CONTROL SYSTEMS - 22/3/2024

[time 3 hours; no textbooks or notes; no programmable calculators; all the mathematical passages must be motivated and clearly explained]

Ex. # 1) Given $\mathbf{P}(s) = \frac{1-0.2s}{s(0.1s+1)}$ design a controller $\mathbf{G}(s)$ such that

(i) the closed-loop system $\mathbf{W}(s) = \frac{\mathbf{PG}(s)}{1+\mathbf{PG}(s)}$ is asymptotically stable (use the Nyquist criterion) with steady state error $|\mathbf{e}_{ss}(t)| \leqslant 0.2$ to inputs $\mathbf{v}(t) = t$

(ii) the open-loop system $\mathbf{PG}(s)$ has crossover frequency $\omega_t^* \leqslant 2$ rad/sec and phase margin $m_\phi^* \geqslant 50°$.

Ex. # 2) Given

$$\dot{\mathbf{x}} = \begin{pmatrix} 0 & 10 \\ 10 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} \mathbf{u}, \ \mathbf{y} = \begin{pmatrix} -0.1 & 0 \end{pmatrix} \mathbf{x} + \mathbf{d},$$

design a controller $\mathbf{G}(s)$, strictly proper and with dimension $\leqslant 2$, such that the feedback system $\mathbf{W}(s) = \frac{\mathbf{PG}(s)}{1+\mathbf{PG}(s)}$ is asymptotically stable with steady state error $|\mathbf{e}_{ss}(t)| \leqslant 10$ to inputs $\mathbf{v}(t) = t$ and steady state response $|\mathbf{y}_{ss}| \leqslant 1/9$ to disturbances $\mathbf{d}(t) = \sin(\omega t)$, $\omega \in [0.01, 0.1]$ rad/sec. Draw the root locus of $\mathbf{PG}(s)$ (using the Routh criterion for determining the curves in the left- and right-half complex plane).

Ex. # 3) Given

$$\dot{\mathbf{x}} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \mathbf{u}, \mathbf{y} = \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} \qquad (1)$$

design a control law $\mathbf{u} = F\mathbf{x} + \mathbf{v}$ such that the closed-loop system is asymptotically stable with eigenvalues characterized by a damping $\zeta = 0.7$ and natural frequency $\omega_n = 1$ rad/sec. For the closed-loop system compute the forced output response $\mathbf{y}(t)$ to a step input $\mathbf{v}(t) = \delta_{-1}(t)$ and its rise time (= time required for $\mathbf{y}(t)$ to rise from 10% to 90% of the steady-state output response $\mathbf{y}_{ss}(t)$).