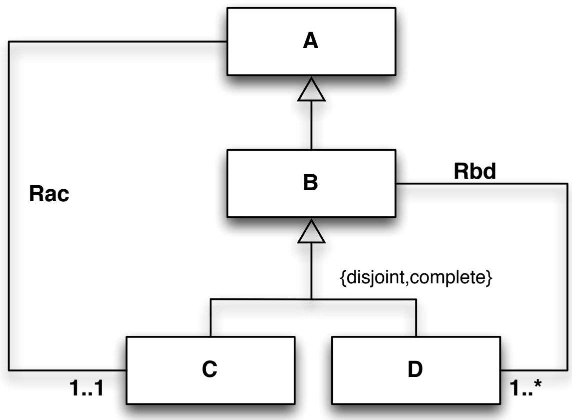
SAPIENZA Università di Roma – MSc. in Engineering in Computer Science
Formal Methods – Feb. 18, 2014
(Time to complete the test: 2 hours)

Exercise 1. Consider the following UML class diagram.

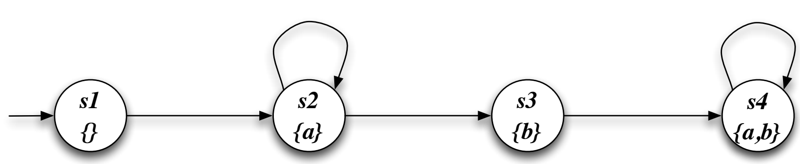


1. Express it in FOL.
2. Express it in *DL-Lite*$_\mathcal{A}$, highlighting parts that are not expressible.
3. Given the ABox $A = \{A(a)\}$ and the conjunctive query $q(x) \leftarrow Rac(x, y), Rbd(y, z), A(z)$, return the certain answer by exploiting the *DL-Lite*$_\mathcal{A}$ rewriting algorithm.

Solution

$$
\begin{array}{ll}
q(x) \leftarrow Rac(x,y), Rbd(y,z), \underline{A(z)} & B \sqsubseteq A \\
q(x) \leftarrow Rac(x,y), Rbd(y,z), \underline{B(z)} & D \sqsubseteq B \\
q(x) \leftarrow Rac(x,y), Rbd(y,z), \underline{D(z)} & \exists Rbd^- \sqsubseteq D \\
q(x) \leftarrow Rac(x,y), \underline{Rbd(y,z)}, \underline{Rbd(w,z)} & w = y, z = z \\
q(x) \leftarrow Rac(x,y), \underline{Rbd(y,z)} & B \sqsubseteq \exists Rbd \\
q(x) \leftarrow Rac(x,y), \underline{B(y)} & C \sqsubseteq B \\
q(x) \leftarrow Rac(x,y), \underline{C(y)} & \exists Rac^- \sqsubseteq C \\
q(x) \leftarrow \underline{Rac(x,y)}, \underline{Rac(v,y)} & v = x, y = y \\
q(x) \leftarrow \underline{Rac(x,y)} & A \subseteq \exists Rac \\
q(x) \leftarrow A(x) & \Longrightarrow x = a
\end{array}
$$

Exercise 2. Model check the Mu-Calculus formula $\nu X.\mu Y.(a \vee \langle next \rangle X) \wedge [next]Y$ and the CTL formula $EG(\neg a \supset AXAFa)$ (showing its translation in Mu-Calculus) against the following transition system:



Solution

We denote by $\mathcal{X}_i$ a set of states and by X_i a new proposition such that $[\![X_i]\!] = \mathcal{X}_i$. Similarly for $\mathcal{Y}_i$ and Y_i. We need to compute $[\![\nu X.\mu Y.(a \vee \langle next \rangle X) \wedge [next]Y]\!]$

