\tilde{x}_i(t,\mu,w)$ instead of $\tilde{x}_i(t)$ to show the dependence of the solution on μ and $w(t)$ for clarity. Suppose that (16) realizes the desired nominal control u^* when $\mu = \mu^*$ and $w \equiv 0$. We are interested in constructing another control

$$u_2(t) = -L_2x_2(t,\mu,w) - L_{c2}x_{c2}(t,\mu,w) + G\tilde{x}^*(t)$$
$$\triangleq -\tilde{L}_2\tilde{x}_2(t,\mu,w) + G\tilde{x}^*(t), \qquad G \in R^{m\times(n+n_c)} \tag{17}$$

which is nominally equivalent to u_1 (i.e., $u_1 = u_2 = u^*$, whenever $\mu = \mu^*$ and $w \equiv 0$) such that the following criterion for sensitivity comparison holds for $t' > 0$

$$\int_0^{t'} \delta\tilde{x}_1^{\mathrm{T}}(t)Z\delta\tilde{x}_1(t)dt > \int_0^{t'} \delta\tilde{x}_2^{\mathrm{T}}(t)Z\delta\tilde{x}_2(t)dt \tag{18}$$

where $\tilde{L}_2 = [L_2 : L_{c2}]$ and $\delta\tilde{x}_1(t)$ and $\delta\tilde{x}_2(t)$ are the trajectory variations in $\tilde{x}_1$ and $\tilde{x}_2$, respectively, caused by parameter perturbations and disturbance input $w(t)$, and Z is a given positive semidefinite matrix. We say that system 2 (given by (10) and (17)) is less sensitive than system 1 (given by (10) and (16)) if (18) is satisfied.

Let $\tilde{x}_i(t,\mu,w) = \tilde{x}_i^* + \delta\tilde{x}_i = \tilde{x}^* + \delta\tilde{x}_i$, $u_i = u_i^* + \delta u_i = u^* - \tilde{L}_i\,\delta\tilde{x}_i$, where $\delta\tilde{x}_i$ and δu_i are state and control variations respectively, x_i^* is the solution of (15), and $u_i^* = u^*$, $\tilde{x}^* = \tilde{x}_i^*$ since u_1 and u_2 are nominally equivalent. It can be seen using (10) and (15)–(17) and neglecting second-order terms in $\delta\tilde{x}_i$ and δu_i that for the given signal r the trajectory variation of system i when $\mu \neq \mu^*$ and $w \not\equiv 0$ is $(i = 1,2)$

$$\delta\dot{\tilde{x}}_i = \tilde{A}^*\,\delta\tilde{x}_i - \tilde{B}^*\tilde{L}_i\delta\tilde{x}_i + \delta\tilde{A}\tilde{x}^* + \delta\tilde{B}u^* + v(w,0)$$
$$\delta\tilde{x}_i(0) = 0 \tag{19}$$

where $\tilde{A}(\mu) = \tilde{A}^* + \delta\tilde{A}$ and $\tilde{B}(\mu) = \tilde{B}^* + \delta\tilde{B}$. Set

$$\tilde{L}_2 = \tilde{L}_1 + \tilde{L}_3 \tag{20}$$

where $\tilde{L}_3$ is an $m \times \tilde{n}$ matrix to be specified later. Using (19) and (20) gives

$$\delta\dot{\tilde{x}}_1 - \delta\dot{\tilde{x}}_2 = (\tilde{A}^* - \tilde{B}^*\tilde{L}_1)(\delta\tilde{x}_1 - \delta\tilde{x}_2) + \tilde{B}^*\tilde{L}_3\delta\tilde{x}_2 \tag{21}$$

which yields

$$\delta\tilde{X}_2(s) = S(s)\,\delta\tilde{X}_1(s) \tag{22}$$

where $\tilde{X}_i(s)$ represents the Laplace transform of $\tilde{x}_i(t)$,

$$S(s) = [I + \phi(s)\,\tilde{B}^*\tilde{L}_3]^{-1} \tag{23}$$

$$\phi(s) = [sI - (\tilde{A}^* - \tilde{B}^*\tilde{L}_1)]^{-1} \tag{24}$$

and I denotes an $\tilde{n} \times \tilde{n}$ identity matrix.

It has been shown in [7–8] that a sufficient condition such that the criterion (18) for sensitivity comparison holds is that for all $\omega \in (-\infty, \infty)$

$$S^{\mathrm{T}}(-j\omega)\,Z\,S(j\omega) \leq Z \tag{25}$$

where the equality sign may apply for some, but not all, ω.

As mentioned in Section II, under the assumptions that (A^*, B^*) is controllable, (C^*, A^*) is observable and (12) is satisfied, it follows that $(\tilde{A}^*, \tilde{B}^*)$ is controllable. Therefore, we can choose a matrix $\tilde{L}_1$ such that $\tilde{A}_{\mathrm{f}}^* \triangleq (\tilde{A}^* - \tilde{B}^*\tilde{L}_1)$ is a Hurwitz matrix (i.e., all the eigenvalues of $\tilde{A}_{\mathrm{f}}^*$ have negative real parts). Now let us consider the Riccati equation

$$P(\tilde{A}^* - \tilde{B}^*\tilde{L}_1) + (\tilde{A}^* - \tilde{B}^*\tilde{L}_1)^{\mathrm{T}}P - P\tilde{B}^*R^{-1}\tilde{B}^{*\mathrm{T}}P + D^{\mathrm{T}}D = 0 \tag{26}$$

where the matrix P is $(n + n_c) \times (n + n_c)$, the matrix D has $(n + n_c)$ columns and is of rank $(n + n_c)$, and R is any $m \times m$ positive definite symmetric matrix. Since state feedback does not change the controllability property of the system, controllability of $(\tilde{A}^*, \tilde{B}^*)$ implies the controllability of $(\tilde{A}_{\mathrm{f}}^*, \tilde{B}^*)$. Furthermore, $(D, \tilde{A}_{\mathrm{f}}^*)$ is observable since rank of $D = n + n_c$. It has been shown in [10] that under these conditions, there exists a unique positive definite symmetric matrix P satisfying (26).

