Adaptive Control and Stabilization of Elastic Spacecraft

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This work treats the question of large angle rotational maneuver and stabilization of an elastic spacecraft (spacecraft-beam-tip body configuration). It is assumed that the parameters of the system are completely unknown. An adaptive control law is derived for the rotational maneuver of the spacecraft. Using the adaptive controller, asymptotically decoupled control of the pitch angle of the space vehicle is accomplished, however this maneuver causes elastic deformation of the beam connecting the orbiter and tip body. For the stabilization of the zero dynamics (flexible dynamics), a stabilizer is designed using elastic mode velocity feedback. In the closed-loop system including the adaptive controller and the stabilizer, reference pitch angle trajectory tracking and vibration suppression are accomplished. Simulation results are presented to show the maneuver capabilty of the control system.

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I. INTRODUCTION

An important class of shuttle-deployed payloads consists of cantilevered beam-like structures with massive tip bodies. Presently, there is considerable interest in the problem of stability and control of such flexible space vehicles. Large angle rotational maneuver of spacecraft induces structural deformation in the flexible appendages. Dynamical models of space vehicles are nonlinear and include the rigid and flexible mode interaction. Moreover the parameters of spacecraft are not precisely known. The nonlinearity of the dynamical model, the parameter uncertainty, and the structural deformation of the elastic parts create considerable difficulty in the design of control systems for rotational maneuvers of spacecraft.

In recent years, several studies related to control of flexible space vehicles have been done, and linear and nonlinear control systems have been designed [1–13]. An excellent survey of research in this area has been published in [13] which provides a good source of references. Optimal controllers for linear and nonlinear models of flexible spacecraft have been designed in [1–3], and bang-bang rest-to-rest maneuver of linear models has been considered in [4–5]. Near-minimum time single axis slewing of flexible vehicle has been treated in [6, 7]. A perturbation method has been used to obtain a feedback controller in [8]. Lyapunov stability theory has been used to design controllers for the maneuver and vibration control of space vehicles in [9, 10]. A controller based on inversion has been also derived [11]. In these studies, it is assumed that the parameters of the spacecraft are exactly known. In the presence of uncertainty, sliding mode control system has been designed [12]. However, the derivation of this controller requires knowledge of the bounds on the uncertain parameters. Thus there is a need to design control systems for the flexible spacecraft, when the system parameters and the bounds on these parameters are unknown.

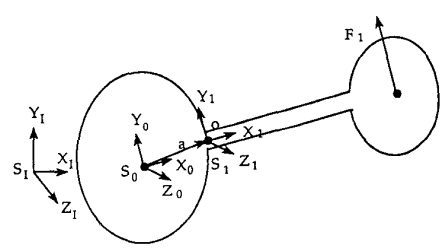

Fig. 1. Model of elastic spacecraft.

In this work, we consider rotational maneuver of an elastic spacecraft (orbiter-beam-tip body configuration) shown in Fig. 1. The motion of flexible spacecraft is governed by a hybrid set of

differential and partial differential equations. For flexible systems, finite dimensional models have been widely used in literature for the design of control systems [1–9, 11–13]. These finite dimensional approximate representations of infinite dimensional flexible systems are obtained by neglecting higher elastic modes. Within the practical limits of spacecraft maneuvers, higher elastic modes are not significantly excited, and their contributions in the elastic deformations of flexible appendages are negligible. For this reason, lumped models of flexible spacecraft obtained by ignoring higher elastic modes have gained much attention in literature and are adequate for designing control systems using relatively well developed control theory for finite dimensional systems. A finite dimensional model of the flexible spacecraft is similary used here. However, it is pointed out that the model is nonlinear since it includes a term which is a nonlinear function of the angular velocity of the spacecraft. Although the approach of this work is applicable to three axis maneuvers, for simplicity, only single-axis control of the pitch angle is considered. The spacecraft is controlled by a torquer on the orbiter, and a torquer and a force actuator on the tip body. When the spacecraft is maneuvered, the beam connecting the orbiter and the end body experiences structural deformation. The problem of interest here is to control the orientation of the spacecraft and to damp the transverse vibration of the beam.

