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Brief Paper

Adaptive Disturbance Attenuation with Global Stability for Rigid and Elastic Joint Robots

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Key Words—Robot control; adaptive control; H^∞ control; stability.

Abstract—The disturbance attenuation problem with global internal stability is solved for both rigid and elastic joint robots in the presence of unknown constant parameters. The proposed dynamic controller combines adaptive and H_∞ control techniques. ©1997 Elsevier Science Ltd. All rights reserved.

1. *Introduction*

In the 1980s much research effort in robotics was spent in the design of adaptive controllers in order to compensate for the effects of not perfectly known system parameters. From the early works of Koivo and Guo (1983), Slotine and Li (1988) and Bayard and Wen (1988), which solved the adaptive control problem for the rigid robot, to the more recent results of Lozano and Brogliato (1992) and Lim *et al.* (1994), which considered the effect of the joint elasticity, much literature is available on the subject. A different approach to cope with parameter uncertainties is robust control; in robotics some interesting results are contained in Spong (1992). Robust control turns out to be a successful approach also in the presence of external disturbances and unmodeled dynamics whereas adaptive controllers behave poorly. Adaptive control algorithms may be modified to cope with the robustness issue, but this yields more complex control structures. More recently the problem of robust control for nonlinear systems has been studied in an H_∞ setting by van der Schaft (1992) and Isidori and Astolfi (1992). The main objective is, in this case, to minimize the effect of the disturbances on a set of penalty variables on one hand and to ensure internal asymptotic stability of the closed-loop system, on the other. The design of an H_∞ controller requires solving a nonlinear Hamilton–Jacobi–Isaacs equation; in general, this solution is difficult to compute. Applications in robotics of these theoretical results have been pursued very recently in the works of Chen *et al.* (1994), Astolfi and Lanari (1994) and Battilotti and Lanari (1995, 1996). A different approach has been pursued, for elastic joint robots, by Tomei (1994), where disturbance attenuation on the tracking link error position is achieved only after some finite time T; moreover, the proposed controller requires the knowledge of some bounds on the parameter uncertainties and external disturbances which may be difficult to compute and may lead to an unnecessarily high control effort. In the work of Chen *et al.* (1994), parameter uncertainties are handled as part of the disturbances. However, this approach has the main drawback that one does not have a direct measure of the attenuation level of the external disturbances on the penalty variable. Moreover, by incorporating the uncertainties in the disturbances, global internal stability is achieved only for a 'nominal' model and additional bounds on the parameter uncertainties must be assumed for stability to hold also for the 'perturbed' model. In Theorem 1, the disturbance attenuation problem with global tracking is solved with no a priori knowledge of the robot's constant parameters. This is achieved using the well-known property of linearity in the parameters of rigid robots. Our result does not share the drawbacks of the approach pursued in Chen *et al.* (1994). In Section 4, Theorem 1 is extended to flexible joint robots, choosing as penalty variables the link position and velocity errors (Theorem 2). The control scheme extends to an H_∞ setting, the adaptive stabilization result contained in Lozano and Brogliato (1992), giving a more systematic procedure even for the case of adaptive stablization. Simulations are given for a two-link rigid robot, showing interesting performances.

2. *Problem formulation*

Given a nonlinear system

$$\dot{x} = f(x) + g(x)u + p(x)w, \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ are the inputs and $w \in \mathbb{R}^q$ the disturbances, assumed to belong to $\mathcal{L}_{2e}$, i.e. any finite time truncation of w is square integrable. Let z be a suitable *penalty variable*, which may include the error between the state $x(t)$ and a desired state trajectory $x_d(t)$. The *disturbance attenuation problem* (or H_∞ control problem) *with global tracking via full-state dynamic feedback* consists in finding a proper positive definite and smooth function $V(x, \sigma)$, $\sigma \in \mathbb{R}^r$, and a smooth control

$$\begin{aligned} \dot{\sigma} &= \eta(x, \sigma), \\ u &= u(x, \sigma), \end{aligned} \tag{2}$$

such that along the trajectories of the closed-loop system (1) and (2), for all $w(t) \in \mathcal{L}_{2e}$ one has

$$\dot{V}(x, \sigma) + \|z\|^2 - \gamma^2\|w\|^2 \le 0 \tag{3}$$

and for $w(t) = 0$ the trajectories of the closed-loop system (1) and (2) are bounded, for all t and for all initial conditions, and $x(t) \to x_d(t)$ as $t \to \infty$. This corresponds to attenuating the effect of the disturbances w on the penalty variable z by a certain amount γ, with global tracking as long as $w(t) = 0$. The interested reader is referred to van der Schaft (1992) and Isidori and Astolfi (1992) for further details and motivations. Note that (3) is equivalent to