Given the control $u_1 = -\tilde{L}_1\tilde{x}_1(t,\mu,w)$, we construct a new control u_2 of the form (17) such that

$$u_2 = -(\tilde{L}_1 + \tilde{L}_3)\,\tilde{x}_2(t,\mu,w) + \tilde{L}_3\tilde{x}^*(t) \tag{27}$$

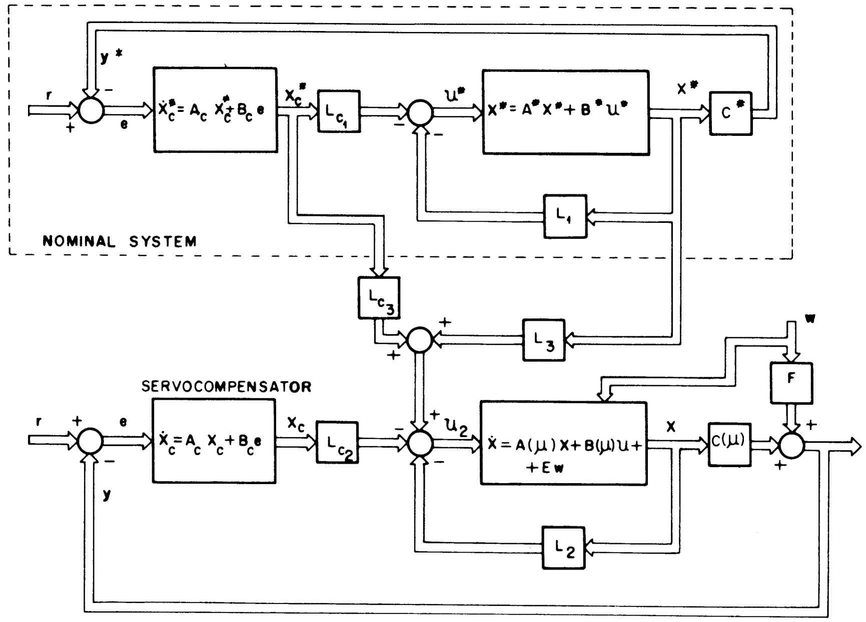


Fig. 1. Closed-loop system showing servocompensator, feedback loop, and feedforward element.

where

$$\tilde{L}_3 = R^{-1}(\tilde{B}^*)^{\mathrm{T}}P. \tag{28}$$

In view of (17) and (27), this gives

$$\tilde{L}_2 = \tilde{L}_1 + \tilde{L}_3, \qquad G = \tilde{L}_3.$$

We notice that controls (16) and (27) are nominally equivalent, since $\tilde{x}_2(t, \mu^*, w=0) = \tilde{x}^*$.

The closed-loop system (10) and (27) is given by

$$\dot{\tilde{x}}_2 = (\tilde{A}(\mu) - \tilde{B}(\mu)\,\tilde{L}_2)\tilde{x}_2 + \tilde{B}(\mu)\,\tilde{L}_3\tilde{x}^* + v(w,r), \qquad \tilde{x}_2(0) = \tilde{x}_0. \tag{29}$$

The complete closed-loop system is shown in Fig. 1.

The following theorem related to asymptotic tracking and sensitivity reduction can be stated.

Theorem 1. Consider the closed-loop system (29) consisting of (1), (5), and (27). Suppose that the matrix pair (A^*, B^*) is controllable, (C^*, A^*) is observable, column rank of $D = n + n_c$, rank $(B^*) = m$, and $(\tilde{A}^* - \tilde{B}^*\tilde{L}_1)$ is a Hurwitz matrix. We also assume that (12) is satisfied and that $\delta\tilde{x}_2(t) \not\equiv \delta\tilde{x}_1(t)$. Then $(\tilde{A}^* - \tilde{B}^*\tilde{L}_2)$ is Hurwitz, and, whenever the parameters vary and arbitrary disturbance input is present, the sensitivity reduction criterion (18) is satisfied with

$$Z = \tilde{L}_3^{\mathrm{T}}\, R\, \tilde{L}_3. \tag{30}$$

Moreover, in the closed-loop system asymptotic tracking and disturbance rejection for the class of reference signals and disturbance inputs given in (4) and (3) are achieved for the values of μ such that $(\tilde{A}(\mu) - \tilde{B}(\mu)\,\tilde{L}_2)$ remains Hurwitz.

Proof. The proof is given in the appendix.

It is interesting to note that trajectory sensitivity is reduced for arbitrary disturbance inputs and parameter variations. However, tracking and disturbance rejection are achieved only for the class of $r(t)$ and $w(t)$ given in (4) and (3). We also note that when $r(t) \equiv 0$, $\tilde{x}^*(t) \to 0$ as $t \to \infty$. Thus, in the presence of disturbance inputs, the feedforward control in (27) is not effective when t is large, and the sensitivity reduction is achieved only by the feedback element. This is natural since when $r(t) \equiv 0$ and $\tilde{x}(t_1) = 0$ at a certain time t_1, ideally one would like to have $\tilde{x}(t) = \tilde{x}^*(t) = 0$ for all $t > t_1$ despite the presence of the disturbances.

Theorem 1 does not give any rule to obtain matrices D and R in order to reduce the sensitivity when the weighting matrix Z in (18) is specified. However, it is important to note that the solution of (26) always exists for any positive definite symmetric matrix R and any matrix D of full column rank. Thus the designer has flexibility in the choice of matrices D and R in (26) to obtain the matrix $\tilde{L}_3$ given in (28) and a suitable weighting matrix Z of the sensitivity reduction criterion (18). In general, several trials are necessary to obtain appropriate matrices D and R and the weighting matrix Z.