The contribution of this work lies in the derivation of a control system for large pitch angle rotational maneuver and vibration suppression of the spacecraft shown in Fig. 1. It is assumed that the system parameters are completely unknown. An adaptive control law is derived to asymptotically decouple the pitch angle dynamics from the elastic dynamics. Using the adaptive controller, reference pitch angle trajectory is tracked. However, slewing of the spacecraft excites the elastic modes of the beam. A stabilizer is designed for the control of vibration based on the asymptotically decoupled flexible dynamics (zero dynamics) [14–15]. The stabilizer is synthesized using measured variables and using colocated actuators and sensors. This way, the knowledge of flexible dynamics is avoided for the design of the stabilizer. In this study it is assumed that all the elastic modes, pitch angle, and pitch rate are available for feedback. In the practical situation one can design a state estimator for constructing the unmeasurable state variables using the signals from the sensors such as accelerometers, and strain gauges, etc.

The mathematical model is presented in Section II. Section III presents the adaptive control law. The stabilizer for vibration suppression is designed in Section IV, and Section V presents the numerical results.

II. MATHEMATICAL MODEL

Fig. 1 shows the elastic spacecraft. Although the design approach of this work can be applied to three-axis maneuver, for simplicity only the maneuver in the pitch plane is considered. A right-handed coordinate frame (denoted as S_0) with axes X_0, Y_0, Z_0 is fixed to the spacecraft with its origin at the mass center of the spacecraft. X_I, Y_I, Z_I are the axes of an inertial frame S_I. The coordinate system S_1 with axes X_1, Y_1, Z_1 is obtained by the translation of the frame S_0 at the attachment point of the beam and the orbiter. The attitude of the spacecraft is given by the pitch angle θ which is the angle of rotation of the spacecraft about the axis Z_0. When the spacecraft is slewed about the Z_0 axis, the beam is allowed to undergo elastic transverse bending in the $X_0 - Y_0$ plane. The attachment point O of the beam and orbiter is located arbitrarily with respect to the mass center of the spacecraft, but it is assumed that the mass center of the tip body is located along the tangent line to the beam at the tip point.

Storch and Gates [16] have obtained a complete derivation of the equations of motion of this spacecraft (Fig. 1) in a rigorous way. The equations of motion are given by

$$a_{00}\ddot{\theta} + \sum_{k=1}^{m} a_{1k}\ddot{p}_k = [b_{01} \quad b_{01} \quad b_{03}]u$$

$$a_{1i}\ddot{\theta} + \sum_{k=1}^{m}\left(\delta_{ik} - \frac{\rho L}{m}u_{3i}u_{3k}\right)\ddot{p}_k = [0 \quad b_{i2} \quad b_{i3}]u + u_{3i}\mu_0 a_2\frac{\dot{\theta}^2}{L} - \Omega_i^2 p_i$$

$$(i = 1,2,\ldots,m) \qquad (1)$$

where the control vector $u = (u_1,u_2,u_3)^{\mathrm{T}} = (T_s,T_p,F)^{\mathrm{T}} \in R^3$ (superscript T denotes transposition); T_s is the external control moment applied to the orbiter; T_p and F are the control moment and force applied on the tip body at its mass center along axes parallel to Z_1 and Y_1, respectively, and p_i denotes the ith elastic mode. In this model, a finite number of elastic modes have been retained assuming that the first m modes are significant and adequately approximate the elastic deformation of the beam. The parameters a_{ij}, b_{ij}, u_{ij}, Ω_i, etc. are given in the Appendix for completeness. The system is nonlinear due to the presence of the $(\dot{\theta}^2)$ term in the model. The transverse elastic deformation δ_e along the axis Y_1 at any point at a distance l from the attachment point O of the spacecraft and the beam is given by

$$\delta_e(l,t) = L\sum_{k=1}^{m} p_k(t)\beta_k(\eta), \qquad \eta = l/L \qquad (2)$$

where $\beta(\eta)$ is the kth mode shape, and L denotes the length of the beam.