$$\begin{aligned} (HJI) = {} & \frac{\partial V(x, \sigma)}{\partial x, \sigma}\left[\begin{pmatrix} f(x) + g(x)u(x, \sigma) \\ \eta(x, \sigma) \end{pmatrix}\right] \\ & + \frac{1}{4\gamma^2}\frac{\partial V(x, \sigma)}{\partial x} p(x)p(x)^{\mathrm{T}} \frac{\partial V(x, \sigma)}{\partial x}^{\mathrm{T}} \\ & + \|z\|^2 \le 0. \end{aligned} \tag{4}$$

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When $r = 0$ we will refer to the *disturbance attenuation problem* (or H_∞ control problem) *with global tracking via full-state static feedback*.

3. *Rigid robot*

Consider the rigid robot's dynamic equations

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + w = u, \qquad (5)$$

where q denotes the joint angular positions, $B(q)$ the inertia matrix, $C(q,\dot{q})\dot{q}$ the centrifugal and Coriolis forces, $g(q)$ the gravity forces acting on the links, $u(t)$ the input torque and $w(t)$ unknown disturbances. For example, $w(t)$ may represent either unmodeled dynamics or external disturbances acting on the system. We make use of the properties:

(P1) $\dot{B}(q) - 2C(q,\dot{q})$ is skew-symmetric for a proper representation of $C(q,\dot{q})$.

(P2) The rigid robot dynamic equations (4) are linear in the parameters, i.e.

$$u = B(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + w = Y_1(q,\dot{q},\ddot{q})\vartheta + w \quad (6)$$

with ϑ a parameter vector function of the robot's constant parameters (e.g. link lengths, masses, etc.). The matrix Y_1 is usually defined as the *regressor*.

Denoting with x_{1d} and x_{2d}, respectively, the desired joint position and velocity profiles and

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} q \\ \dot{q} \end{pmatrix}, \quad e(t) = \begin{pmatrix} e_1(t) \\ e_2(t) \end{pmatrix} = \begin{pmatrix} x_1(t) - x_{1d}(t) \\ x_2(t) - x_{2d}(t) \end{pmatrix}, \quad (7)$$

one has the following error system:

$$\begin{aligned} \dot{e}_1 &= e_2, \\ \dot{e}_2 &= -B^{-1}(e_1 + x_{1d})\Big[C(e_1 + x_{1d}, e_2 + x_{2d})(e_2 + x_{2d}) \\ &\quad + g(e_1 + x_{1d}) - u + w\Big] - \ddot{x}_{1d}. \end{aligned} \qquad (8)$$

For simplicity we will omit the arguments of the functions $B(\cdot)$, $C(\cdot,\cdot)$ and $g(\cdot)$ unless otherwise stated. Define

$$\begin{aligned} \mu_1 &= -(1 - \alpha_2\beta_1 + \alpha_2^2), \\ Y(e_1, e_2, s, \dot{s}, t)\vartheta &= B\dot{s} + Cs + g, \\ \mu_2(e_1, e_2) &= \frac{1}{4\gamma^2}(e_2 + \alpha_2 e_1) + (\beta_1 - \alpha_2)e_1 + e_2, \\ s(e_1, t) &= x_{2d} - \alpha_2 e_1. \end{aligned} \qquad (9)$$

In the next theorem we present an iterative procedure which uses properties (P1) and (P2) and where only an estimate of the robot's constant parameters as link's lengths, masses, inertias, etc., is required.

Theorem 1. Let $z = \left(e_1^{\mathrm{T}} \;\; e_2^{\mathrm{T}}\right)^{\mathrm{T}}$. The disturbance attenuation problem with global tracking is solvable for the rigid robot (5) via full-state dynamic feedback. The dynamic controller, for any $\alpha_3 > 0$, $\beta_1 > 2$ and $\alpha_2 \in \left(\frac{\beta_1 - \sqrt{\beta_1^2 - 4}}{2}, \frac{\beta_1 + \sqrt{\beta_1^2 - 4}}{2}\right)$, is given by

$$\begin{aligned} \dot{\hat{\vartheta}} &= -Y^{\mathrm{T}}(e_2 + \alpha_2 e_1), \\ u &= Y\hat{\vartheta} - \mu_2(e_1, e_2) - (e_2 + \alpha_2 e_1)\alpha_3. \end{aligned} \qquad (10)$$

Proof.

