CONTROL SYSTEMS - 23/9/2022

[time 2 hours and 30 minutes; no textbooks; no programmable calculators]

Ex. # 1) Consider the process $\mathbf{P}(s) = \frac{1}{s-1}$. Design a controller $\mathbf{G}(s)$ with minimal dimension such that
(i) the feedback system $\frac{\mathbf{PG}(s)}{1+\mathbf{PG}(s)}$ is asymptotically stable (check with Nyquist criterion) with steady-state error response $\mathbf{e}_{ss}(t) = 0$ to ramp inputs $\mathbf{v}(t) = t$,
(ii) the open-loop system $\mathbf{PG}(s)$ has crossover frequency $\omega_t^* = 0.1$ rad/sec and phase margin $m_\phi^* \geqslant 30°$.

Ex. # 2) Given the plant $\mathbf{P}(s) = \frac{(s+a)^2}{(s^2+1)(s+1)}$:
(i) for each case $a < 0$, $a \in [0, 1)$ and $a \geqslant 1$ draw the root locus of $\mathbf{P}(s)$
(ii) for $a < 0$ determine if there exists a controller $\mathbf{G}(s) = K$ such that the feedback system $W(s) = \frac{\mathbf{PG}(s)}{1+\mathbf{PG}(s)}$ is asymptotically stable
(ii) for $a \in [0, 1)$ determine if there exists a controller $\mathbf{G}(s)$ with minimal dimension such that the feedback system $W(s) = \frac{\mathbf{PG}(s)}{1+\mathbf{PG}(s)}$ is asymptotically stable with poles in the region $\{s \in \mathbb{C} : \mathrm{Re}(s) < -1\}$
(ii) for $a \geqslant 1$ determine if there exists a controller $\mathbf{G}(s)$ with minimal dimension such that the feedback system $W(s) = \frac{\mathbf{PG}(s)}{1+\mathbf{PG}(s)}$ is asymptotically stable with poles in the region $\{s \in \mathbb{C} : \mathrm{Re}(s) < -1\}$ and steady-state error $\mathbf{e}_{ss}(t) = 0$ to constant inputs

Ex. # 3) Given the system $\dot{x} = Ax + Bu$, $y = Cx$, where

$$A = \begin{pmatrix} 0 & 1 \\ 0 & -1 \end{pmatrix}, B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, C = \begin{pmatrix} 1 & 1 \end{pmatrix}, \tag{1}$$

and $x(t, x_0, u)$ the state solution with input u and initial condition x_0 and $y(t, x_0, u)$ the corresponding output, determine
(i) calculate the reachable state from $x_0 = (-1, 1)^\top$
(ii) find, if any, a control $u(\cdot)$ and $t_f > 0$ such that $x(t_f, x_0, u) = (-2, 0.5)^\top$ with $x_0 = (0, 1)^\top$
(iii) find, if any, a control $u(\cdot)$ and $t_f > 0$ such that $y(t_f, x_0, u) = 0$ with $x_0 = (0, 1)^\top$.