

## Impact Angle Control for Planar Engagements

**A new and practical terminal guidance law is presented for impact angle control for two-dimensional active homing engagement scenarios. Impact angle control is required to enhance terminal effectiveness of antitank and antiship missile systems in particular. The proposed guidance law is developed to cope with missile velocity reduction due to aerodynamic drag and target maneuver. In actual applications, the proposed guidance law is combined in cascade with a target tracking filter under the certainty equivalence principle. The proposed guidance law in conjunction with the target tracking filter is shown to be effective in antiship active homing engagement scenarios by a series of Monte Carlo simulation runs.**

### I. INTRODUCTION

To increase terminal effectiveness of tactical missile systems such as antitank and antiship missile systems, impact angle control as a part of trajectory modulation is required in the terminal phase together with selection of an effective warhead type. Impact angle control is also needed to enhance survivability of the missiles against increased capability of ship defense systems.

Previous research activities related to impact angle control are briefly reviewed. An optimal impact guidance law for an air-to-surface missile system has been proposed in [1], however, nonlinear system dynamic equations are formulated in an inertial coordinate system such that the linearization suggested in [1] may not be valid for large missile attitude variations necessary to cope with target maneuvers. A guidance law composed of pure pursuit and proportional navigation guidance (PNG) [2] is proposed in [3] to guide the missile on a collision course. Later the guidance law is extended to cope with constant target maneuver and varying missile velocity [4]. The principle of the suggested guidance is to reduce course deviation from a predetermined collision course without direct control of the impact angle. An integrated PNG law with missile attitude control is studied in [5] to satisfy the same principle. An optimal guidance law derived with a quadratic performance index subject to impact angle and

missile maneuver constraints is suggested in [6] for a stationary target, and an optimal guidance law for missiles with varying velocity is extensively studied in [7]. However, the guidance laws may not be suitable for impact angle control in case of a maneuvering target and varying missile velocity.

A new optimal guidance law for impact angle control in the terminal phase is proposed here for planar active homing engagements involving a maneuvering target and a missile with varying velocity. It is presumed in the guidance law development that the proposed active homing guidance law follows successful guidance handover from a proper midcourse guidance scheme so that the prerequisite conditions for the proposed guidance such as small heading errors, and continuous target tracking throughout the midcourse and terminal phases by the same seeker with practical gimbal angle limits, are not violated. The desired impact angle can be made arbitrary if the prerequisite conditions are satisfied. Since the proposed guidance law is derived by using missile-target relative kinematics in an aiming frame which is rotating according to target maneuvers; decoupling and linearization of the system dynamic equations are possible with valid assumptions. The guidance can be extended to spatial engagements without complexities. A suboptimal filter for target state estimation [8] is combined with the guidance law using the certainty equivalence principle [9] which indicates that a guidance law and an estimator can be designed independently. The proposed closed-loop guidance law is tested for antiship engagements by a series of Monte Carlo simulation runs.

### II. GUIDANCE

In this section, system dynamic equations based on missile-target engagement geometry are established and an optimal impact angle control law needed in the terminal phase to increase system effectiveness is derived by minimizing a quadratic performance index with final state constraints. The guidance law is combined with a target state estimator introduced in the latter part of this section.

#### A. System Dynamics for Guidance

Consider the initial missile-target engagement configuration for the terminal phase depicted in Fig. 1, where the target is tracked continuously by an active seeker on board the missile from the midcourse phase. The seeker considered here has mechanical gimbal angle limits inherent to current seekers in practice. The seekers on-board the current homing missiles have mechanical gimbal angle limits of approximately  $\pm 45^\circ$  in general. However, the engagements with gimbal angle well within the limits are required in practice for guidance effectiveness with limited missile

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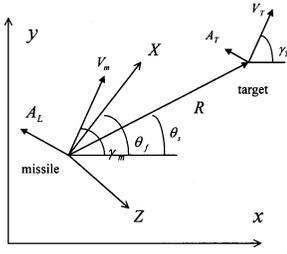


Fig. 1. Missile-target engagement configuration.

