Exercise 1. Consider the following UML class diagram.

1. Express it in FOL.
2. Express it in DL-Lite\(_A\), highlighting the parts that are not expressible.
3. Given the ABox \( A = \{ A(c) \} \), compute the certain answer to the query \( q(x) : -Rcd(x, y), D(y) \), using the rewriting technique for DL-Lite\(_A\).

Exercise 2. Model check the Mu-Calculus formula \( \nu X.\mu Y.((b \land \langle\text{next}\rangle X) \lor \langle\text{next}\rangle Y) \) and the CTL formula \( AG(AFa \land EFb \land EG\neg b) \) (showing its translation in Mu-Calculus) against the following transition system:

Exercise 3. Consider the following predicates: \( \text{Supplier}(x, y) \), saying that \( x \) is a supplier in city \( y \); \( \text{Item}(x, y) \), saying that item \( x \) has color \( y \); and \( Sells(x, y, z) \) saying that supplier \( x \) sells item \( y \) at price \( z \). Express in FOL the following boolean queries, stating which ones are CQs (do not use abbreviations for cardinalities):

1. There exists a supplier in NY selling a blue item.
2. There exists a supplier in NY selling at least two blue items.
3. There exists a supplier in NY selling only blue items.
4. There exists a supplier in NY selling all blue items.
5. Return the pairs of suppliers such that the first supplier sells at least one item at a cheaper price than the second one.
6. Return the pairs of suppliers such that the first supplier sells all items that the second one sells, and at a cheaper price.

Exercise 4. Compute the certain answers to the following CQs over the following incomplete database (naive tables), and discuss how you obtained the result:

\[ q(x) \leftarrow Sells(x, y), \text{Item}(y, z) \quad q(x, z) \leftarrow Sells(x, y), \text{Item}(y, z) \]

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Item</th>
<th>Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smith</td>
<td>null2</td>
<td>item1</td>
</tr>
<tr>
<td>Brown</td>
<td>null3</td>
<td>item2</td>
</tr>
<tr>
<td>Green</td>
<td>null4</td>
<td>null5</td>
</tr>
<tr>
<td>White</td>
<td>null5</td>
<td>null6</td>
</tr>
</tbody>
</table>

Exercise 5. Check the truth of the following Hoare triple, assuming as invariant: \( i \leq 64 \), explaining in details the technique used:

\[ \{i=1\} \text{ while}(i<64) \text{ do } i:= i*2 \quad \{i=64\} \]