IV. APPLICATION TO AIRCRAFT CONTROL SYSTEM

In this section the design procedure described in Sections II and III is applied to a specific VTOL aircraft (helicopter). The model used is that considered by Narendra and Tripathi [5]. The linearized model of the VTOL aircraft in a vertical plane is described by

$$\dot{x} = A(\mu)\,x + B(\mu)\,u + Ew \tag{31}$$

where A is the (4×4) system matrix, B is the (4×2) input matrix, and x and u are 4×1 and 2×1 state and

TABLE I
Set of Matrices for Desirable Sensitivity Reduction

$$Q_n = \begin{bmatrix} 0.04 & 0.01 & 0.0 & 0.01 & 0.0 & 0.0 \\ 0.01 & 0.25 & 0.01 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.01 & 0.01 & 0.0 & 0.0 & 0.0 \\ 0.01 & 0.0 & 0.0 & 0.01 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 3.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 2.5 \end{bmatrix} \qquad R_n = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}$$

$$D^T D = \begin{bmatrix} 3.2195 & 0.2343 & 0.1505 & -1.2670 & -1.5938 & 0.1097 \\ 0.2343 & 0.0877 & 0.1252 & 0.1050 & -0.1845 & -0.0620 \\ 0.1505 & 0.1252 & 0.1951 & 0.2680 & -0.1785 & -0.1047 \\ -1.2670 & 0.1050 & 0.2680 & 1.1097 & 0.4511 & -0.2291 \\ -1.5938 & -0.1845 & -0.1785 & 0.4511 & 0.9206 & 0.0381 \\ 0.1097 & -0.0620 & -0.1047 & -0.2291 & 0.0381 & 0.0887 \end{bmatrix} \qquad R = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}$$

$$\hat{L}_1 = \begin{bmatrix} 2.5377 & 0.1067 & -0.3282 & -1.0847 & -1.7014 & -0.2963 \\ 0.5334 & -0.7155 & -0.0840 & 0.2697 & -0.3245 & 1.5531 \end{bmatrix}$$

$$\hat{L}_3 = \begin{bmatrix} 1.1577 & -0.0663 & -0.1513 & -0.7889 & -0.2786 & 0.2905 \\ -0.4498 & -0.0831 & -0.0944 & 0.0421 & 0.3411 & 0.0558 \end{bmatrix}$$

control vectors, respectively, and E is the suitable disturbance input matrix. The state variables are

x_1 horizontal velocity in knots
x_2 vertical velocity is knots
x_3 pitch rate in degrees per second
x_4 pitch angle in degrees.

The control inputs are

u_1 collective in degrees
u_2 longitudinal in degrees.

Essentially, control is achieved by varying the pitch (angle of attack with respect to air) of the rotor blades. The collective control alters the angle of attack of all the rotor blades simultaneously by the same amount, thus producing lift. Its main use is control over the vertical velocity of the VTOL aircraft. However, this control also has some effect on the horizontal velocity. The longitudinal cyclic control progressively alters the angle of attack of each rotor blade as it sweeps around, thus changing the lift of the VTOL aircraft from a purely vertical direction to a combination of vertical and horizontal directions; this in turn produces a forward motion. Here u_1 and u_2 denote the components of control vector u and should not be confused with u_1 and u_2, the two different control laws of Section III.

Although other parameter variations can be considered, here we consider the airspeed variations during the flight for the longitudinal motion of the aircraft. Following [5], the nominal airspeed is assumed to be 135 knots. At this nominal airspeed the matrices A and B are

$$A^* = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.01 & 0.0024 & -4.0208 \\ 0.1002 & 0.3681 & -0.707 & 1.42 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B^* = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ -5.52 & 4.49 \\ 0 & 0 \end{bmatrix}$$

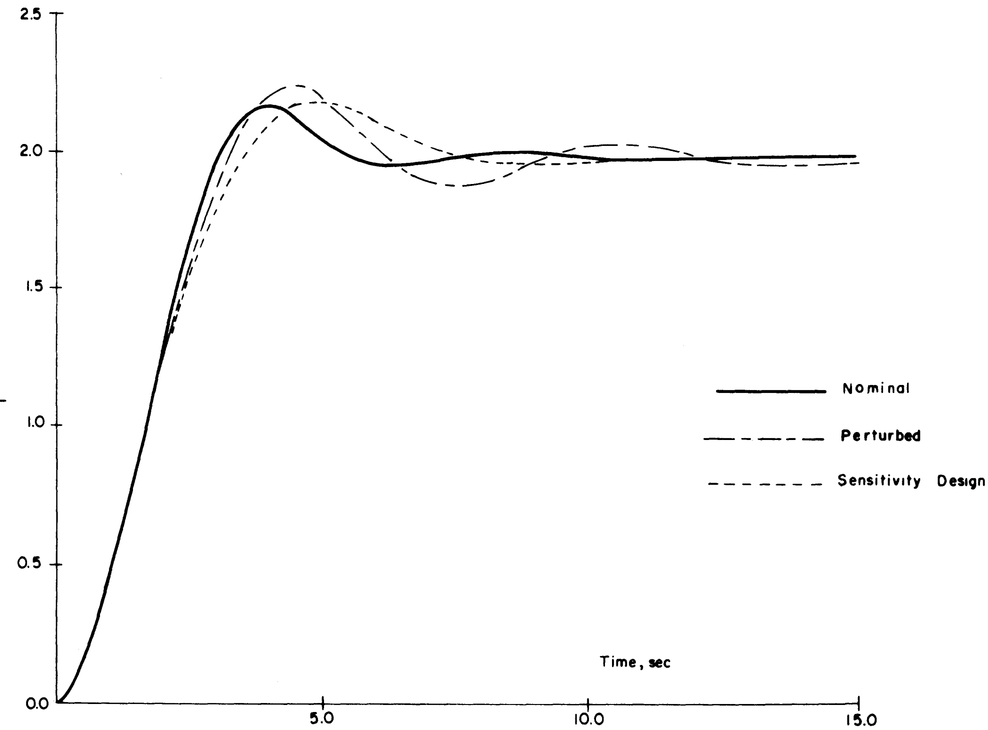
We assume that the range of variation of the airspeed is from 60 knots to 170 knots. It is found that when the airspeed changes, significant changes occur only in the elements a_{32}, a_{34}, and b_{21}. Hence it is assumed that all the other elements remain constant during the parameter variations. In the range of 60 knots to 170 knots, it has been shown in [5] that the bounds for these elements are $0.06635 \le a_{32}(\mu) \le 0.5047$, $0.1198 \le a_{34}(\mu) \le 2.526$, and $0.9775 \le b_{21}(\mu) \le 5.112$.

