Control Systems - 3/11/2022

[time 150 minutes; no textbooks; no programmable calculators]

1) With $\mathbf{P}(s) = \frac{1}{1-10s}$ design a controller $\mathbf{G}(s)$ with minimal dimension such that we have for the feedback system $\frac{\mathbf{PG}(s)}{1+\mathbf{PG}(s)}$

   (i) asymptotic stability (use the Nyquist criterion)

   (ii) steady-state error response $\mathbf{e}_{ss}(t)$ to inputs $\mathbf{v}(t) = t$ such that $|\mathbf{e}_{ss}(t)| \leq 0.1$

   and for the open loop system $\mathbf{PG}(s)$

   (iii) crossover frequency $\omega_t^* = 10$ rad/sec and phase margin $m_\phi^* \geq 40°$.

2) Given $\mathbf{P}(s) = \frac{3}{(s-1)(s+10)}$

   i) determine a one-dimensional controller $\mathbf{G}(s)$ such that the closed-loop system $\frac{\mathbf{PG}(s)}{1+\mathbf{PG}(s)}$ has two complex conjugate poles with real part ≤ -0.5 and damping in $[0.5, 1)$

   ii) determine a minimal dimensional controller $\mathbf{G}(s)$ such that the closed-loop system $\frac{\mathbf{PG}(s)}{1+\mathbf{PG}(s)}$ has steady-state error response $\mathbf{e}_{ss}(t) \equiv 0$ to inputs $\mathbf{v}(t) = \delta^{(-1)}(t)$ and poles with real part ≤ -1. Draw the root locus of $\mathbf{PG}(s)$.

   If $\mathbf{P}(s) = \frac{3}{(s-1)(s+a)}$, $a > 0$, and $\mathbf{G}(s)$ designed as in ii), for which values of $a > 0$ the closed-loop system $\frac{\mathbf{PG}(s)}{1+\mathbf{PG}(s)}$ maintains the property of a steady-state error response $\mathbf{e}_{ss}(t) \equiv 0$ to inputs $\mathbf{v}(t) = \delta^{(-1)}(t)$ and poles with real part ≤ -1?

3) Given the system

$$\dot{\mathbf{x}}_1 = -\mathbf{x}_1 + \mathbf{x}_2$$
$$\dot{\mathbf{x}}_2 = \mathbf{u} - \mathbf{x}_1 - \mathbf{x}_2, \ \mathbf{y} = \mathbf{x}_1$$

   (i) determine the forced output response to the input $\mathbf{u}(t) = \delta^{(-1)}(t)$

   (ii) determine the steady-state output response to the input $\mathbf{u}(t)$

   (iii) give an approximate value of the 5% settling time and maximal overshooting of the forced output response.