The set of equations (1) can be written in the compact form as

$$\begin{pmatrix} D_{11} & D_{12} \\ D_{12}^{\mathrm{T}} & D_{22} \end{pmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{p} \end{bmatrix} = \begin{bmatrix} 0 \\ g \end{bmatrix} \dot{\theta}^2 - \begin{bmatrix} 0 \\ K \end{bmatrix} p + B_0 u \tag{3}$$

where, $p = (p_1, \ldots, p_m) \in R^m$ and the positive definite symmetric inertia matrix is $D \triangleq (D_{ij})$, $i,j = 1,2$. Here the matrix B_0 can be obtained by equating similar terms in (1) and (3), and

$$\begin{aligned} K &= \mathrm{diag}(\Omega_i^2), \qquad i = 1, \ldots, m \\ g &= (\mu_0 a_2 / L)(u_{31}, \ldots, u_{3m})^{\mathrm{T}}. \end{aligned} \tag{4}$$

Define $H \triangleq D^{-1}$, and $z = (\theta, p^{\mathrm{T}})^{\mathrm{T}} \in R^{(m+1)}$. Then solving (3), gives

$$\ddot{z} = H_2 g \dot{\theta}^2 - H_2 K p + H B_0 u \tag{5}$$

where $H = [H_1, H_2]$, and H_1 denotes the the first column of H.

The reference pitch angle trajectory is assumed to be generated by a third-order system

$$\dddot{\theta}_r + 3\lambda_c \ddot{\theta}_r + 3\lambda_c^2 \dot{\theta}_r + \lambda_c^3 (\theta - \theta^\star) = 0 \tag{6}$$

where $\lambda_c > 0$ and $\theta^\star$ is the desired terminal value of the pitch angle. Since $\lambda_c > 0$, the system (6) is asymptotically stable.

We are interested in deriving a control system such that in the closed-loop system, the attitude angle $\theta(t)$ tracks the reference trajectory $\theta_r(t)$, and the elastic oscillations are damped out. For the design of controller, it is assumed that the system parameters are completely unknown. For a proper choice of reference trajectory, the spacecraft attains the desired orientation, as $\theta(t)$ converges to $\theta_r(t)$.

III. ADAPTIVE ATTITUDE CONTROL

In this section, an adaptive control law for the attitude control of the spacecraft is derived. Using (5), one obtains the differential equation for θ given by

$$\ddot{\theta} = H_{21} g \dot{\theta}^2 - H_{21} K p + \alpha_0 u_1 + \alpha_1 u_2 + \alpha_2 u_3 \tag{7}$$

where H_{21} is the first row of the matrix H_2, and the first row of HB_0 is $[\alpha_0, \alpha_1, \alpha_2]$. Since it is desired to control the pitch angle, define a controlled variable s which is a linear function of the tracking error and its derivative of the form

$$s = \dot{\tilde{\theta}} + a_0 \tilde{\theta} \tag{8}$$

where $a_0 > 0$ and $\tilde{\theta} = \theta - \theta_r$ is the pitch angle tracking error.

It is desired to regulate s to 0. In view of (7), this in turn will lead to the convergence of the tracking error to zero. Consider the derivative of s along the solution of (7) given by

$$\begin{aligned} \dot{s} &= H_{21} g \dot{\theta}^2 - H_{21} K p + \alpha_0 u_1 + \alpha_1 u_2 + \alpha_2 u_3 - \ddot{\theta}_r + a_0 \dot{\tilde{\theta}} \\ &\doteq \alpha_0 [\psi(p, u_2, u_3, \dot{\theta}, t)\beta + u_1] \end{aligned} \tag{9}$$

where for the spacecraft model the parameter α_0 is a positive number and

$$\begin{aligned} \psi &= [u_2, u_3, \dot{\theta}^2, (a_0 \dot{\tilde{\theta}} - \ddot{\theta}_r), p^{\mathrm{T}}] \\ \beta &= \alpha_0^{-1} [\alpha_1, \alpha_2, H_{21} g, 1, -H_{21} K]^{\mathrm{T}}. \end{aligned}$$

Here $\psi \in R^{m+4}$ is the regressor vector and $\beta = [\beta_1, \ldots, \beta_{m+4}]^{\mathrm{T}}$ is the vector of constant parameters.

