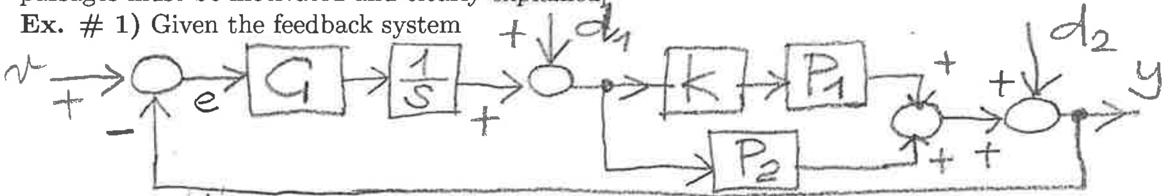
CONTROL SYSTEMS - 4/9/2023

[time 3 hours; no textbooks; no programmable calculators; all the mathematical passages must be motivated and clearly explained]

Ex. # 1) Given the feedback system

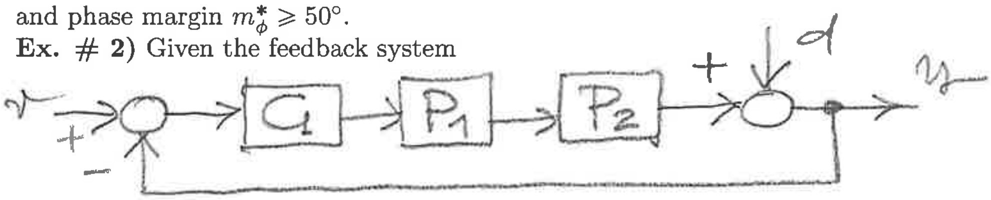


with $\mathbf{P}_1(s) = \frac{1}{s+10}$, $K = 30$, $\mathbf{P}_2(s) = -2$, design a controller $\mathbf{G}(s)$ with minimal dimension and such that

(i) the feedback system (from $\mathbf{v}$ to $\mathbf{y}$) is asymptotically stable (use Nyquist criterion for assessing stability) with steady-state error $|\mathbf{e}_{ss}(t)| \leqslant 0.2$ to inputs $\mathbf{v}(t) = t$ and steady-state output $\mathbf{y}_{ss}(t) \equiv 0$ to constant disturbances $\mathbf{d}_1(t)$ and $\mathbf{d}_2(t)$

(ii) the open-loop system (from $\mathbf{e}$ to $\mathbf{y}$) has crossover frequency $\omega_t^* = 2$ rad/sec and phase margin $m_\phi^* \geqslant 50°$.

Ex. # 2) Given the feedback system



with $\mathbf{P}_1(s) = 3$ and $\mathbf{P}_2(s) = \frac{1}{(s-1)(s+2)}$,

(i) draw accurately the root locus of $\mathbf{P}_1(s)\mathbf{P}_2(s)$, using the Routh table for determining the crossing points of the imaginary axis and finding analytically the singular points, if any

(ii) design a minimal dimensional controller $\mathbf{G}(s)$ such that the feedback system is asymptotically stable with all real poles $\leqslant -1$

(iii) for the resulting feedback system calculate the steady state output response $\mathbf{y}_{ss}(t)$ to disturbances $\mathbf{d}(t) = t$

(iv) is it possible to find a one-dimensional controller $\mathbf{G}(s)$ such that the feedback system is asymptotically stable with poles $p = -1 \pm j$? If yes, find it.

Ex. # 3) Given the feedback system

with $\mathbf{P}(s) = \frac{1}{s+1}$ find all the values $K_d, K_r \in \mathbb{R}$ such that the feedback system is asymptotically stable with steady-state output response $\mathbf{y}_{ss}(t) = -t + a$, $|a| \leqslant 0.1$, to inputs $\mathbf{v}(t) = -t + 1$.