In general terms, the objectives of the design are (1) to enable the aircraft to follow the desired reference trajectories, (2) to minimize the perturbations in state of the aircraft from the nominal steady state when it is perturbed by disturbance forces, and (3) to maintain constant response characteristics of the aircraft in different flight regimes.

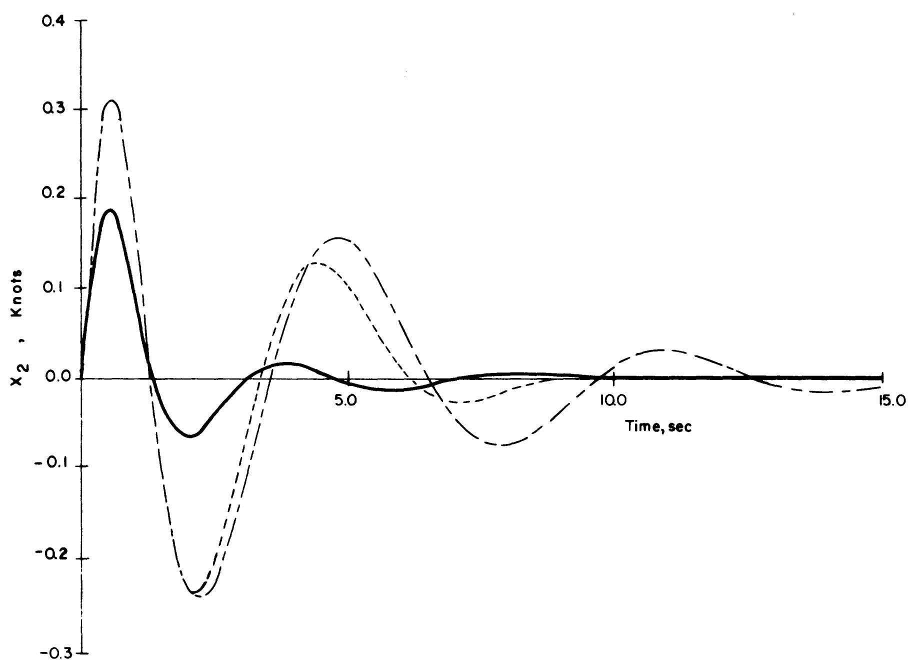
By proper selection of controlled variables, a suitable controller can be designed for achieving various practical objectives. In this paper, the controlled variables chosen are the horizontal velocity and the vertical velocity; that is, we are interested in designing a controller such that the horizontal and vertical velocities can be precisely controlled to track desired trajectories of a given class even in the presence of disturbance inputs. Thus the output equation in (1) takes the form

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} x \triangleq C\,x. \tag{32}$$

Following Section II, we construct the precompensator, once the modes of the disturbance input $w(t)$ and the command signal $r(t)$ are known. Although we can choose

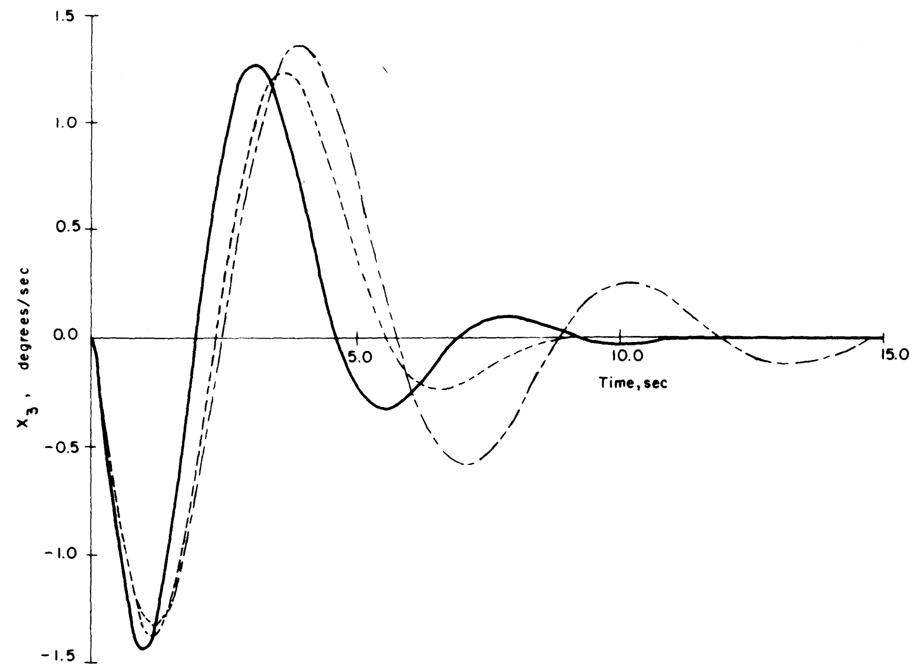


(a)



(b)

Fig. 2. Control of horizontal velocity: command $r = [2,0]^T$, $\bar{x}(0) = 0$, $w \equiv 0$, and airspeed = 170 knots. (a) Horizontal velocity, x_1. (b) Vertical velocity, x_2. (c) Pitch rate, x_3. (d) Pitch angle, x_4.