is the angular velocity of the G system with respect to the I system. Note that  $\bar{\omega}_G$  is equivalent to  $\dot{\gamma}_T$ . The relative velocity can be expressed as

$$\begin{pmatrix} \dot{X} \\ \dot{Z} \end{pmatrix} = \begin{pmatrix} V_T \cos(\gamma_T - \theta_f) - V_m \cos(\gamma_m - \theta_f) - \dot{\gamma}_T Z \\ -V_T \sin(\gamma_T - \theta_f) + V_m \sin(\gamma_m - \theta_f) + \dot{\gamma}_T X \end{pmatrix}. \quad (2)$$

Similarly, the relative acceleration with respect to the G system can be written as

$$\begin{pmatrix} \ddot{X} \\ \ddot{Z} \end{pmatrix} = \begin{pmatrix} -A_T \sin(\gamma_T - \theta_f) - \dot{V}_m \cos(\gamma_m - \theta_f) + A_L \sin(\gamma_m - \theta_f) - 2\dot{\gamma}_T \dot{Z} + X\dot{\gamma}_T^2 \\ -A_T \cos(\gamma_T - \theta_f) + \dot{V}_m \sin(\gamma_m - \theta_f) + A_L \cos(\gamma_m - \theta_f) + 2\dot{\gamma}_T \dot{X} + Z\dot{\gamma}_T^2 \end{pmatrix} \quad (3)$$

maneuverability regarding target maneuvers, initial heading errors, and seeker measurement noises. It is assumed that the target has a constant speed  $V_T$  and a constant acceleration  $A_T$  and the missile has varying velocity  $V_m$  due to aerodynamic drag. The missile achieves a lateral acceleration  $A_L$  perpendicular to the current  $V_m$  to change the flight path angle. The flight path angles of the target and missile are denoted as  $\gamma_T$  and  $\gamma_m$ , respectively. Note that in this work, the sign of an angle is positive if it is measured counterclockwise from the  $x$  axis, and a counterclockwise rotation results in a positive angular rate.

In Fig. 1,  $(x, y)$  denotes an inertial coordinate system (denoted in this paper as I system for brevity), and  $(X, Z)$  represents a guidance reference coordinate system (G system for brevity) where the  $X$  axis is established along the desired missile flight direction with the angle  $\theta_f$  from the  $x$  axis such that the desired impact angle is  $\gamma_T - \theta_f$ . In this way, the sign of the desired impact angle is defined to be positive if it is measured clockwise from  $-V_T$  vector. The actual impact angle is  $\gamma_T - \gamma_m$ . Note that the G system is rotating with an angular rate  $\dot{\gamma}_T$  to keep the desired impact angle constant. The missile to target line-of-sight (LOS) angle formed by the seeker is denoted as  $\theta_s$ .

The relative position vector between the missile and the target can be expressed in the G system as

$$\bar{R} = \begin{pmatrix} X \\ Z \end{pmatrix} = \begin{pmatrix} R \cos(\theta_f - \theta_s) \\ R \sin(\theta_f - \theta_s) \end{pmatrix} \quad (1)$$

and the relative velocity with respect to the G system coordinatized in the G system can be derived from differentiating (1) with respect to time or by using the fundamental relation [11] of  $d\bar{R}/dt|_G = d\bar{R}/dt|_I - \bar{\omega}_G \times \bar{R}$  where  $d\bar{R}/dt|_G$  is the missile-target relative velocity with respect to the rotating G system,  $d\bar{R}/dt|_I$  is the relative velocity with respect to the I system, and  $\bar{\omega}_G$

where the last two terms are the Coriolis acceleration and the centrifugal acceleration components, respectively [11].