$$
\begin{aligned}
&\mathcal{X}_0 = \{1,2,3,4\}\\
&\mathcal{X}_1 = [\![\mu Y.(a \vee \langle next\rangle X_0) \wedge [next]Y]\!]\\
&\qquad \mathcal{Y}_{10} = \emptyset\\
&\qquad \mathcal{Y}_{11} = [\![(a \vee \langle next\rangle X_0) \wedge [next]Y_0]\!]\\
&\qquad = \{s1, s2, s3, s4\} \cap PreA(\emptyset) = \emptyset\\
&\mathcal{X}_1 = \emptyset\\
&\mathcal{X}_2 = [\![\mu Y.(a \vee \langle next\rangle X_1) \wedge [next]Y]\!]\\
&\qquad \mathcal{Y}_{20} = \emptyset\\
&\qquad \mathcal{Y}_{21} = [\![(a \vee \langle next\rangle X_1) \wedge [next]Y_0]\!] = \{s1, s2, s3, s4\} \cap PreA(\emptyset) = \emptyset\\
&X_2 = \emptyset
\end{aligned}
$$

So the TS does not satisfy the formula, since its initial state $s1$ is not in $[\![\nu X.\mu Y.(a \vee \langle next\rangle X) \wedge [next]Y]\!]$.

Notice that the fact that we are looking for a least fixpoint of a variable (Y) in a next operator $[next]Y$ that occurs in AND with a complex expression $(\mu Y.(a \vee \langle next\rangle X_i) \wedge [next]Y$ trivializes the computation of the fixpoint to the empty set

Checking the CTL formula and translating it into mu-calculus is left as an exercise.

Exercise 3. Consider the following predicates $Employee(x)$ saying that x is an employee, $Manages(x, y)$ saying that x manages y, and $MSc(x)$ saying that x is a person with master degree. Express in FOL the following boolean queries (stating which ones are CQs):

1. *There exists an employee with master degree that manages someone with the master degree.*
2. *There exists an employee with master degree that manages at least two people with the master degree.*
3. *There exists an employee that manages someone with the master degree and someone without the master degree.*
4. *There exists an employee that manages only people with master degree.*
5. *There exists an employee that manages all the people with master degree.*

Solution

1. *There exists an employee with master degree that manages someone with the master degree*

$$\exists x.Employee(x) \wedge MSc(x) \wedge Manages(x, y) \wedge MSc(y) \quad (\textit{CQ})$$

2. *There exists an employee with master degree that manages at least two people with the master degree*

$$\exists x.Employee(x) \wedge MSc(x) \wedge Manages(x, y) \wedge MSc(y) \wedge Manages(x, z) \wedge MSc(z) \wedge y \neq z$$

3. *There exists an employee that manages someone with the master degree and someone without the master degree*

$$\exists x.Employee(x) \wedge Manages(x, y) \wedge MSc(y) \wedge Manages(x, z) \wedge \neg MSc(z)$$

4. *There exists an employee that manages only people with master degree.*

$$\exists x.Employee(x) \wedge (\forall y.Manages(x, y) \supset MSc(y))$$

5. *There exists an employee that manages all the people with master degree.*

$$\exists x.Employee(x) \wedge (\forall y.Msc(y) \supset Manages(x, y))$$

Exercise 4. Compute the certain answers to the CQ $q(x) \leftarrow Employee(x), Manages(x, y)$ over the incomplete database (naive tables):

$Employee$

$name$
Smith
$null_1$
Brown

$Manages$

mgr	mgd
Green	Smith
Smith	$null_1$
$null_1$	Brown
Brown	$null_2$

Solution

- Evaluate q over the database as it was a complete database
- Filter out all answers where null appears (certain answers are constituted by tuples of constants in $Cons$)

Answer: $\{$*Smith,Brown*$\}$

Exercise 5. Compute the weakest precondition for getting $\{x = y\}$ by executing the following program:

```
x := y + 1;
if (y > 0) then
  x := x + y
else x := y + 100;
x := x + y;
```

Solution

```
{(y > 0 & y + 1 + y = 0) | (y = -100)} = {( y > 0 & y = -0.5 | y = -100} = {false | y = -100}

x := y + 1;


{(y > 0 & x + y = 0) | (y =< 0 & y=-100)} = {(y > 0 & x + y = 0) | (y = -100)}

if (y > 0) then

      {x + y = 0}

  x := x + y

      {x=0}


      {y + 200 = 0} = {y = -100}

else x := y + 100;

      {x=0}


{x+y = y} = {x=0}

x := x + y;

{x=y}
```