Step 1. Consider first the system $\dot{e}_1 = u_2$ with the Lyapunov function $V_1(e_1) = \frac{1}{2}e_1^{\mathrm{T}}\beta_1 e_1$ with $\beta_1 > 0$. The control input $u_2 = -\alpha_2 e_1$, with $\alpha_2 > 0$, is such that $\dot{V}_1 = -\alpha_2\beta_1 e_1^{\mathrm{T}} e_1$.

Step 2. Now rewrite (8) as

$$\dot{e}_1 = (e_2 - u_2) + u_2,$$

$$\dot{e}_2 = -B^{-1}\left(C(e_2 + x_{2d}) + g - u + w\right) - \ddot{x}_{1d}.$$

Choose as the Lyapunov function

$$\begin{aligned} V(e_1, e_2, \hat{\vartheta}, t) &= V_1(e_1) + \frac{1}{2}(e_2 - u_2)^{\mathrm{T}}B(e_2 - u_2) \\ &\quad + \frac{1}{2}(\hat{\vartheta} - \vartheta)^{\mathrm{T}}(\hat{\vartheta} - \vartheta). \end{aligned} \qquad (11)$$

With the inertia matrix B bounded from below and above by constant matrices, we have

$$V_I(e_1, e_2, \hat{\vartheta}) \le V(e_1, e_2, \hat{\vartheta}, t) \le V_S(e_1, e_2, \hat{\vartheta})$$

for some positive definite quadratic functions $V_I(e_1, e_2, \hat{\vartheta})$ and $V_S(e_1, e_2, \hat{\vartheta})$. Choosing the controller as in (10) and using (P1) and (P2), one has

$$\begin{aligned} (HJI) &= -\mu_1 e_1^{\mathrm{T}} e_1 \\ &\quad - \alpha_3(e_2 + \alpha_2 e_1)^{\mathrm{T}}(e_2 + \alpha_2 e_1) \\ &\le 0, \end{aligned} \qquad (12)$$

which proves disturbance attenuation. To prove internal stability note that from (12) it follows, for $w(t) = 0$

$$\dot{V} \le (HJI) \le 0. \qquad (13)$$

This implies that

$$e_1,\; e_2 + \alpha_2 e_1,\; \dot{e}_1,\; \dot{e}_2 + \alpha_2\dot{e}_1,\; \hat{\vartheta} - \vartheta,\; \hat{\vartheta} \in \mathcal{L}_\infty. \quad (14)$$

Moreover, from (13), we conclude that

$$\begin{aligned} &\int_0^\infty (\mu_1\|e_1(\tau)\|^2 + \alpha_3\|e_2(\tau) + \alpha_2 e_1(\tau)\|^2)\mathrm{d}\tau \\ &\le V(0) - V(\infty) \le M, \end{aligned}$$

which implies e_1, $e_2 + \alpha_2 e_1 \in \mathcal{L}_2$. This, together with (14), implies by Barbalat's Lemma that e_1, $e_2 + \alpha_2 e_1 \to 0$ as $t \to \infty$, or equivalently, $e_1, e_2 \to 0$ as $t \to \infty$.

■

An interesting comparison can be done when exact knowledge of the robot's parameters is guaranteed. It turns out that the control is a computed-torque one (independent from γ) plus a linear feedback one (dependent from γ). It is well-known that the computed-torque nonlinear state-feedback

$$u(x_1, x_2) = B(x_1)(\ddot{x}_{1d} + v) + C(x_1, x_2)x_2 + g(x_1)$$

applied to (8) leads to

$$\begin{aligned} \dot{e}_1 &= e_2, \\ \dot{e}_2 &= v - B^{-1}w. \end{aligned} \qquad (15)$$

To solve the disturbance attenuation control problem with global tracking for (15), choose

$$V(e_1, e_2, t) = \frac{1}{2}e_1^{\mathrm{T}}\beta_1 e_1 + \frac{1}{2}(e_2 + \alpha_2 e_1)^{\mathrm{T}}B(e_2 + \alpha_2 e_1).$$

Note that this Lyapunov function derives directly from the step-by-step procedure stated in Theorem 1. It is easy to show that, with $\beta_1 > 2$, $\alpha_3 > 0$ and $\alpha_2 \in \left(\frac{\beta_1 - \sqrt{\beta_1^2 - 4}}{2}, \frac{\beta_1 + \sqrt{\beta_1^2 - 4}}{2}\right)$, the final control law is $u = B(x_1)s_2 + C(x_1, x_2)s_1 + g(x_1) - \mu_2(e_1, e_2) - \alpha_3(e_2 + \alpha_2 e_1)$ which coincides with (10), when $\vartheta = \hat{\vartheta}$. The obtained dynamic controller (10) contains the well-known adaptive controller of Slotine and Li (1988) plus a term independent of the robot's parameters ϑ.

