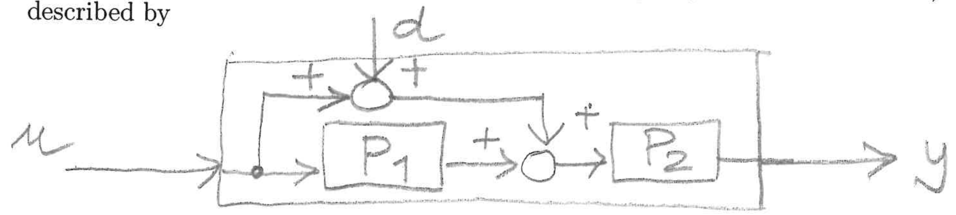
CONTROL SYSTEMS - 18/02/2022

[time 2 hours and 30 minutes; no textbooks; no programmable calculators]

Ex. # 1) Given the system **P**, with input **u**, output **y** and disturbance **d**, described by



where $\mathbf{P}_1(s) = \frac{1}{s(s+2)}$ and $\mathbf{P}_2(s) = \frac{s-2}{(s+1)^2}$, design a feedback control system around **P** such that

(i) the closed-loop system is asymptotically stable (check with Nyquist criterion) with steady-state response $\mathbf{y}_{ss}(t) = 0$ to constant disturbances $\mathbf{d}(t)$ and steady-state error $|\mathbf{e}_{ss}(t)| \leqslant 0.1$ to ramp inputs $\mathbf{v}(t) = t$,

(ii) the open-loop system has crossover frequency $\omega_t \geqslant 0.5$ rad/sec with phase margin $m_\varphi \geqslant 45°$.

Ex. # 2) Given the system **P**, with input **u**, output **y** and disturbance **d**, described by

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}, \ \mathbf{y} = C\mathbf{x} + \mathbf{d},$$
$$A = \begin{pmatrix} 0 & 2 \\ -1 & -3 \end{pmatrix}, \ B = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, C = \begin{pmatrix} -1 & -2 \end{pmatrix},$$

design a feedback control system around **P** such that the closed-loop system

(i) has zero steady state output response for constant disturbances and sinusoidal disturbances $\sin(t)$

(ii) has all poles with real part $\leqslant -0.3$.

Draw the root locus of the open loop system (use the Routh criterion to determine the intersections with the imaginary axis and discuss, also qualitatively, the existence and position of the singular points).

Ex. # 3) Given

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2 - \mathbf{x}_1$$
$$\dot{\mathbf{x}}_2 = -sin(\mathbf{x}_1) - \mathbf{x}_2 + \mathbf{x}_1 + \mathbf{u} \qquad (1)$$

discuss the existence of equilibrium points (with $\mathbf{u} = 0$) and their stability, using the linearization of (1) around each equilibrium point. For each unstable linearization of (1) find a state feedback control which stabilizes the closed-loop system.