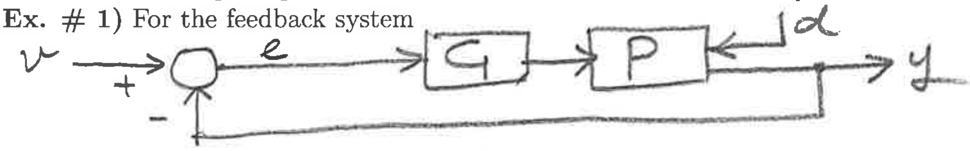
CONTROL SYSTEMS (B) - 9/1/2023

[time 2 hours and 30 minutes; no textbooks; no programmable calculators; all the mathematical passages must be motivated and clearly explained]

Ex. # 1) For the feedback system



where $\mathbf{P} : \dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u},\ \mathbf{y} = C\mathbf{x} + \mathbf{d}$ with

$$A = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix},\ B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, C = (0\ ,\ 1),$$

design a controller $\mathbf{G}(s)$ such that:

i) $\mathbf{G}(s)$ has minimal dimension and $|\mathbf{G}(j\omega)|_{dB} \leqslant 60$dB for all $\omega > 0$

ii) the feedback system is asymptotically stable (use Nyquist criterion as stability test) with steady-state error $\mathbf{y}_{ss}(t) = 0$ to disturbances $\mathbf{d}(t) = t$ and steady-state output response $|\mathbf{e}_{ss}(t)| \leqslant 0.01$ to inputs $\mathbf{v}(t) = t^2/2$.

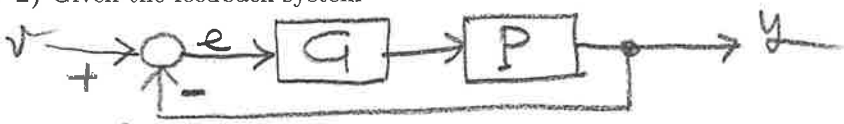
iii) the open-loop system $\mathbf{PG}(s)$ has maximal phase margin

Finally, design $\mathbf{G}(s)$ in such a way to satisfy (ii) together with

iv) $|\mathbf{PG}(j\omega)|_{dB} \geqslant 20$dB for all $\omega \in [0, 0.1]$

v) the open-loop system $\mathbf{PG}(s)$ has minimal crossover frequency.

Ex. # 2) Given the feedback system



with $\mathbf{P}(s) = \frac{3-s}{s(s+4)^2(s+2)}$

i) draw the root locus of the open loop system (use the Routh criterion to determine the intersections with the imaginary axis and discuss, also qualitatively, the existence and position of the singular points)

ii) design a minimal dimensional controller $G(s)$ such that the feedback system has all poles with real part $\leqslant -0.2$; is it possible to find a minimal dimensional controller $G(s)$ such that the feedback system has all real negative poles? If yes, motivate the answer

(iii) design a controller $G(s)$ such that the feedback system is asymptotically stable with steady-state error $\mathbf{e}_{ss}(t) = 0$ to inputs $\mathbf{v}(t) = t$; is it possible to design $G(s)$ in such a way that additionally $|\mathbf{e}_{ss}(t)| \leqslant 0.5$ to inputs $\mathbf{v}(t) = t^2/2$? If yes, motivate the answer

Ex. # 3) Given the system $S : \dot{\mathbf{x}}_1 = \mathbf{x}_1 + \mathbf{u}\ , \dot{\mathbf{x}}_2 = \mathbf{x}_2^2 + \mathbf{x}_1\mathbf{u}$:

(i) find all the constant solutions $(\mathbf{x}(t), \mathbf{u}(t)) \equiv (\mathbf{x}_e, \mathbf{u}_e)$ of S

(ii) determine which $(\mathbf{x}_e^*, \mathbf{u}_e^*)$ among these constant solutions are such that the linearization of S around $(\mathbf{x}_e^*, \mathbf{u}_e^*)$ is controllable

(iii) choose any such solution $(\mathbf{x}_e^*, \mathbf{u}_e^*)$ and compute the linearization of S around $(\mathbf{x}_e^*, \mathbf{u}_e^*)$. Let $\dot{\mathbf{z}} = A\mathbf{z} + B\mathbf{v}$ be the linearized system and find a control $\mathbf{v}(t)$ such that $\mathbf{z}(T) = (0,0)^\top$ at $T = 1$ sec with $\mathbf{z}(0) = (1,1)^\top$.