The varying missile velocity is assumed to satisfy the following deceleration equation [2]

$$\dot{V}_m = -\lambda V_m, \quad V_m(0) = V_{m_0} \quad (4)$$

where  $\lambda$  is a positive constant, and  $\dot{\gamma}_T = A_L/V_T$  is small such that  $\dot{\gamma}_T^2$  is negligible. By using  $X$  in (2) and denoting missile control acceleration in the  $Z$  direction as  $U_Z \triangleq A_L \cos(\gamma_m - \theta_f)$ , then  $\ddot{Z}$  in (3) becomes

$$\begin{aligned} \ddot{Z} = & -\lambda \dot{Z} - \lambda V_T \sin(\gamma_T - \theta_f) + U_Z + A_T \cos(\gamma_T - \theta_f) \\ & + \lambda \dot{\gamma}_T X - 2V_m \dot{\gamma}_T \cos(\gamma_m - \theta_f). \end{aligned} \quad (5)$$

For a missile with small  $\lambda$ , the coupling between  $X$  and  $Z$  channel is negligible, and the missile heading error  $\gamma_m - \theta_f$  should be small for the working impact angle control algorithm. Since impact angle control is required in the terminal guidance phase, and the time required for the terminal phase is relatively short compared with the initial and midcourse guidance phases, there is enough time for trajectory shaping through proper midcourse guidance such as biased pure pursuit and biased proportional navigation guidance to satisfy the small heading error requirement. The situation is particularly true for antiship engagements. Note that the initial missile-target engagement configuration of Fig. 1 for the proposed guidance law development should be considered within the context of practical seeker gimbal angles and small heading errors.

However, the proposed guidance law may work at the expense of large accelerations in the case where the missile heading error  $\gamma_m - \theta_f$  is not small. The large acceleration requirement for this case can be seen from the relation  $A_L = U_Z / \cos(\gamma_m - \theta_f)$ . The above fact is verified in Section III by a simulation study. With the small missile heading error assumption, the

X-Z channel decoupling, and the desired impact angle selected as an arbitrary constant, (5) can be written as

$$\frac{d}{dt} \begin{pmatrix} Z \\ \dot{Z} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & -\lambda \end{pmatrix} \begin{pmatrix} Z \\ \dot{Z} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} U_Z + \begin{pmatrix} 0 \\ A_T \cos(\gamma_T - \theta_f) - \lambda V_T \sin(\gamma_T - \theta_f) - 2V_m \dot{\gamma}_T \end{pmatrix} \quad (6)$$

which is a linear time-invariant system state equation used for derivation of  $U_Z$ .

### B. Optimal Impact Angle Control

With the linear system dynamics along the Z axis described in (6), an analytic optimizing solution  $U_Z$  is obtained with the performance index

$$J = \frac{1}{2} \int_t^{t_f} U_Z^2 d\tau \quad (7)$$

subject to the given final time  $t_f$ , and constraints  $Z(t_f) = 0$ , and  $\dot{Z}(t_f) = -V_T \sin(\gamma_T - \theta_f)$ . The constraints are formulated here to reduce the miss distance with a small missile attitude angle error. If the constraint  $\dot{Z}(t_f) = -V_T \sin(\gamma_T - \theta_f)$  is not included in the problem formulation as in the derivation of optimal PNG in [9], the missile needs to have a non-zero inertial velocity in the Z direction to form a collision course so that high velocity superiority over the target is required for small errors at large impact angle. However for the period near the impact, the missile velocity superiority is not needed in this optimal impact angle control formulation since the constraint eventually makes the missile velocity lie in the X axis only. The optimal control  $U_Z$  can be derived by either the Schwartz inequality [10] or modern control theory [9] such that the resulting control becomes

$$U_Z = c_1(t)Z(t) + c_2(t)\dot{Z}(t) + c_3(t)V_T \sin(\gamma_T - \theta_f) + c_4(t)V_m(t)\dot{\gamma}_T + c_5(t)A_T \cos(\gamma_T - \theta_f) \quad (8)$$

where the coefficients  $c_i(t)$ s are summarized in Appendix A, and the state variables  $Z$  and  $\dot{Z}$  involved in the guidance law are calculated by using (1) and (2). Note that the  $t_f$  in the coefficients  $c_i(t)$ s, which is assumed to be given in the optimal control problem formulation, is calculated with the estimated missile-to-target range and the estimated closing velocity from the proposed filter algorithm of Section IIC. The other methods for  $t_f$  calculation can be found in [2, 3, and 7].