4. *Flexible joint robot*

Consider the flexible joint robot's simplified dynamic equations (Spong, 1987)

$$B_1(q_\ell)\ddot{q}_\ell + C(q_\ell, \dot{q}_\ell)\dot{q}_\ell + g(q_\ell) + K(q_\ell - q_m) + w_1 = 0,$$
$$B_2\ddot{q}_m - K(q_\ell - q_m) + w_2 = u, \quad (16)$$

where $q_\ell \in \mathbb{R}^n$ and $q_m \in \mathbb{R}^n$ denote, respectively, the link and motor angular positions, $B_1(q_\ell)$ and B_2 the inertia matrices, $C(q_\ell, \dot{q}_\ell)\dot{q}_\ell$ the centrifugal and Coriolis forces, $g(q_\ell)$ the gravity force acting on the links, K the joint stiffness matrix, $u(t)$ the input torque and $w_1(t)$ and $w_2(t)$ unknown disturbances belonging to L_{2e} acting on the system. In what follows, we use the properties:

(P3) $\dot{B}_1(q_\ell) - 2C(q_\ell, \dot{q}_\ell)$ is skew-symmetric for a proper representation of $C(q_\ell, \dot{q}_\ell)$.

(P4) The flexible joint robot dynamic equations (16) are linear in the parameters.

By defining $x = \left(q_\ell^T \quad \dot{q}_\ell^T \quad q_m^T \quad \dot{q}_m^T\right)^T$, we obtain the following state-space representation:

$$\begin{aligned}
\dot{x}_1 &= x_2,\\
\dot{x}_2 &= -B_1^{-1}(x_1)\left\{C(x_1, x_2)x_2 + g(x_1) + K(x_1 - x_3) + w_1\right\},\\
\dot{x}_3 &= x_4,\\
\dot{x}_4 &= B_2^{-1}\left\{K(x_1 - x_3) + u - w_2\right\}. \qquad (17)
\end{aligned}$$

Let the desired state trajectory $x_d(t)$ be defined as

$$x_d^T = \left(x_{1d}^T \quad x_{2d}^T \quad x_{3d}^T \quad x_{4d}^T\right)^T = \left(q_{\ell d}^T \quad \dot{q}_{\ell d}^T \quad q_{md}^T \quad \dot{q}_{md}^T\right)^T$$

with q_{md} defined as

$$q_{md} = K^{-1}\left(B_1(q_{\ell d})\ddot{q}_{\ell d} + C(q_{\ell d}, \dot{q}_{\ell d})\dot{q}_{\ell d} + g(q_{\ell d}) + Kq_{\ell d}\right),$$

i.e. the nominal trajectory corresponding to the assigned desired link trajectory $q_{\ell d}(t)$; the error vector is $e(t) = x(t) - x_d(t)$. The proof of the following theorem is based on a step-by-step procedure in which we will denote by $(HJI)_i$ and V_i the corresponding Hamilton–Jacobi equation (HJI) and the Lyapunov function, respectively, at the ith step. This procedure has also been used in Dawson *et al.* (1991) for solving a robust stabilization problem. Note that the constant stiffness matrix is positive definite and, therefore

$$\exists \delta_i > 0 \quad \text{such that } K > \mathrm{diag}\{\delta_i\}. \qquad (18)$$

Moreover, note that $\Delta(e_1, t) = \det(B_1(e_1 + x_{1d})) = \vartheta_4^T Y_4(e_1, t)$ with ϑ_4 a constant parameter vector, i.e. Δ is linear in the parameters. In what follows, by L.P. we will mean 'linear in the parameters'. Following the idea of Lozano and Brogliato (1992), we can define the subspace H as $H = \left\{v : v = Y_4(x_1), \text{ for some } x_1\right\}$, and the domain D as $D = \left\{s : s^T v \geq \alpha^n, \ \forall v \in H\right\}$ with αI a lower bound of B_1. It can be proved (Lozano and Brogliato, 1992) that D is convex and $\vartheta_4 \in D$. These two properties will be used in the next theorem. Let μ_1 and $\mu_2(e_1, e_2)$ as in (9).

Theorem 2. Let $z = \left(e_1^T \quad e_2^T\right)^T$. The disturbance attenuation problem with global tracking is solvable for the elastic joint robot (17) via full-state dynamic feedback.

Proof.

Step 1. The first step is identical to the one of Theorem 1.