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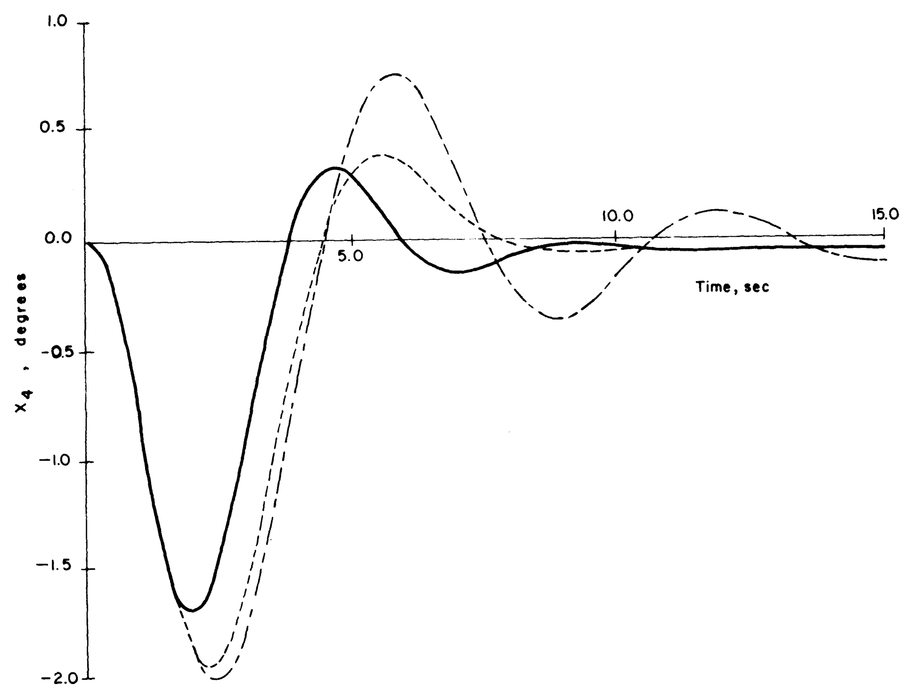
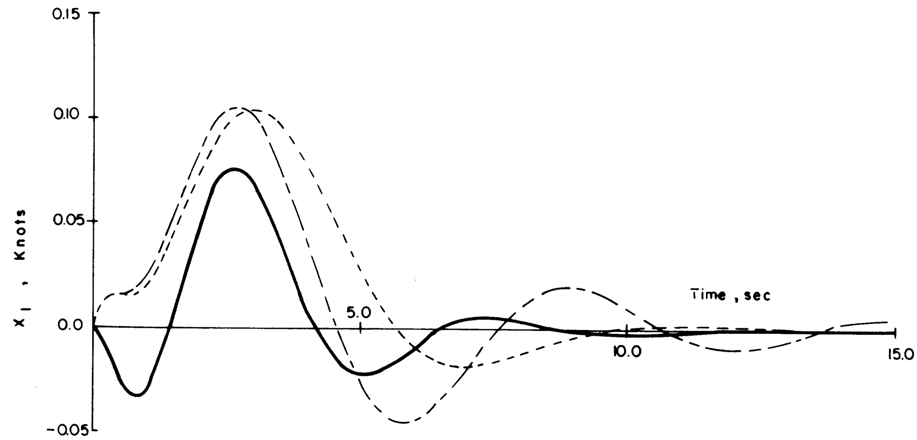
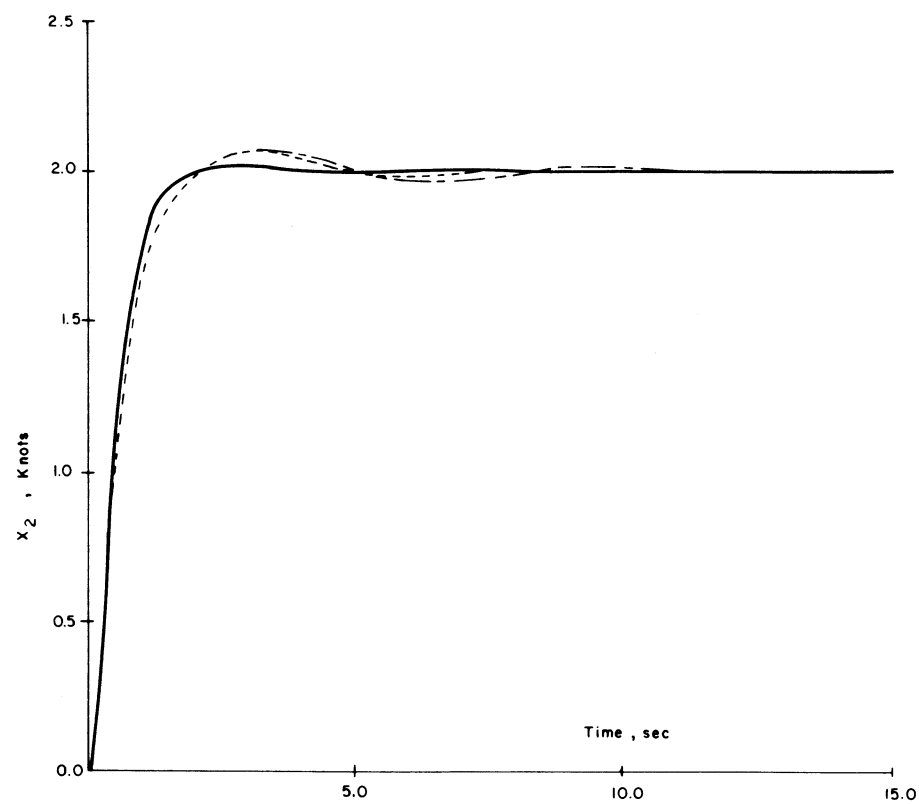