Besides the optimal guidance law, an accurate target state estimator is needed to provide estimates of the target state variables involved in the guidance algorithm. A suboptimal estimator used in the study is introduced in the next section.

### C. Suboptimal Filter

The system state vector  $(Z, \dot{Z})^T$ ,  $t_{go} \triangleq t_f - t$ , and the target state variables such as  $V_T$ ,  $A_T$ ,  $\gamma_T$ , and  $\dot{\gamma}_T$  involved in the optimal guidance law should be estimated by a proper filter in actual applications. A suboptimal filter previously used in radar target tracking [5] is applied in this work with a slight modification in target acceleration modeling.

The six-element state vector  $x^M$  in the filter dynamic model is composed of missile-target relative position, relative velocity, and the Singer model target acceleration expressed in the I system. Note that the system dynamics for guidance law derivation are established in the G system while the filter dynamic model is formulated in the I system such that transformation of state estimates is required before applying to the guidance law. The continuous dynamic model is represented as follows

$$\dot{x}^M = A^M x^M + B^M A_m + D w \quad (9)$$

where  $x^M = (x, y, \dot{x}, \dot{y}, A_T, A_T)^T$ ,

$$A^M = \begin{bmatrix} 0 & I_2 & 0 \\ 0 & 0 & I_2 \\ 0 & 0 & -\frac{1}{\tau} I_2 \end{bmatrix}, \quad B^M = \begin{bmatrix} 0 \\ -I_2 \\ 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau} I_2 \end{bmatrix} \quad (10)$$

and the process noise  $w = (w_x, w_y)^T$  is a white Gaussian noise vector with zero-mean and power spectral density of  $2\tau\sigma_{A_T}^2$ . Note that  $\sigma_{A_T}^2$  is the assumed variance of the target acceleration and  $\tau$  is the time-constant of target maneuver. Note also that the missile acceleration  $A_m$  in (9) satisfies

$$A_m = \begin{pmatrix} A_{m_x} \\ A_{m_y} \end{pmatrix} = \begin{pmatrix} \dot{V}_m \cos \gamma_m - A_L \sin \gamma_m \\ \dot{V}_m \sin \gamma_m + A_L \cos \gamma_m \end{pmatrix} \quad (11)$$

where  $A_L = U_Z / \cos(\gamma_m - \theta_f)$ . The measurements from the active seeker at time  $t = t_k$  comprised of noise-corrupted missile-target relative range and LOS angle can be expressed as

$$\begin{pmatrix} z_R \\ z_\theta \end{pmatrix}_k = \begin{pmatrix} \sqrt{x^2 + y^2} + v_R \\ \tan^{-1} \frac{y}{x} + v_\theta \end{pmatrix}_k \quad (12)$$

where  $v_R$  and  $v_\theta$  are zero-mean white Gaussian noises with variances  $\sigma_R^2$  and  $\sigma_\theta^2$  respectively. The suboptimal filter used in this study utilizes pseudomeasurements obtained by algebraic manipulation of the original

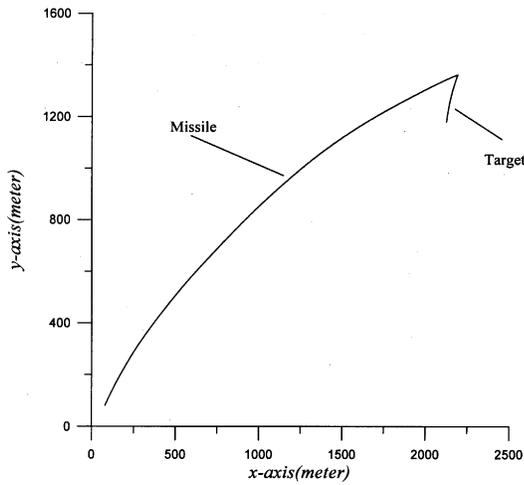


Fig. 2. Trajectories of missile and target.

nonlinear measurements of (12), and state estimates of the filter are calculated in the I system while covariance matrices and filter gain are calculated in the Cartesian coordinate system based on the LOS of the seeker to maintain a decoupled structure in covariance calculation. The state estimates and the filter gain are combined by proper coordinate transformation matrices which are formulated to reduce estimation biases. Due to the decoupled structure of covariance matrices, the filter has computational effectiveness which is important in real-time guidance applications. The detailed filter algorithm is referred to [5].