Step 2. Consider the system

$$\begin{aligned}
\dot{e}_1 &= e_2,\\
\dot{e}_2 &= -B_1^{-1}\left(C(e_2 + x_{2d}) + g + K(e_1 + x_{1d})\right)\\
&\quad + B_1^{-1}K\hat{u}_3 - \ddot{x}_{1d} + p_{21}w_1
\end{aligned}$$

with $\hat{u}_3$ to be determined and $p_{21}(e_1, t) = -B_1^{-1}(e_1 + x_{1d})$. Note first that, being K diagonal i.e. $K = \mathrm{diag}\{k_i\}$, one can write

$$K\hat{u}_3 = \mathrm{diag}\{\hat{u}_{3i}\}k \quad \text{with} \quad k^T = (k_1 \quad k_2 \quad \cdots \quad k_n)^T,$$

where the subscript i denotes the ith component of the vector. Proceeding as for the rigid case, choose

$$\begin{aligned}
V_2(e_1, e_2, \hat{\vartheta}_1, \hat{\vartheta}_2, t) &= \frac{1}{2}\beta_1 e_1^T e_1\\
&\quad + \frac{1}{2}(e_2 + \alpha_2 e_1)^T B_1 (e_2 + \alpha_2 e_1)\\
&\quad + \frac{1}{2}(\hat{\vartheta}_1 - \vartheta_1)^T(\hat{\vartheta}_1 - \vartheta_1)\\
&\quad + \frac{1}{2}(\hat{\vartheta}_2 - k)^T(\hat{\vartheta}_2 - k).
\end{aligned}$$

Choosing $\alpha_3 > 0$ and by (P4)

$$B_1(\ddot{x}_{1d} - \alpha_2 e_2) + C(\dot{x}_{1d} - \alpha_2 e_1) + g + K(\dot{x}_{1d} + e_1) = Y_1(e_1, e_2, t)\vartheta_1$$

with

$$\begin{aligned}
\dot{\hat{\vartheta}}_1 &= -Y_1^T(e_2 + \alpha_2 e_1),\\
\hat{u}_{3i} &= \frac{1}{\hat{\vartheta}_{2i}}\left((Y_1\hat{\vartheta}_1)_i - \mu_{2i} - \alpha_3(e_2 + \alpha_2 e_1)_i\right) \quad (19)
\end{aligned}$$

one obtains

$$\begin{aligned}
(HJI)_2 &= -\mu_1 e_1^T e_1 - \alpha_3\|e_2 + \alpha_2 e_1\|^2\\
&\quad + (\hat{\vartheta}_2 - k)^T\dot{\hat{\vartheta}}_2. \qquad (20)
\end{aligned}$$

Now, choose $\dot{\hat{\vartheta}}_{2i}$ as (see Lozano and Brogliato, 1992, for a comparison)

$$\dot{\hat{\vartheta}}_{2i} = \begin{cases} \hat{u}_{3i}(e_2 + \alpha_2 e_1)_i \\ \quad \text{if } \hat{\vartheta}_{2i} \geq \delta_i \text{ or } \hat{u}_{3i}(e_2 + \alpha_2 e_1)_i \geq 0, \\ \left[f(\hat{\vartheta}_{2i})\right]^{-\hat{u}_{3i}(e_2+\alpha_2 e_1)_i}\hat{u}_{3i}(e_2 + \alpha_2 e_1)_i \\ \quad \text{else} \end{cases} \quad (21)$$

as the ith component, with $f(\hat{\vartheta}_{2i})$ a suitable smooth function, such that $0 \leq f(\hat{\vartheta}_{2i}) \leq 1$ with $f(\delta_i/2) =$

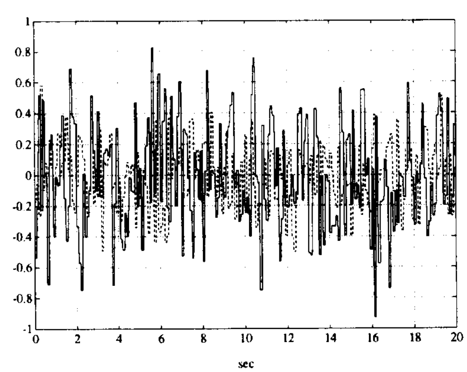

Fig. 1. Disturbances w_1 (–), w_2 (- -).

0 and $f(\delta_i) = 1$. If the dynamic controller (21) is initialized with a $\hat{\vartheta}_{2i}(0) \geq \frac{\delta_i}{2}$, the control $\hat{u}_{3i}$ is well-defined since, by the structure of (21), $\hat{\vartheta}_{2i}$ can never reach zero. Moreover, the right-hand side of (21) is continuously differentiable. With this choice (20) is less than or equal to zero.