(d)

Fig. 2 (contd.). Control of horizontal velocity: command $r = [2,0]^T$, $\tilde{x}(0) = 0$, $w \equiv 0$, and airspeed = 170 knots. (a) Horizontal velocity, x_1. (b) Vertical velocity, x_2. (c) Pitch rate, x_3. (d) Pitch angle, x_4.

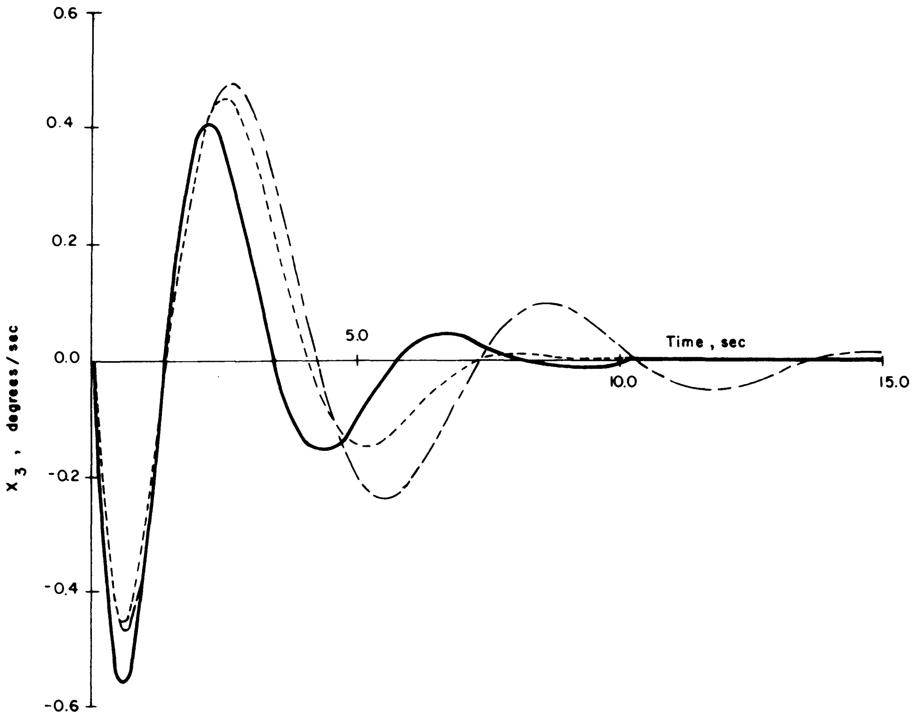


(a)

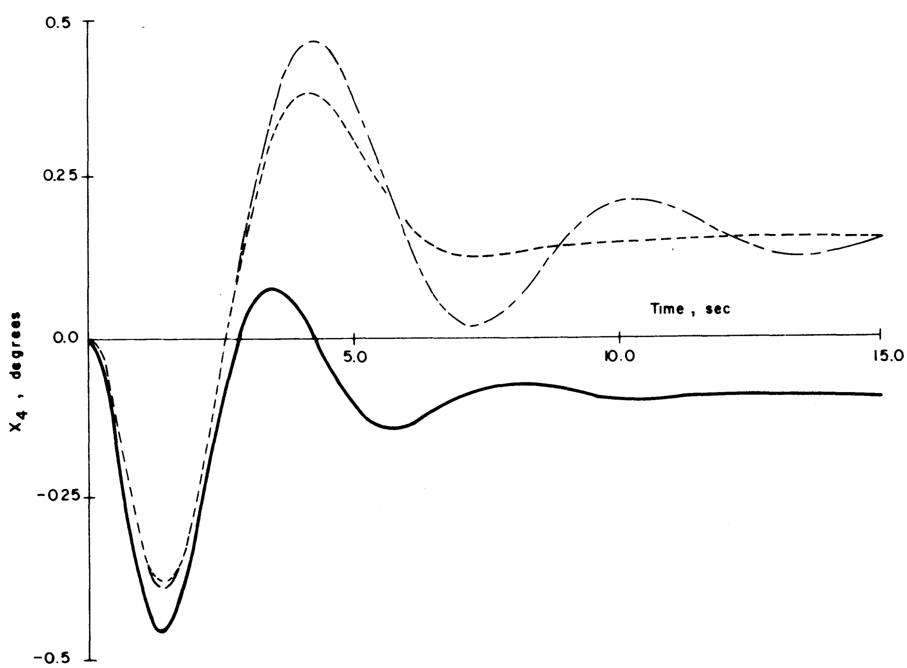


(b)

Fig. 3. Control of vertical velocity and disturbance rejection: command input $r = [0,2]^T$, $\bar{x}(0) = 0$, w = constant function, and airspeed = 170 knots. (a) Horizontal velocity, x_1. (b) Vertical velocity, x_2. (c) Pitch rate, x_3. (d) Pitch angle, x_4.