### III. SIMULATION RESULTS

The optimal impact control law and the suboptimal filter of the previous section are placed in cascade to form a closed loop under the assumption of the certainty equivalence principle. A series of Monte Carlo simulation runs is carried out to analyze performance of the proposed guidance law in the terminal phase. Since the simulation starts from the beginning of the terminal phase, the initial time is the time of guidance handover at the end of the midcourse phase. The missile used in this planar engagement study has an initial speed of 272 m/s with the deceleration time-constant  $1/\lambda$  equal to 100 (s). The target is a ship with a constant speed of 20.3 m/s and the lateral acceleration  $A_T$  of  $-0.725 \text{ m/s}^2$  so that the ship is changing its heading with  $-2^\circ/\text{s}$  angular rate. Initial inertial positions of the missile and the target are set to be (76.2 m, 76.2 m) and (2124.2 m, 1180 m). For this engagement scenario, initial values of  $\gamma_m$  and  $\gamma_T$  are  $55^\circ$  and  $80^\circ$ , respectively, and the desired impact angle is  $45^\circ$ . The spectral density of the process noise  $q = 2\tau\sigma_{A_T}^2$  is calculated with the assumed target maneuver time-constant  $\tau = 100$  (s) and the assumed target acceleration variance  $\sigma_{A_T}^2 = 1 \text{ (m}^2/\text{s}^4)$ . The measurement noise

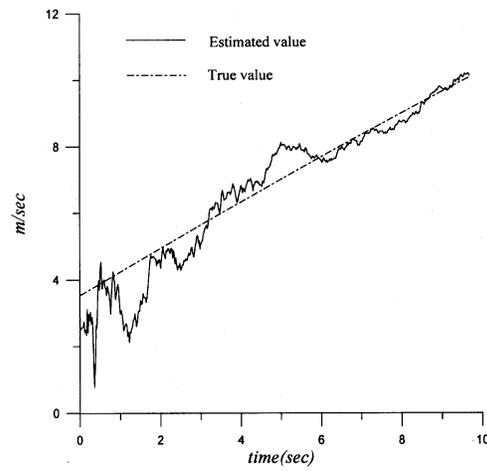


Fig. 3. True and estimated  $\dot{x}_T$ .

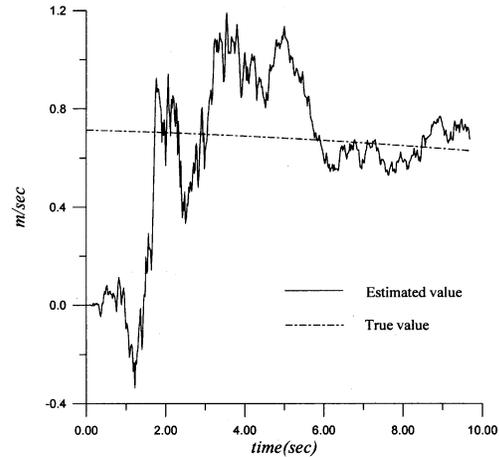


Fig. 4. True and estimated  $A_{T_x}$ .

variances used in the study are  $\sigma_R^2 = 4 \text{ (m}^2)$  and  $\sigma_\theta^2 = 1 \text{ (mrad}^2)$ . The initial state estimate  $\hat{x}^M(0)$  and covariance  $\hat{P}(0)$  for the suboptimal filter are assumed to be  $\hat{x}^M(0) = (1948, 1003.8, -153.5, -203.8, 0, 0)^T$  and  $\hat{P}(0) = \text{diag}(10^4, 10^4, 10, 10, 1, 1)$ . The guidance and filter are operating at 50 Hz. Fig. 2 shows the trajectories of the missile and the target for the engagement scenario. Figs. 3–4 show the true and estimated target velocity and acceleration in the  $x$  axis, respectively. The results together with state estimates in the  $y$  axis indicate good tracking performance of the suboptimal filter. Fig. 5 shows the actual impact angle history which indicates the missile heading error  $\gamma_m - \theta_f$  remains less than the initial heading error of  $20^\circ$  throughout the terminal phase satisfying the small heading error assumption. Fig. 6 shows the required missile lateral acceleration  $A_L$  well within current missile capabilities. Fig. 7 shows history of the seeker gimbal angle  $\gamma_m - \theta_s$  for the engagement scenario indicating that max gimbal angle of  $27^\circ$  is required at the initial time and the target is engaged with gimbal angles within the practical limits. Impact angle and miss distance calculations by averaging