Step 3. Define $\hat{\eta}_1 = x_3 - \hat{u}_3$. Consider the system

$$\dot{e}_1 = e_2,$$
$$\dot{e}_2 = -B_1^{-1}\left(C(e_2 + x_{2d}) + g + K(e_1 + x_{1d} - \hat{u}_3)\right) + B_1^{-1}K\hat{\eta}_1 - \ddot{x}_{1d} + p_{21}w_1,$$
$$\dot{\hat{\eta}}_1 = \hat{u}_4 + \hat{p}_{31}w_1 - \dot{\hat{u}}_3\Big|_{w_1=0} + \dot{\hat{u}}_3\Big|_{w_1=0,\vartheta_i=\hat{\vartheta}_i},$$

together with (19) and (21), with some smooth $\hat{p}_{31}$ and $\hat{u}_4$ to be determined. As will become clear, the introduction of $-\dot{\hat{u}}_3\Big|_{w_1=0} + \dot{\hat{u}}_3\Big|_{w_1=0,\vartheta_i=\hat{\vartheta}_i}$ allows the achievement of the L.P. property for $\hat{u}_4$. Choosing

$$V_3(e_1, e_2, \hat{\eta}_1, \hat{\vartheta}_1, \hat{\vartheta}_2, \hat{\vartheta}_3, \hat{\vartheta}_4, t) = V_2(e_1, e_2, \hat{\vartheta}_1, \hat{\vartheta}_2, t) + \frac{1}{2}\hat{\eta}_1^{\mathrm{T}}\Delta\hat{\eta}_1 + \frac{1}{2}(\hat{\vartheta}_3 - \vartheta_3)^{\mathrm{T}}(\hat{\vartheta}_3 - \vartheta_3) + \frac{1}{2}(\hat{\vartheta}_4 - \vartheta_4)^{\mathrm{T}}(\hat{\vartheta}_4 - \vartheta_4)$$

one obtains, for a suitable parameter vector ϑ_3

$$(HJI)_3 = (HJI)_2 + \hat{\eta}_1^{\mathrm{T}}(\Delta\hat{u}_4 - Y_3(e_1, e_2, \hat{\eta}_1, t, \hat{\vartheta}_1, \hat{\vartheta}_2)\vartheta_3) + (\hat{\vartheta}_3 - \vartheta_3)^{\mathrm{T}}\dot{\hat{\vartheta}}_3 + (\hat{\vartheta}_4 - \vartheta_4)^{\mathrm{T}}\dot{\hat{\vartheta}}_4.$$

Let $\alpha_4 > 0$ and

$$\hat{u}_4 = \frac{1}{\hat{\vartheta}_4^{\mathrm{T}}Y_4}(Y_3\hat{\vartheta}_3 - \alpha_4\hat{\eta}_1), \tag{22}$$

$$\dot{\hat{\vartheta}}_3 = -Y_3^{\mathrm{T}}\hat{\eta}_1, \tag{23}$$

$$\dot{\hat{\vartheta}}_4 = \begin{cases} Y_4\hat{\eta}_1^{\mathrm{T}}\hat{u}_4 \\ \quad \text{if } \hat{\vartheta}_4 \in int(D) \text{ or } (Y_4\hat{\eta}_1^{\mathrm{T}}\hat{u}_4)^{\mathrm{T}}\hat{\vartheta}_4^{\perp} \leq 0, \\ Pr(Y_4\hat{\eta}_1^{\mathrm{T}}\hat{u}_4) \\ \quad \text{else,} \end{cases} \tag{24}$$

where $Pr(\cdot)$ denotes the orthogonal projection onto D, ∂D the boundary of D, $int(D)$ the interior of D and $\hat{\vartheta}_4^{\perp}$ the outward vector normal to ∂D at $\hat{\vartheta}_4^{\perp}$. The definition of (24) guarantees that $\hat{\vartheta}_4^{\mathrm{T}}Y_4$ is bounded away from zero. Moreover, unlike (21), (24) is only continuous. Reasoning as in Step 2 one obtains

$$(HJI)_3 \leq 0.$$

Step 4. Define $\hat{\eta}_2 = x_4 - \dot{\hat{u}}_3\Big|_{w_1=0,\vartheta_i=\hat{\vartheta}_i} - \hat{u}_4$, and rewrite the system (17) as

