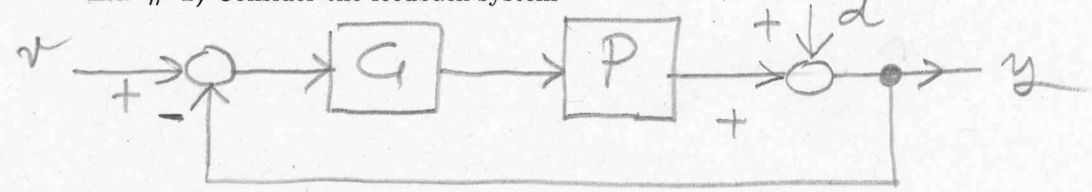
CONTROL SYSTEMS - 2/2/2021

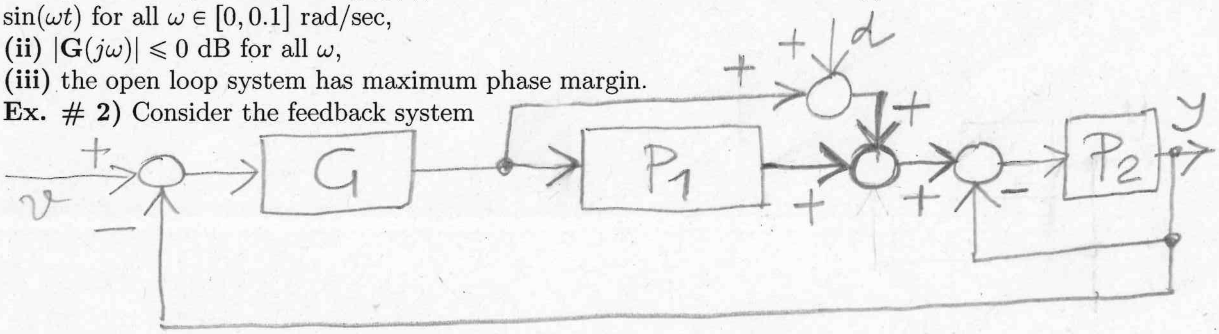
[time 2 hours and 30 minutes; no textbooks; no programmable calculators]

Ex. # 1) Consider the feedback system



with $\mathbf{P}(s) = \frac{1}{s^3}$. Design a controller $\mathbf{G}(s)$, with dimension 2, such that
(i) the closed-loop system is asymptotically stable (use Nyquist criterion) with steady state output response $|\mathbf{y}_{ss}(t)| \leqslant 0.11$ to sinusoidal disturbances $\mathbf{d}(t) = \sin(\omega t)$ for all $\omega \in [0, 0.1]$ rad/sec,
(ii) $|\mathbf{G}(j\omega)| \leqslant 0$ dB for all ω,
(iii) the open loop system has maximum phase margin.

Ex. # 2) Consider the feedback system



with $\mathbf{P}_1(s) = \frac{1}{s(s+2)}$ and $\mathbf{P}_2(s) = \frac{s-2}{s^2+s+3}$. Design a controller $\mathbf{G}(s)$, with dimension $\leqslant 2$, such that the closed-loop system is asymptotically stable with
(i) steady state error response $|\mathbf{e}_{ss}(t)| \leqslant 0.1$ to ramp inputs $\mathbf{v}(t) = t$,
(i)zero steady state output response $\mathbf{y}_{ss}(t) \equiv 0$ to both constant $\mathbf{d}(t)$ and sinusoidal $\mathbf{d}(t) = \sin(t)$.

Draw the root locus of $\mathbf{PG}(s)$ using the Routh table for an accurate study of the crossing points of the imaginary axis.

Ex. # 3) Given $\dot{\mathbf{x}} = A\mathbf{x} = \begin{pmatrix} -1 & 1 \\ 0 & -2 \end{pmatrix} \mathbf{x}$, $\mathbf{y} = C\mathbf{x} = \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x}$,
(i) determine the indistinguishable states x from the output $\mathbf{y}$,
(ii) decompose the system into observable and unobservable subsystems, and find, if possible, K such that $A - KC$ has the spectrum $\{-2, -3\}$,
(iii) for the system $\dot{\mathbf{x}} = A\mathbf{x} - K\mathbf{y}$, $\mathbf{y} = C\mathbf{x} + \mathbf{d}$, with disturbances $\mathbf{d}$ and K as in **(ii)**, compute the steady state output response $\mathbf{y}_{ss}(t)$ to constant $\mathbf{d}(t)$.