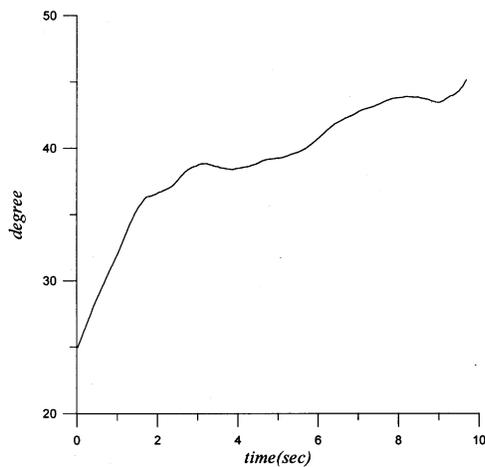


Fig. 5. Actual impact angle history.

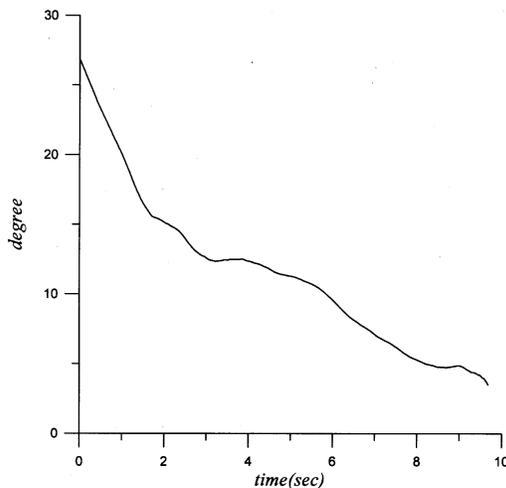


Fig. 7. Seeker gimbal angle history.

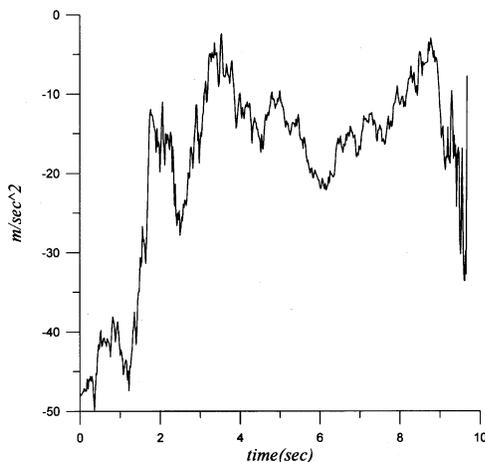


Fig. 6. Required missile lateral acceleration.

25 runs of Monte Carlo simulation indicate that the mean and the variance of the actual impact angle at the final engagement time are  $45.09^\circ$  and  $0.195(^{\circ})^2$ , respectively and the mean and the variance of the miss distance are 2.2 m and  $2.06 \text{ (m)}^2$ , representing excellent performance.