$$\dot{e}_1 = e_2,$$
$$\dot{e}_2 = -B_1^{-1}\left(C(e_2 + x_{2d}) + g + K(e_1 + x_{1d} - \hat{\eta}_1 - \hat{u}_3)\right) - \ddot{x}_{1d} + p_{21}w_1,$$
$$\dot{\hat{\eta}}_1 = \hat{u}_4 + \hat{p}_{31}(e_1, e_2, \hat{\vartheta}_1, \hat{\vartheta}_2, t)w_1 - \dot{\hat{u}}_3\Big|_{w_1=0} + \dot{\hat{u}}_3\Big|_{w_1=0,\vartheta_i=\hat{\vartheta}_i} + \hat{\eta}_2,$$
$$\dot{\hat{\eta}}_2 = B_2^{-1}(K(x_1 - x_3) + \hat{u}) - \left[\frac{\mathrm{d}}{\mathrm{d}t}\left(\dot{\hat{u}}_3\Big|_{w_1=0}\right)\right]_{w_1=0} - \dot{\hat{u}}_4\Big|_{w_1=0} + \hat{p}_{41}w_1 + p_{42}w_2,$$

together with the controller (19), (21) and (22)–(24), for some functions $\hat{p}_{41}$ and p_{42}, with $\hat{u}$ to be determined. Note that $\dot{\hat{\eta}}_2$ is well-defined since the right-hand side of (21) is continuously differentiable. Choosing

$$V(e_1, e_2, \hat{\eta}_1, \hat{\eta}_2, \hat{\vartheta}_1, \hat{\vartheta}_2, \hat{\vartheta}_3, \hat{\vartheta}_4, \hat{\vartheta}_5, \hat{\vartheta}_6, t) = V_3(e_1, e_2, \eta_1, \hat{\vartheta}_1, \hat{\vartheta}_2, \hat{\vartheta}_3, \hat{\vartheta}_4, t) + \frac{1}{2}\hat{\eta}_2^{\mathrm{T}}\Delta B_2\hat{\eta}_2 + \frac{1}{2}(\hat{\vartheta}_5 - \vartheta_5)^{\mathrm{T}}(\hat{\vartheta}_5 - \vartheta_5) + \frac{1}{2}(\hat{\vartheta}_6 - \vartheta_6)^{\mathrm{T}}(\hat{\vartheta}_6 - \vartheta_6)$$

one obtains

$$(HJI) = (HJI)_3 + \hat{\eta}_2^{\mathrm{T}}(\Delta\hat{u} - Y_5(e_1, e_2, \hat{\eta}_1, t, \hat{\vartheta}_1, \hat{\vartheta}_2, \hat{\vartheta}_3, \hat{\vartheta}_4)\vartheta_5) + (\hat{\vartheta}_5 - \vartheta_5)^{\mathrm{T}}\dot{\hat{\vartheta}}_5 + (\hat{\vartheta}_6 - \vartheta_6)^{\mathrm{T}}\dot{\hat{\vartheta}}_6$$

for some parameter vector ϑ_5. Rewrite Δ as $\hat{\vartheta}_6^{\mathrm{T}}Y_4$ and let

$$\hat{u} = \frac{1}{\hat{\vartheta}_6^{\mathrm{T}}Y_4}(Y_5\hat{\vartheta}_5 - \alpha_5\hat{\eta}_2), \tag{25}$$

$$\dot{\hat{\vartheta}}_5 = -Y_5^{\mathrm{T}}\hat{\eta}_2, \tag{26}$$

$$\dot{\hat{\vartheta}}_6 = \begin{cases} Y_4\hat{\eta}_2^{\mathrm{T}}\hat{u} \\ \quad \text{if } \hat{\vartheta}_6 \in int(D) \\ \quad \text{or } (Y_4\hat{\eta}_2^{\mathrm{T}}\hat{u})^{\mathrm{T}}\hat{\vartheta}_6^{\perp} \leq 0, \\ Pr(Y_4\hat{\eta}_2^{\mathrm{T}}\hat{u}) \\ \quad \text{else,} \end{cases} \tag{27}$$

with $\alpha_5 > 0$ (see Lozano and Brogliato, 1992, for a comparison). Reasoning as in Step 3, one has

$$(HJI) \leq 0. \tag{28}$$

This proves that disturbance attenuation is achieved. Now, we will prove that for $w(t) = 0$ one has global tracking with bounded trajectories. For, from (28) we have that, if $w(t) = 0$

$$\dot{V} \leq -\mu_1 e_1^{\mathrm{T}}e_1 - \alpha_3(e_2 + \alpha_2 e_1)^{\mathrm{T}}(e_2 + \alpha_2 e_1) - \alpha_4\hat{\eta}_1^{\mathrm{T}}\hat{\eta}_1 - \alpha_5\hat{\eta}_2^{\mathrm{T}}\hat{\eta}_2 \leq 0. \tag{29}$$