Now the derivation of a control law for the input u_1 for the regulation of s is considered. Here it is assumed that the control input $u_f = (u_2, u_3)^{\mathrm{T}}$ is a function of $(p, \dot{p})$. (The derivation of control law for u_f is given in Section IV.) In view of (9), for the control of s, one can choose an inverse control law of the form

$$u_1 = -c_0 s - \psi\beta \tag{10}$$

where $c_0 > 0$. However, the parameter vector β is unknown. Therefore, one chooses a control law using an estimated parameter vector β_e given by

$$u_1 = -c_0 s - \psi\beta_e. \tag{11}$$

Substituting (11) in (9), gives

$$\dot{s} = \alpha_0(-c_0 s + \psi\tilde{\beta}) \tag{12}$$

where $\tilde{\beta}_e = \beta - \beta_e$ is the vector of parameter error. When β is known, one chooses $\beta_e = \beta$ in (12) to obtain asymptotically stable decoupled responses for the tracking error according to

$$\begin{aligned} \dot{s} &= -\alpha_0 c_0 s \\ \dot{\tilde{\theta}} &= -a_0 \tilde{\theta} + s. \end{aligned} \tag{13}$$

For the derivation of the control law (11) using the estimated parameters, one uses the Lyapunov approach. Now consider a quadratic Lyapunov function

$$W_0 = [s^2 + \alpha_0 \tilde{\beta}^{\mathrm{T}} \Gamma^{-1} \tilde{\beta}]/2 \tag{14}$$

where Γ^{-1} is a positive definite symmetric matrix. The derivative of W_0 along the trajectory of (12) is given by

$$\dot{W}_0 = s\alpha_0(-c_0 s + \psi\tilde{\beta}) + \alpha_0 \tilde{\beta}^{\mathrm{T}} \Gamma^{-1} \dot{\tilde{\beta}}. \tag{15}$$

In view of (15), one chooses an adaptation law for the parameter vector β_e of the form

$$\dot{\tilde{\beta}} = -\dot{\beta}_e = -s\Gamma\psi^{\mathrm{T}} \tag{16}$$

Substituting (16) in (15) gives

$$\dot{W}_0 = -\alpha_0 c_0 s^2 \leq 0. \tag{17}$$

Since W_0 is positive definite and its derivative is negative semidefinite, it follows that s and $\tilde{\beta}$ are bounded, and $s \in L_2$, the set of square integrable functions. Since s is bounded, in view of (8), it follows that $\tilde{\theta}$ and $\dot{\tilde{\theta}}$ are bounded. Under the assumption that the reference trajectory θ_r, its derivatives, and $(p, \dot{p})$ are bounded, it follows that the function ψ is bounded. This in view of (12) implies that $\dot{s}$ is bounded, and therefore, $\dot{s}$ is uniformly continuous. Then using Barbalat's Lemma [14–15], one concludes that s tends to zero as t tends to ∞. In view of (13), this implies that the tracking error $\tilde{\theta}$ and its derivative converge to zero as t tend to ∞.

The adaptive controller can accomplish attitude angle trajectory tracking provided that the control input u_f is such that p and $\dot{p}$ remain bounded during the manuever. In order to obtain boundedness of the elastic modes, one must design a control law for u_f such that the zero dynamics of the system are stable. The zero dynamics are defined to be the residual motion of the spacecraft when $s \equiv 0$ and $\theta \equiv \theta_r \equiv \theta^*$, the desired terminal value of the pitch angle. Furthermore, this implies that θ and its derivatives are identically zero. Obviously, when the attitue angle is held constant, the zero dynamics essentially are the elastic dynamics of the system. In the next section, stabilization of the zero dynamics is considered.