(c)



(d)

Fig. 3 (contd.). Control of vertical velocity and disturbance rejection: command input $r = [0, 2]^T$, $\bar{x}(0) = 0$, w = constant function, and airspeed = 170 knots. (a) Horizontal velocity, x_1. (b) Vertical velocity, x_2. (c) Pitch rate, x_3. (d) Pitch angle, x_4.

a precompensator of higher dimension to track and reject polynomial, exponential, or some other types of velocity commands and disturbance inputs, respectively, for simplicity, we assume that the disturbance inputs and the command $r(t)$ are constant functions of time. Thus in this case, it is sufficient to introduce a servocompensator (5) of the form

$$\dot{x}_c = \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & 0 \end{array}\right] x_c + \left[\begin{array}{c|c} 1 & 0 \\ \hline 0 & 1 \end{array}\right] [r - Cx] \quad (33)$$

since $\phi_{wr} = s$.

In this case, $\tilde{x} \in R^6$. The airspeed value of 135 knots is chosen as the "nominal" condition and the "fixed feedback gains" are obtained at this nominal condition by minimizing a quadratic performance index

$$J = \int_0^\infty \tilde{x}^T(t)\, Q_n \tilde{x}(t) + u^T(t) R_n u(t)\, dt$$

for the nominal system (15) with $r(t) \equiv 0$. The feedback matrix $\tilde{L}_1$ is given by

$$\tilde{L}_1 = R_n^{-1} (\tilde{B}^*)^T P_n \quad (34)$$

where P_n is the solution of the Riccati equation

$$P_n \tilde{A}^* + (\tilde{A}^*)^T P_n - P_n \tilde{B}^* R_n^{-1} (\tilde{B}^*)^T P_n + Q_n = 0. \quad (35)$$

The weighting matrices were selected to produce desirable handling characteristics at the nominal airspeed of 135 knots. The resulting feedback gains and matrices Q_n, R_n, and $\tilde{L}_1$ are given in Table I. For obtaining a suitable matrix Z, (26) was solved for several choices of D and R. A set of matrices D and R for desirable sensitivity reduction are given in Table I.

V. SIMULATION RESULTS

In this section we present the results of digital simulation.

Control of Horizontal Velocity ($w(t) \equiv 0$)

Fig. 2 shows the response of the aircraft when reference command input $r = [2.0, 0]^T$ is applied. The initial condition is $\tilde{x}(0) = 0$, and disturbance input $w(t) \equiv 0$. The response of the closed-loop system (15) with $u^*(t) = -\tilde{L}_1 \tilde{x}_1^*(t)$ is marked "nominal" in the figures. When the fixed gain control $u(t) = -\tilde{L}_1 \tilde{x}_1(t, \mu, w)$ is used at other speeds, the response curves (marked "perturbed" in the figures) are found to be different. The response of the system (10) with the fixed gain control (16) with $w(t) \equiv 0$ at the airspeed of 170 knots is shown in Fig. 2. It was observed that the fixed gain controller produced underdamped response at 60 knots and an overdamped response at 170 knots. The trajectories marked "sensitivity design" correspond to the response of the closed-loop system (29) resulting from the use of sensitivity reduction controller (27). It is seen that, unlike [2] and [5], the steady state errors in x_i, $t = 1, \ldots, 4$, are zero. The control magnitudes are bounded as $|u_1(t)| < 1$ and $-0.2 < u_2(t) < 1$ (the time histories are not shown here). Since the response of the aircraft at the airspeed of 60 knots is still better, these results are not presented here to save space.

The response to command $r = [0, 2]^T$ with $w(t) \equiv 0$ at 170 knots was also obtained. As predicted, good vertical velocity command following was observed, and, therefore, the results are not shown.

Remark 1. We observe from Fig. 2 that the responses in variables x_2, x_3, and x_4, using the sensitivity design method, stay closer to the "nominal" trajectories than the perturbed trajectories. However, in Fig. 2(a), it is seen that over certain subintervals of time the "perturbed" response is closer to the nominal than the sensitivity design trajectory. The reason for this is the particular form of the weighting matrix Z. However, this is not a serious problem, since the designer can choose the matrices D and R to obtain suitable trajectory sensitivity reduction.

Control of Vertical Velocity ($w(t)$ = constant)

To show the tracking and disturbance rejection capability of the controller, simulation results for $r(t) = [0, 2]^T$ and constant $w(t) = [1, 1]^T$ at 170 knots are givei in Fig. 3. Although disturbance rejection is achieved for any matrix E, simulation results are shown for

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T.$$

We observe that despite disturbance $w(t)$, the desired vertical velocity is achieved. The steady state errors in x_i, $i = 1,2,3$, are zero. Also, response to command input $r = [2, 0]^T$ with the same $w(t)$ was obtained and good horizontal velocity tracking and disturbance rejection were observed.

Extensive simulations with disturbance inputs at different airspeeds for different command inputs were performed. From these simulation results we conclude that compared with the fixed gain controller, the new controller has a better capability for reducing the sensitivity, tracking, and disturbance rejection. Furthermore, tradeoff between response characteristics and control magnitudes is possible by the proper selection of the weighting matrices Q_n, R_n, D, and R. We also note that the designer has some flexibility in keeping the sensitivity of a particular choice of state variables less by the proper selection of weighting matrices R and D.

VI. CONCLUSIONS

An approach for sensitivity reduction, robust tracking, and disturbance rejection in multivariable time-invariant linear systems has been presented. The controller consists of a servocompensator containing the modes of the reference signals and disturbance inputs, a feedback compensator, and a feedforward compensator. Given a specific stabilizing feedback gain matrix, a procedure was given for finding another feedback gain matrix and a feedforward signal as a function of the nominal trajectory such that the closed-loop system is less sensitive to parameter variations and disturbance inputs than the original closed-loop system. Furthermore, the controller achieves robust tracking and disturbance rejection in the closed-loop system.

The above method was used in designing a VTOL aircraft control system. It was shown that in comparison to the existing controllers used in the literature for reducing the trajectory sensitivity of the VTOL aircraft, the controller of this paper performs better; and it has good tracking, sensitivity reduction, and disturbance rejection capabilities. Of course, the adaptive controllers are the best of all, but for the synthesis of controllers, onboard computers are required. The controller of this paper is easy to synthesize.