Another scenario which violates the small heading error assumption is also studied with initial values of  $\gamma_m$  and  $\gamma_T$  at  $45^\circ$  and  $160^\circ$ , and the desired impact angle is  $90^\circ$ . The other parameters remain the same as the first scenario but the initial state estimate is changed as  $\hat{x}^M(0) = (1948, 1003.8, -212.4, -186.4, 0, 0)^T$  due to the different engagement geometry. Simulation studies for this case indicate that the small heading error assumption is violated since the maximum missile heading error becomes  $-45^\circ$  larger than the initial heading error of  $-25^\circ$  in magnitude, and that the missile needs to have high maneuverability for the second scenario since it requires an acceleration variation of nearly  $12g$  in magnitude during the terminal phase. As analyzed in Section IIB, high missile lateral accelerations are expected for large

missile heading errors. The seeker gimbal angle for this scenario remains well within the practical limits. Impact angle and miss distance calculations by 25 Monte Carlo simulation runs indicate that the mean and the variance of the actual impact angle at the final engagement time are  $89.96^\circ$  and  $0.325(^{\circ})^2$ , respectively, and the mean and the variance of the miss distance are 2.46 m and  $4.12 \text{ (m)}^2$ . The above results show that if the small heading error assumption is violated, the proposed guidance law works at the expense of large lateral accelerations. Therefore, it is important for missiles with limited maneuverability to have trajectory shaping through proper midcourse guidance before the proposed guidance law is applied in the terminal phase with a careful choice of the desired impact angle to form the initial engagement configuration rendering small heading errors and gimbal angles within the physical limits.

#### IV. CONCLUSIONS

An optimal active homing guidance law for impact angle control is studied in this paper by missile-target relative dynamics established in a Cartesian coordinate system rotating according to target maneuver to maintain a desired impact angle. The guidance law is a control energy minimizing solution with constraints to reduce miss distance with a small missile attitude error at the final engagement time. To reduce the impact angle error, the desired  $\dot{Z}(t_f)$  at the final time is constrained to be  $-V_T \sin(\gamma_T - \theta_f)$  in the optimal control formulation. A suboptimal filter is introduced and connected to the optimal guidance in cascade to form a closed loop. The results of a series of Monte Carlo simulation runs indicate that the guidance law together with the suboptimal filter is effective without computational burden for antiship engagements. In actual applications with limited missile maneuverability, it is required to have proper trajectory shaping to satisfy the small missile heading

error during relatively long midcourse guidance phase before the proposed guidance law is applied in the terminal phase.

#### APPENDIX A. COEFFICIENTS OF $U_Z$

In this Appendix, the coefficients of the optimal impact angle control specified in (8) are listed

$$c_1(t) = -\frac{1}{2\lambda^2\Delta}(1-E)^2 \quad (13a)$$

$$c_2(t) = -\frac{1}{2\lambda^3\Delta}(1-4E+(2\lambda t_{go}+3)E^2) \quad (13b)$$

$$c_3(t) = -\frac{1}{2\lambda^3\Delta}(1-\lambda t_{go}+2(\lambda t_{go}-2)E+(\lambda t_{go}+3)E^2) \quad (13c)$$

$$c_4(t) = \frac{1}{\lambda^3\Delta}\left(\frac{1}{\lambda}-\left(\frac{3}{\lambda}+2t_{go}\right)E+\left(\frac{3}{\lambda}+2t_{go}+2\lambda t_{go}^2\right)E^2-\frac{1}{\lambda}E^3\right) \quad (13d)$$

$$c_5(t) = \frac{1}{2\lambda^4\Delta}(2-\lambda t_{go}-4E+(\lambda t_{go}+2)E^2) = -1 \quad (13e)$$

where

$$\Delta = \frac{1}{4\lambda^4}(-4+2\lambda t_{go}+8E-(2\lambda t_{go}+4)E^2) \quad (14a)$$

$$E = e^{-\lambda t_{go}} \quad (14b)$$

$$t_{go} \triangleq t_f - t = \frac{R}{V_c} \quad (14c)$$

and

$$V_c = V_m \cos(\gamma_m - \theta_s) - V_T \cos(\gamma_T - \theta_s). \quad (14d)$$

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#### Wideband Quadrature Error Correction (using SVD) for Stepped-Frequency Radar Receivers

We present a new technique which corrects the wideband quadrature errors associated with homodyne stepped-frequency radar receivers. The correction algorithm is derived using singular value decomposition (SVD) which diagonalizes and scales the covariance matrix of a test signal while preserving the coherent phasor alignment across all frequency steps of the homodyne receiver. Using our technique, the wideband

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