IV. VIBRATION CONTROL

For obtaining the zero dynamics, one sets $\theta = \theta_r = \theta^*$ and its derivatives to zero in (3). Then the resulting elastic motion obtained from (3) is described by

$$D_{22}\ddot{p} = -Kp + B_f u_f \tag{18}$$

where B_f denotes the $m \times 2$ submartix formed by the bottom m rows and last two columns of the matrix B_0.

For the vibration suppression, colocated actuators and sensors are assumed to be mounted on the beam. The sensors are used to measure the velocity of the elastic modes. In this case, the measurement equation is

$$y = B_f^{\mathrm{T}} \dot{p}. \tag{19}$$

Then a simple output feedback control law for the stabilization of the elastic modes is chosen of the form

$$u_f = -Fy \tag{20}$$

where $F = \mathrm{diag}(f_{11}, f_{22})$ is diagonal matrix with positive elements f_{ii}.

For establishing stability of the zero dynamics (18), one considers a quadratic positive definite Lyapunov function of the form

$$W_f = (\dot{p}^{\mathrm{T}} D_{22} \dot{p} + p^{\mathrm{T}} K p)/2. \tag{21}$$

The derivative of W_f along the solution of (18) and (20) is given by

$$\dot{W}_f = -\dot{p}^{\mathrm{T}} B_f F B_f^{\mathrm{T}} \dot{p} \leq 0. \tag{22}$$

Using a stability result [14], one concludes that the trajectory of the system (18) eventually lies in the set E, where

$$E = \{(p^{\mathrm{T}}, \dot{p}^{\mathrm{T}})^{\mathrm{T}} \in R^{2m} : B_f^{\mathrm{T}} \dot{p} = 0\}.$$

The motion of (18) confined to the set E evolves according to

$$\dot{x}_f = A_f x_f \tag{23}$$

where

$$A_f = \begin{bmatrix} 0 & I \\ -D_{22}^{-1} K & 0 \end{bmatrix}.$$

Here 0 and I denote null and identity matrices of appropriate dimensions. Associated with the system (23), define an output vector y_f as

$$y_f = C_f x_f = [0, B_f^{\mathrm{T}}] x_f. \tag{24}$$

According to this definition of y_f, y_f is identically zero whenever x_f belongs to the set E. Now if the matrix pair (C_f, A_f) is observable [17], then $y_f \equiv 0$ implies that $x_f \equiv 0$. Thus under the assumption that (C_f, A_f) is observable, it follows that the equilibrium point $x_f = (p^{\mathrm{T}}, \dot{p}^{\mathrm{T}})^{\mathrm{T}} = 0$ of the zero dynamics (18) with the control law (20), is globally exponentially stable.

For the spacecraft model under consideration, the observability condition is satisfied for the choice of location of the sensors at the end body. In physical terms, the observability condition means that any flexible motion of the beam must be detected by the sensors. For exponentially stable zero dynamics, as described in the previous section, s and $\tilde{\theta}$ tend to zero. Moreover, as θ converges to θ^*, the derivatives of θ tend to zero. Thus, in the closed-loop system including the adaptive controller and the stabilizer, as t tends to ∞, the elastic dynamics are described by the system (18), and therefore, elastic modes converge to zero as well. This establishes the convergence of the pitch angle to θ^* and of elastic modes to zero in the closed-loop system.

V. SIMULATION RESULTS

In this section, numerical results for the closed-loop system are presented. For obtaining numerical results, five significant modes are retained in the model (1), thus we have $m = 5$. For $m = 5$,