APPENDIX. PROOF OF THEOREM 1

First we show that the sensitivity comparison criterion (18) is satisfied. Since the proof is similar to that of [7] and [9], it is only briefly described. Under the hypotheses of the theorem, beginning with (26) and following the steps in the proof of [7, theorem 3] and [9, corollary 2] gives

$$[I + \phi(-j\omega)\tilde{B}^*\tilde{L}_3]^{\mathrm{T}} \tilde{L}_3^{\mathrm{T}} R\tilde{L}_3 [I + \phi(j\omega)\tilde{B}^*\tilde{L}_3]$$

$$= \tilde{L}_3^{\mathrm{T}} R\tilde{L}_3 + V^{\mathrm{T}}(-j\omega)\, V(j\omega), \qquad \omega \in (-\infty,\infty) \quad (36)$$

$$\delta\tilde{X}_2^{\mathrm{T}}(-j\omega)\; V^{\mathrm{T}}(-j\omega)\; V(j\omega)\; \delta\tilde{X}_2(j\omega) > 0 \quad (37)$$

over some interval in ω where $V(j\omega) = D\phi(j\omega)\,\tilde{B}^*\tilde{L}_3$.

From (36) and (37) it follows that

$$\delta\tilde{X}_2^{\mathrm{T}}(-j\omega)\; [(S^{\mathrm{T}}(-j\omega))^{-1}\tilde{L}_3^{\mathrm{T}} R\tilde{L}_3\, S(j\omega) - \tilde{L}_3^{\mathrm{T}} R\tilde{L}_3]$$

$$\delta\tilde{X}_2(j\omega) > 0 \quad (38)$$

for all ω in some interval. Now sensitivity comparison criterion (18) follows from (38).

Now we shall show that asymptotic tracking and disturbance rejection are achieved for the class of $r(t)$ and $w(t)$ given in (3) and (4). Noting that $(\tilde{A}^* - \tilde{B}^*\tilde{L}_1)$ is Hurwitz, it follows from (15) that

$$\tilde{X}^*(s) = \phi(s)\,\tilde{x}^*(0) + \phi(s)\,\hat{V}(s) \quad (39)$$

where $\hat{V}(s)$ and $\tilde{X}^*(s)$ denote Laplace transforms of $v(0,r)$ and $\tilde{x}^*(t)$, respectively. Let the element in the ith row and jth column of $\phi(s)$ be $(p_{ij}(s)/q_{ij}(s))$. We note that the roots of the polynomial $q_{ij}(s) = 0$ have negative real parts. Apparently, as $t \rightarrow \infty$, the contribution of $\tilde{x}^*(0)$ in (39) tends to zero, and the only modes present in $\tilde{x}^*(t)$ are those of signal $r(t)$ resulting from the product term $\phi(s)\,\hat{V}(s)$. Thus in the closed-loop system (29), we note that modes of the additive terms $\tilde{B}(\mu)\tilde{L}_3\tilde{x}^* + v(w,r)$ asymptotically tend to the modes present in the servocompensator. Now according to [1], robust tracking and disturbance rejection are achieved and the proof of Theorem 1 is complete.

REFERENCES

[1] Davison, E.J. (1976)
The robust control of a servomechanism problem for linear time-invariant multivariable systems.
IEEE Transactions on Automatic Control, Feb. 1976, *AC-21*, 25–34.

[2] Sundararajan, N. (1978)
Sensitivity reduction in aircraft control systems.
IEEE Transactions on Aerospace and Electronic Systems, Mar. 1978, *AES-14*, 292–297.

[3] Gourishankar, V., and Zackowski, G.V. (1980)
Minimum sensitivity controllers with application to VTOL aircraft.
IEEE Transactions on Aerospace and Electronic Systems, Mar. 1980, *AES-16*, 217–226.

[4] Okada, T., Kihara, M., and Motoyama, K. (1982)
Sensitivity reduction by double perfect model following.
IEEE Transactions on Aerospace and Electronic Systems, Jan. 1982, *AES-18*, 29–38.

[5] Narendra, K.S., and Tripathi, S.S. (1973)
Identification and optimization of aircraft dynamics.
Journal of Aircraft, Apr. 1973, *10*, 193–199.

[6] Stengel, R.F., Broussad, J.R., and Berry, P.W. (1978)
Digital controllers for VTOL aircraft.
IEEE Transactions on Aerospace and Electronic Systems, Jan. 1978, *AES-14*, 54–63.

[7] Kreindler, E. (1968)
Closed-loop sensitivity reduction of linear optimal control systems.
IEEE Transactions on Automatic Control, June 1968, *AC-13*, 254–262.

[8] Perkins, W.R., and Cruz, J.B., Jr. (1965)
The parameter variation problem in state feedback control systems.
Journal of Basic Engineering. Transactions ASME, Series D, 1965, *87*, 120–124.

[9] Sundararajan, N., and Cruz, J.B., Jr. (1971)
Sensitivity improvement of semi-closed loop systems.
Presented at the 2nd IFAC Symposium on Multivariable Technical Control Systems, Duesseldorf, Germany, 1971.

[10] Athans, M., and Falb, P. (1966)
Optimal Control.
New York: McGraw-Hill, 1966.

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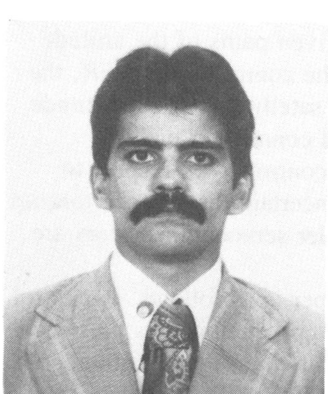

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