From (29), it follows as usual that the trajectories of the closed-loop system are bounded. Moreover, using Barbalat's Lemma as in Theorem 1, one concludes that e_1, e_2, $\hat{\eta}_1$, $\hat{\eta}_2 \to 0$ as $t \to \infty$. That is x_1 and x_2 tend, respectively, to $x_{1d}(t)$ and $x_{2d}(t)$, while $\hat{\eta}_1 \to 0$ implies

$$x_3 \to \operatorname{diag}\left\{\frac{1}{\hat{\vartheta}_{2i}}\right\} Y_1\hat{\vartheta}_1,$$

i.e. the desired trajectory x_{3d} evaluated in the estimated parameters $\hat{\vartheta}_1$ and $\hat{\vartheta}_2$. As $\hat{\eta}_1$, $\hat{\eta}_2$, $\hat{u}_3$, $\dot{\hat{u}}_3$ and $\hat{u}_4$ are all bounded functions, x_3 and $x_4 \in \mathcal{L}_\infty$. ∎

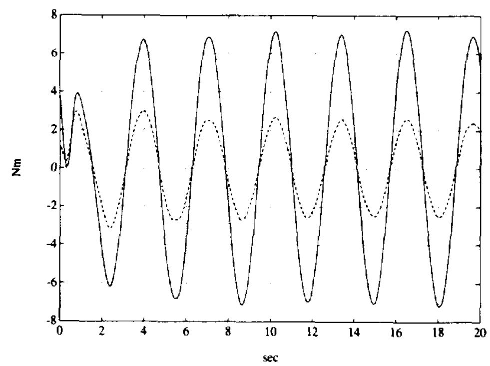

Fig. 2. Control input u_1 (–), u_2 (- -).

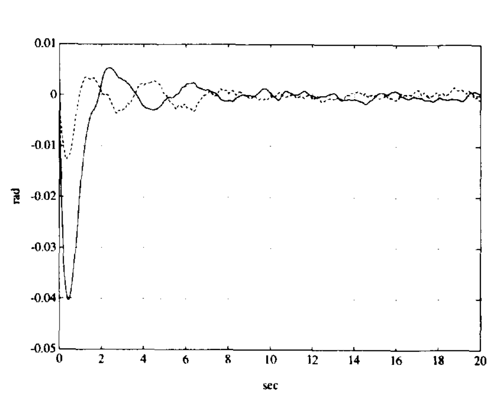

Fig. 3. Position errors e_1 (–), e_2 (- -).

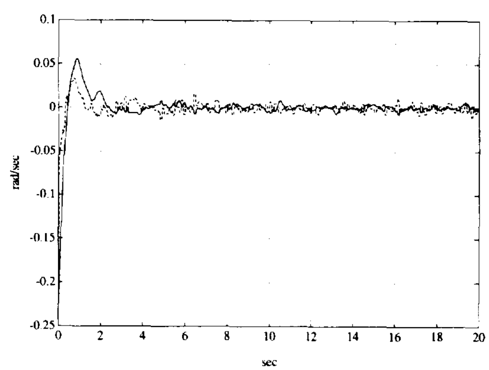

Fig. 4. Velocity errors $\dot{e}_1$ (–), $\dot{e}_2$ (- -).

5. *Simulations*

To show the behavior of the proposed control strategies, we apply our results to a two-link rigid robot arm. The robot's parameters are as in Spong (1992). The desired reference trajectory is the same for both links and is given by $q_{1d}(t) = q_{2d}(t) = 0.1\sin(2t)$. The initial condition for the robot is the origin while the dynamic controller is initialized with values equal to half the true values of the parameters. The results are obtained for $\gamma = 0.1$. The control input, relative to the disturbances shown in Fig. 1, is reported in Fig. 2. The attenuation obtained is illustrated, for both the position and velocity errors, in Figs 3 and 4.

6. *Conclusions*

In this paper, the disturbance attenuation for both the rigid and elastic joint robots is solved via state-feedback laws. We choose the tracking error as penalty variables. First, we recover a result due to Chen *et al.* (1994), giving a general procedure in which the H_∞ control for both the rigid and elastic joint robots can be treated. In Theorem 1, we assume no a priori knowledge of the robot's constant parameters, using the L.P. property. Theorem 1 is extended to the flexible joint robots case, choosing as penalty variables the link position and velocity vector (Theorem 2). Simulations are given for a two-link rigid robot, showing interesting performances. Further study will be devoted to the case of partial state feedback.

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