SAPIENZA Università di Roma – MSc. in Engineering in Computer Science
Formal Methods – September 5, 2022
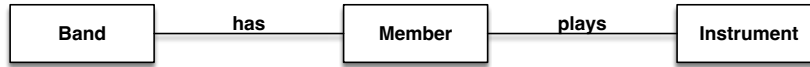*(Time to complete the test: 2 hours)*

Exercise 1. Consider the following simple UML class diagram, and express in FOL the following boolean queries (stating which ones are CQs):


1. *Return bands that have at least one member who plays guitar.*
2. *Return bands that have at exactly one member who plays guitar.*
3. *Return bands that have exactly two members who play guitar.*
4. *Return bands that have only members who play guitar.*
5. *Check if there exists a band whose members play (collectively) all instruments.*

Exercise 2. Check whether the following FOL formula is valid, by using tableaux:
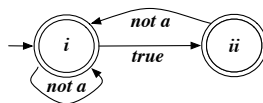
$$(\forall x.(A(x) \equiv (\exists y.B(y)))) \supset ((\forall x.A(x)) \vee (\forall x.\neg A(x)))$$

Exercise 3. Consider the following transition system:
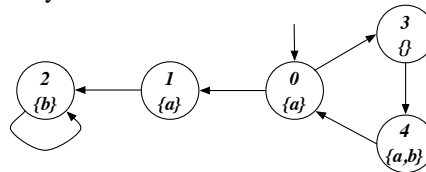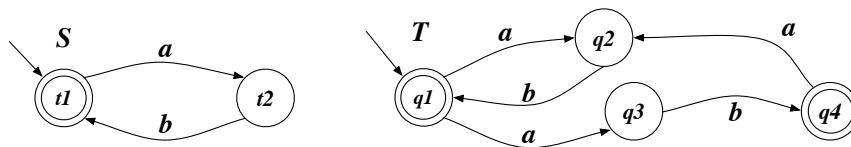

1. Model check the Mu-Calculus formula: $\nu X.\mu Y.((a \wedge \langle next \rangle X) \vee \langle next \rangle Y)$

2. Model check (by translating it in Mu-Calculus) the CTL formula: $AF(a \wedge AXa)$

Exercise 4. Consider the transition system of Exercise 3. Model check the LTL formula $\Diamond(a \wedge \bigcirc a)$, by considering that the Büchi automaton for $\neg \Diamond(a \wedge \bigcirc a)$ is the one below:

Exercise 5. Consider the following two transition systems:

Write the definition of bisimilarity and compute the bisimilarity relation for the two transition systems.

Exercise 6. Compute the weakest precondition for getting $\{x = y\}$ by executing the following program:

```
x := y + 1;
if (y < 0) then
   x := x + y
else {
    if (x > 0) then
        x := y + 100
    else
        x := -x;
}
x := x - y
```