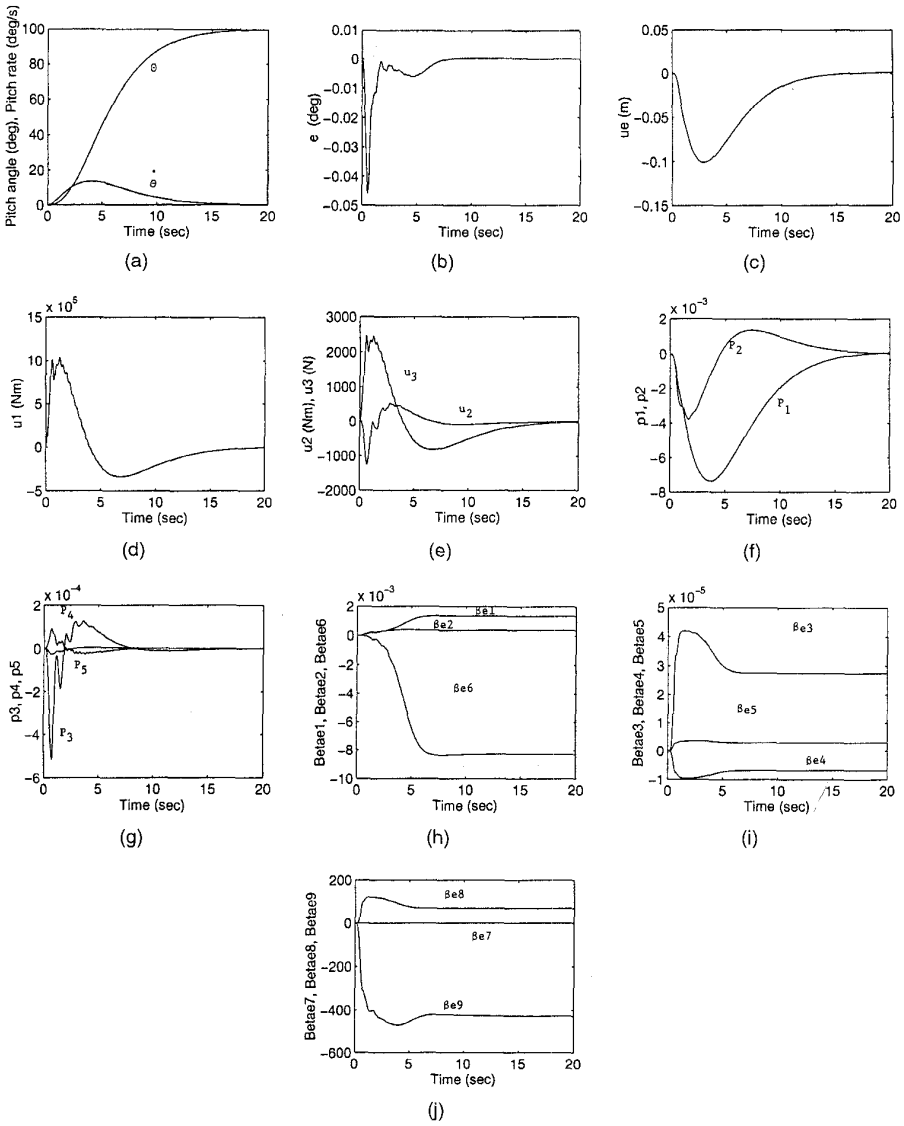

Fig. 2. Adaptive control and stabilization. (a) Pitch angle θ, angular velocity $\dot{\theta}$. (b) Pitch angle tracking error e. (c) End point deflection u_e. (d) Control torque u_1. (e) Control torque u_2, control force u_3. (f) Elastic modes p_1, p_2. (g) Elastic modes p_3, p_4, p_5. (h) Parameters β_{e1}, β_{e2}, and β_{e6}. (i) Parameters β_{e3}, β_{e4}, and β_{e5}. (j) Parameters β_{e7}, β_{e8}, and β_{e9}.

the dimension of the parameter vector β_e is 9. The parameters of the spacecraft are given in the Appendix. In the figures, $u_e = \delta_e(L,t)$ is the transverse deflection of the end point of the beam, and the pitch angle tracking error is $e = \theta - \theta_r$.

A. Rotational Maneuver and Vibration Damping

The complete closed-loop system (1), (12), (16), and (19) was simulated. The initial conditions were assumed to be $z(0) = 0$, $\dot{z}(0) = 0$, $\theta_r(0) = 0$, $\dot{\theta}_r(0) = 0$, and $\ddot{\theta}_r = 0$. The initial value $\beta_e(0)$ of the estimated parameter was arbitrarily set to zero. It should be noted that it is rather a worse choice of estimate of the parameter vector β. The controller parameters were selected as $a_0 = 5$, $c_0 = 10^8$, $\Gamma = 100I$, $F = \mathrm{diag}(10^8, 8 \times 10^8)$, and the parameter of the command generator was assumed be at $\lambda_c = 0.5$. It was desired to control the pitch angle to 100 deg.

Selected responses are shown in Fig. 2. Smooth control of attitude and vibration supression were accomplished in the closed-loop system. The response time of the pitch angle is of the order of 15–20 s. It is seen that the elastic modes are asymptotically converging to zero as well. The estimated parameters β_{ek} $(k = 1, \ldots, 9)$ also converge to some constant values. It should be noted that in general, these parameters do not converge to their actual values unless persistent excitation condition of certain signals in the closed-loop system is satisfied

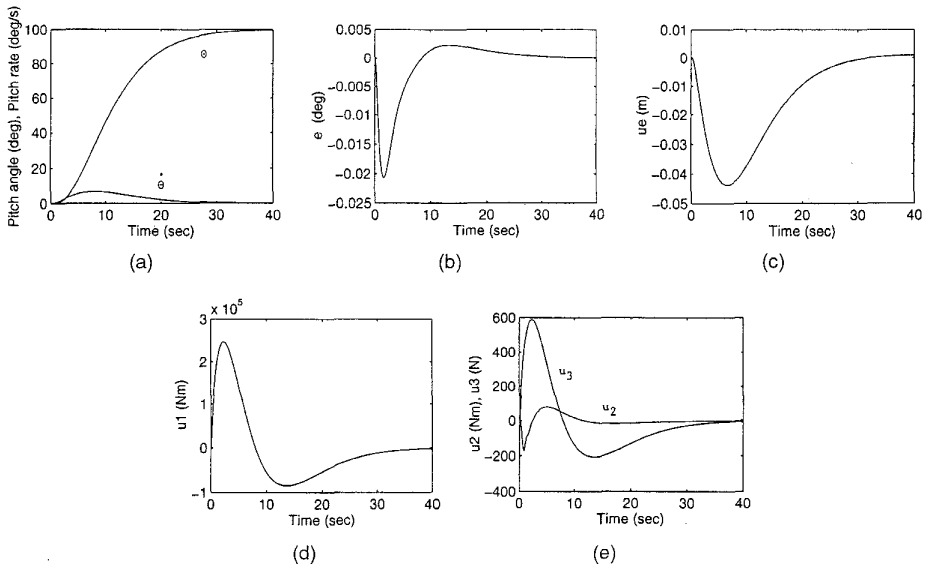

Fig. 3. Adaptive control and stabilization: faster command. (a) Pitch angle θ, angular velocity $\dot{\theta}$. (b) Pitch angle tracking error e. (c) End point deflection u_e. (d) Control torque u_1. (e) Control torque u_2, control force u_3.

[14–15]. The control inputs also vary smoothly. The maximum end point deflection was 1.0 m.

B. Rotational Maneuver and Vibration Damping: Faster Command

In order to obtain a quicker regulation of the pitch angle, a faster command trajectory was generated using $\lambda_c = 1$. The remaining parameters and initial conditions of the previous simulation were retained. Selected responses are shown in Fig. 3. We observe smooth control of the attitude angle and stabilization of the elastic modes. The pitch angle response time is about half of the value of case 1. However, larger control magnitudes were required as predicted. Moreover, maximum deflection of the end point increased to 2 m. Only a small tracking error (less than 0.18 deg) was observed.

Extensive simulations showed that the control system accomplishes large angle rotational maneuvers and vibration suppression. There are several design parameters which can be properly selected to accomplish rotational maneuver with reasonable control input magnitude and elastic deformation of the beam.

VI. CONCLUSIONS

In this paper, rotational maneuver and vibration suppression of an elastic spacecraft was considered. An adaptive control system was designed for the regulation of the pitch angle to any desired value. Using the adaptive controller, reference pitch angle trajectories were asymptotically tracked. For the control of elastic modes, a stabilizer was designed using only measured output and colocated actuators and sensors. For the design of elastic mode stabilizer, zero dynamics which describe only the flexible motion, was used. For the design of the attitude controller and the stabilizer, the knowledge of system parameters was not assumed. Simulation results showed that in the closed-loop system, smooth control of pitch angle and vibration suppression can be accomplished using the designed control system.

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APPENDIX

System Parameters

m_0, m_t = (98739.5, 875.32)—Spacecraft, tip body mass (kg).

I_0, I_t = (9769869.5, 1400.512)—Spacecraft and tip body moment of inertia about their mass centers perpendicular to plane of motion (kg $- m^2$).

a = $(a_1, a_2) = (2, 0.1)$—Vector from spacecraft mass center to beam attachment point (m).

d = 2—Distance of tip body mass center from the attachment point of tip body with beam (m).

ρ = 21.883—Mass per unit length of beam (kg/m).

EI = 353520—Bending rigidity ($N - m^2$).

$$
\begin{aligned}
m_1 &= \rho L + m_t,\ m = m_0 + m_1,\ \mu_0 = m_0/m, \\
&\quad \mu_1 = m_1/m. \\
b_1 &= \{(\rho L^2/2) + m_t(L+d)\}/m_1,\ L = 20.0 \text{ m}. \\
a_{00} &= \frac{I_0 + J_0}{\rho L^3} + (m/\rho L)\mu_0\mu_1 \\
&\quad \times \left(\frac{a_1^2}{L^2} + \frac{a_2^2}{L^2} + \frac{2a_1b_1}{L^2}\right) - \frac{m}{\rho L^3}\mu_1^2 b_1^2. \\
a_{1k} &= \frac{\mu_0 a_1 u_{3k}}{L} + u_{4k} - \frac{\mu_1 b_1 u_{3k}}{L}\ (k = 1,2,\ldots). \\
b_{01} &= 1/(\rho L^3). \\
b_{03} &= \left(\mu_0\frac{a_1}{L} + 1 + \frac{d}{L} - \mu_1\frac{b_1}{L}\right)\Big/\rho L^2. \\
b_{i2} &= u_{1i}/\rho L^3,\ b_{i3} = \{u_{2i}/\rho L^3\} - \{u_{3i}/mL\}.
\end{aligned}
$$

Modal Parameters

$$
\begin{aligned}
(u_{11},\ldots,u_{15}) &= (0.9087, -4.835, 6.0703, -4.966, 3.5599) \\
(u_{21},\ldots,u_{25}) &= (0.676, -0.1266, -0.0027, 0.0552, -0.0608) \\
(u_{31},\ldots,u_{35}) &= (1.5691, 0.5224, 0.298, 0.2204, 0.1707) \\
(u_{41},\ldots,u_{45}) &= (1.654, 0.14854, 0.05058, 0.02590, 0.015)
\end{aligned}
$$

Frequencies

$$(\Omega_1,\ldots,\Omega_5) = (0.323, 3.805, 11.10, 22.987, 41.162).$$

Elastic Deflection

$$u_e = \sum_{j=1}^{5}(Lu_{2k} - du_{1k})p_k(t).$$

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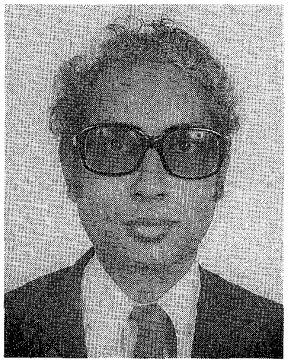

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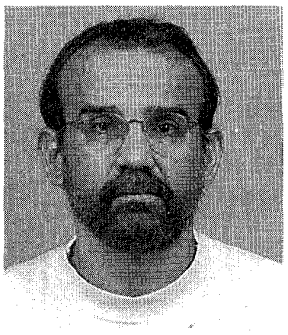

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